A computational framework for sustainable geothermal energy production in a fracture-controlled reservoir based on well placement optimization

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Our topic in a nutshell

- We apply physical theories, the corresponding mathematical models and numerical methods to improve sustainable geothermal energy production.
- We model transient non-isothermal fluid flow in a fracture-dominated geothermal reservoir with wells.
- The fractured rock ≈ 3D layered porous medium containing fracture networks represented by 2D manifolds. The fluid inside ≈ water.

Our goals

- To help with sustainable and optimized geothermal energy production in complex geological settings.
- To find the optimal placements of multi-well geothermal facilities using gradient-based optimization algorithms.

Mathematical model for fluid flow in fractured rock [1,3,4]

Model in layers ($\boldsymbol{la}$):

$$\varepsilon_r \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \boldsymbol{v}) = S_{\mathrm{M}} \qquad \text{(mass balance)}$$

$$\boldsymbol{v} = -\frac{1}{\mu}\boldsymbol{k}(\nabla p - \rho \boldsymbol{g}) \qquad \text{(momentum balance)}$$

$$\varepsilon_r \frac{\partial(\rho e)}{\partial t} + (1-\varepsilon_r)\frac{\partial E_r}{\partial t} + \nabla \cdot (\rho h \boldsymbol{v} - \lambda_{\mathrm{eff}} \nabla T) = h \nabla \cdot (\rho \boldsymbol{v}) + S_{\mathrm{E}} \qquad \text{(energy balance)}$$

Model in fractures ($\boldsymbol{fr}$):

$$d_{fr}\varepsilon_{fr} \frac{\partial \rho}{\partial t} + d_{fr} \nabla_{\boldsymbol{t}} \cdot (\rho \boldsymbol{v}) = S_{\mathrm{M}} + (\rho \boldsymbol{v})_{la} \cdot \boldsymbol{n}^+ + (\rho \boldsymbol{v})_{la} \cdot \boldsymbol{n}^- \qquad \text{(mass balance)}$$

$$\boldsymbol{v} = -\frac{1}{\mu}\boldsymbol{k}(\nabla_{\boldsymbol{t}} p - \rho \boldsymbol{g}) \qquad \text{(momentum balance)}$$

$$d_{fr}\varepsilon_r \frac{\partial(\rho e)}{\partial t} + d_{fr}(1-\varepsilon_r)\frac{\partial E_r}{\partial t} + d_{fr}\nabla_{\boldsymbol{t}} \cdot (\rho h \boldsymbol{v} - \lambda_{\mathrm{eff}} \nabla_{\boldsymbol{t}} T)$$
$$= S_{\mathrm{E}} + d_{fr} h \nabla_{\boldsymbol{t}} \cdot (\rho \boldsymbol{v}) + \boldsymbol{q}_{la} \cdot \boldsymbol{n}^+ + \boldsymbol{q}_{la} \cdot \boldsymbol{n}^- \qquad \text{(energy balance)}$$

Intersections of 2 fractures:
$$\sum_{i\in\{1,2\}} \sum_{s\in\{+,-\}} (\rho \boldsymbol{v})_{fr} \boldsymbol{n}_i^s = 0 \quad \text{and} \quad \sum_{i\in\{1,2\}} \sum_{s\in\{+,-\}} \boldsymbol{q}_{fr} \boldsymbol{n}_i^s = 0$$

- The functions ρ, μ, e, h, λ depend on p, T: These definitions are based on [2].
- The fracture network is created using Frackit.

