International Workshop

6th Singular Days on Asymptotic Methods for PDEs

Program
Abstracts
Participants

April 29 – May 1, 2010

Weierstrass Institute for Applied Analysis and Stochastics
Berlin, Germany

International Workshop

6th Singular Days on Asymptotic Methods for PDEs

WIAS, April 29 – May 1, 2010

Scientific Board

M. Costabel (Rennes) · M. Dauge (Rennes) · A. Glitzky (Berlin)
D. Knees (Berlin) · S. Nicaise (Valenciennes) · A.-M. Sändig (Stuttgart)

Organizers

Annegret Glitzky · Dorothee Knees

Local Organizing Committee

Annegret Glitzky · Hauke Hanke · Dorothee Knees · Olga Kuphal

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Weierstrass Institute for Applied Analysis and Stochastics

Scope

Partial differential equations play a fundamental role in many applications in physics and mechanics. There, PDEs usually have changing boundary conditions or nonsmooth coefficients and have to be solved on nonsmooth domains. Furthermore, small inclusions or thin layers have to be taken into account. We aim for a better understanding of the influence of these effects on the overall behavior of the solutions. This knowledge is relevant, in particular, for the construction of efficient numerical algorithms for solving elliptic PDEs. The workshop focuses on the following topics:

- regularity for elliptic PDEs in nonsmooth domains or with nonsmooth coefficients,
- asymptotic expansion methods and energetic methods in mechanics and physics (e.g. fracture mechanics, thin layers, small defects, homogenization, singular perturbations),
- computational issues in this field.

Thursday, 29.04.2010, 08:00 - 18:00

08:00 - 09:00	Registration
09:00 - 09:15	Opening
09:15	Weighted analytic regularity in corner domains: The two-dimensional case **Monique Dauge**
10:00	Analytic regularity on polyhedra **Martin Costabel**
10:30 - 11:00	Coffee Break
11:00	Stationary solutions to the Vlasov-Poisson System in Singular Geometries **Simon Labrunie**
11:30	New regularity results for nonlinear elliptic diagonal systems **Jens Frehse**
12:00	New regularity theorems for non-autonomous anisotropic variational integrals **Dominic Breit**
12:30 - 14:00	Lunch Break
14:00 - 15:30	Poster Presentation
	Finite element error estimates for boundary control problems **Thomas Apel**
	Approximate Models for Wave Propagation Across Thin Periodic Interfaces **Bérangère Delourme**
	Optimal elliptic regularity for some polygonal spatio–material constellations in three–dimensional real space **Hans–Christoph Kaiser**
	Asymptotic of capacity of a system of closely placed bodies **Aleksandr Kolpakov**
	Internal layers for transmission problems in thin shell theory: Rigid junction case **Ismail Merabet**
	Propagation of acoustic waves in junction of thin slots **Adrien Semin**
	Extracting Generalized Flux/Stress Intensity Functions along circular singular edges for the Laplace equation and the elasticity system in 3-D domains **Samuel Shannon**
15:30 - 16:00	Coffee Break
16:00	Optimal elliptic regularity near 3-dimensional, heterogeneous Neumann vertices **Joachim Rehberg**
16:30	Singular behavior of the solution of the Helmholtz equation in weighted L^p-Sobolev spaces **Serge Nicaise**
17:00	Singular behavior of the solution of the heat equation in weighted L^p-Sobolev spaces **Colette De Coster**
17:30 - 18:00	The Dirichlet problem for non-divergence parabolic equations with discontinuous in time coefficients in a wedge **Vladimir Kozlov**

Friday, 30.04.2010, 09:00 - 22:00

09:00	Discontinuous Galerkin Methods on Graded Meshes **Susanne Brenner**
09:45	Interactions between moderately close inclusions for the Laplace equation and applications in mechanics **Virginie Bonnaillie-Noël**
10:15 - 10:45	Coffee Break
10:45	Modeling and Shape Optimization for Compressible Navier-Stokes Equations - Singular Limits in Compressible Flows and Applications **Jan Sokolowski**
11:15	A mathematical point of view in Electrowetting **Claire Scheid**
11:45	Inhomogeneous and anisotropic phase-field quantities in the sharp interface limit **Christiane Kraus**
12:15 - 14:00	Lunch Break
14:00	Circular edge singularities for the Laplace equation and the elasticity system in 3-D domains **Zohar Yosibash**
14:45	Singularities at the Tip of a Crack in Anisotropic Composites **Martin Steigemann**
15:15	Modeling of nonlinear effects at the tip zones for a crack onset **Maria Specovius-Neugebauer**
15:45 - 16:15	Coffee Break
16:15	Wave-crack interaction in finite elastic bodies **Anna-Margarete Sändig**
16:45	Overlapping domain problems with cracks and rigid inclusions **Alexander Khludnev, Evgeny Rudoy**
17:15 - 17:45	A logarithmic singularity for the end of bonded joints **Dominique Leguillon**
19:00 - 22:00	Dinner