Nomenclature

ε_r	porosity $[-]$
ρ	density $[\mathrm{kg \cdot m^{-3}}]$
t	time [s]
$\boldsymbol{v}$	velocity [m/s]
S_{M}	source of mass $[\mathrm{kg \cdot m^{-3} \cdot s^{-1}}]$
μ	dynamic viscosity $[\mathrm{Pa \cdot s}]$
$\boldsymbol{k}$	permeability tensor $[\mathrm{m^2}]$
p	pressure [Pa]
$\boldsymbol{g}$	gravitational acc. vector $[\mathrm{m \cdot s^{-2}}]$
e	specific internal energy [J/kg]
E	internal energy [J/kg]
h	specific enthalpy [J/kg]
λ	thermal conductivity coefficient $[\mathrm{W/(m \cdot K)}]$
T	thermodynamic temperature [K]
S_{E}	source of energy $[\mathrm{J/(kg \cdot s)}]$
d_{fr}	aperture $[-]$
$\boldsymbol{n}$	unit outward normal $[-]$
$\boldsymbol{t}$	unit tangential vector $[-]$
$\boldsymbol{q}$	$\boldsymbol{q} = (\rho h \boldsymbol{v} - \lambda_{\mathrm{eff}} \nabla T)$
λ_{eff}	$\lambda_{\mathrm{eff}} = (1-\varepsilon_r)\lambda_r + \varepsilon_r \lambda$
$\nabla_{\boldsymbol{t}}$	$\nabla_{\boldsymbol{t}} f = \nabla f - (\nabla f \cdot \boldsymbol{n}^+)\,\boldsymbol{n}^+$

Numerical solution + Example

- Primary variables: p and T
- Discretization in space:
  - Finite element method with P_1 elements
  - Mesh generated using Gmsh
- Discretization in time:
  - 2D and 3D decoupled
  - Backward Euler + linearization
  - Balance equations decoupled at each time step

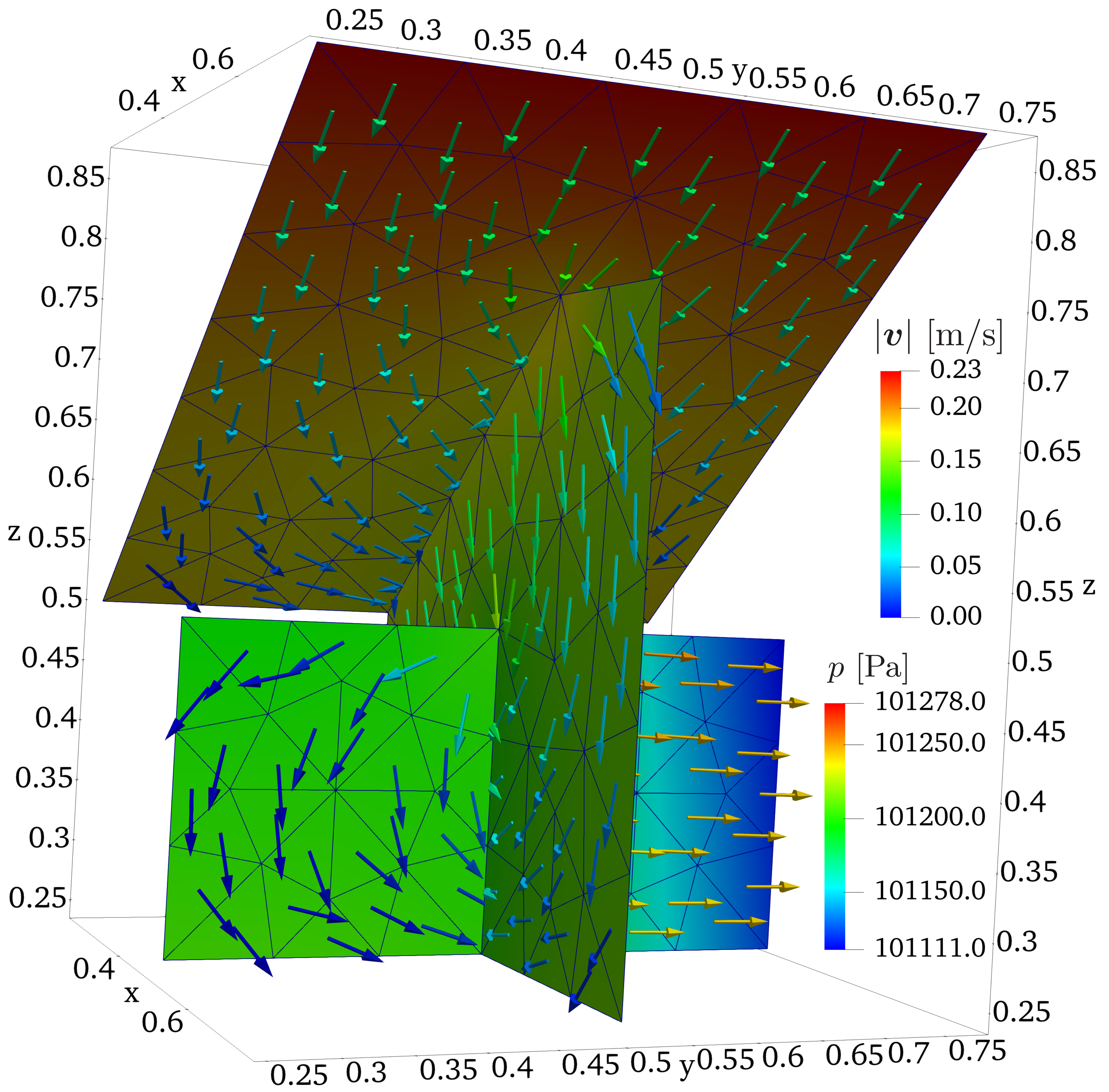
$$d_{fr}\varepsilon_r \left(\partial\rho/\partial p\right)^n \left(p^{n+1} - p^n\right)/\Delta t + d_{fr}\varepsilon_r \left(\partial\rho/\partial T\right)^n \left(T^n - T^{n-1}\right)/\Delta t$$
$$+ d_{fr}\nabla_{\boldsymbol{t}} \cdot \left(\rho^n \boldsymbol{v}^{n+1}\right) = S_{\mathrm{M}}^{n+1} + (\rho\boldsymbol{v})_{la}^n \cdot \boldsymbol{n}^+ + (\rho\boldsymbol{v})_{la}^n \cdot \boldsymbol{n}^-$$

$$d_{fr}\left(\varepsilon_r(e-h)\partial\rho/\partial p + \varepsilon_r\rho\,\partial e/\partial p + (1-\varepsilon_r)\partial E_r/\partial p\right)^n (p^{n+1} - p^n)/\Delta t$$
$$+ d_{fr}\left(\varepsilon_r(e-h)\partial\rho/\partial T + \varepsilon_r\rho\,\partial e/\partial T + (1-\varepsilon_r)\partial E_r/\partial T\right)^n (T^{n+1} - T^n)/\Delta t$$
$$+ d_{fr}\nabla_{\boldsymbol{t}} \cdot \left(\rho^n h^{n+1} \boldsymbol{v}^n - \lambda_{\mathrm{eff}}^n \nabla_{\boldsymbol{t}} T^{n+1}\right) = S_{\mathrm{E}}^{n+1}$$
$$- h^{n+1}\left(S_{\mathrm{M}}^{n+1} + (\rho\boldsymbol{v})_{la}^n \cdot \boldsymbol{n}^+ + (\rho\boldsymbol{v})_{la}^n \cdot \boldsymbol{n}^-\right) + \boldsymbol{q}_{la}^n \cdot \boldsymbol{n}^+ + \boldsymbol{q}_{la}^n \cdot \boldsymbol{n}^-$$

where

$$\boldsymbol{v}^{n+1} = -(1/\mu^n)\boldsymbol{k}\left(\nabla_{\boldsymbol{t}} p^{n+1} - \rho^{n+1}\boldsymbol{g}\right)$$
$$\rho^{n+1} = \rho^n + \Delta t\left((\partial\rho/\partial p)^n \left(p^{n+1} - p^n\right)/\Delta t + (\partial\rho/\partial T)^n \left(T^n - T^{n-1}\right)/\Delta t\right)$$
$$h^{n+1} = h^n + \Delta t\left((\partial h/\partial p)^n \left(p^{n+1} - p^n\right)/\Delta t + (\partial h/\partial T)^n \left(T^{n+1} - T^n\right)/\Delta t\right)$$

- Stabilization via algebraic flux correction (under construction)



Example: Steady-state flow field for isothermal flow of ideal gas

References

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