Saturday, 01.05.2010, 09:00 - 11:45

09:00	Viscous Acoustic Equations in periodically perforated chamber - A Modelling by Surface Homogenisation and Matched Asymptotic Expansions **Kersten Schmidt**
09:30	Asymptotic expansion of the solution of a transmission problem in electromagnetism with a singular interface **Victor Péron**
10:00 - 10:30	Coffee Break
10:30	From Damage to Delamination in Nonlinearly Elastic Materials **Marita Thomas**
11:00	Approximation of dynamic boundary conditions **Matthias Liero**
11:30 - 11:45	Closing

Poster

Finite element error estimates for boundary control problems

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In recent time we derived local error estimates for the finite element solution of optimal control problems with a scalar, elliptic state equation and pointwise bounds of the control variable. The underlying domain is polygonal, and our interest is in problems with corner singularities. They are treated by local mesh grading. In the presentation we focus on problems with Neumann boundary control. It appears that optimal estimates for the finite element error on the boundary are not yet available even in the case of linear elliptic boundary value problems which limits the results for the optimal control problem. This is joint work with Arnd Rösch and Johannes Pfefferer.

Lecture

Interactions between moderately close inclusions for the Laplace equation and applications in mechanics

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The presence of small inclusions or surface defects alters the solution of the Laplace equation posed in a reference domain. If the characteristic size of the perturbation is small, one can expect the solution of the problem posed on the perturbed geometry to be close to the solution of the reference shape. An asymptotic expansion with respect to that small parameter (the characteristic size of the perturbation) can then be performed.

The case of a single inclusion, centered either in the reference domain either on its boundary has been deeply studied. The techniques rely on the notion of profile, a normalized solution of the Laplace equation in the exterior domain obtained by blow-up of the perturbation. It is used in a fast variable to describe the local behavior of the solution in the perturbed domain. Convergence of the asymptotic expansion is obtained thanks to the decay of the profile at infinity.

In this talk, I will first briefly talk about the case of multiple inclusions, in particular about the intermediate cases, where the inclusions are moderately close, i.e. the distance between them is a third intermediary scale between the size of the inclusion and the size of the whole object. One can expect to have a weak interaction between the two inclusions. In a joint work with M. Dambrine, S. Tordeux and G. Vial, we have quantified this effect and provided a completed asymptotic description of the solution of Laplace equation.

In a second time, I will talk about the numerical computation of the profile. In order to have an accurate computation, we have looked for a transparent boundary condition for an exterior boundary value problem in planar linear elasticity. The goal is to bound the infinite domain by a large "box" to make numerical approximations possible (typically a large ball of radius R). The solution of the problem set in this bounded domain has to be close to the original solution; the convergence is expected as R goes to infinity. Precisely, with D. Brancherie, M. Dambrine and G. Vial, we have considered the case of a linear elastic material in the exterior of a bounded domain on the boundary of which the displacement is prescribed. In that case, cancelling the leading singular parts at infinity of the solution leads to the approximate boundary condition of Ventcell's type set on the circle of radius R. The physical parameters E and ν are such that the quantity in front of the Laplace-Beltrami operator is nonnegative: Young's modulus E is nonnegative and Poisson's coefficient ν takes values in the interval $(-1, 0.5)$. In that case, the usual variational approach for treating Ventcell's boundary condition are not at hands. With M. Dambrine, F. Hérau and G. Vial, we proved that this problem is well posed provided the truncation radius R is chosen big enough.

Lecture

New regularity theorems for non-autonomous anisotropic variational integrals

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In the calculus of variations one prominent problem is minimizing anisotropic integrals with a (p, q)-elliptic density $F : \mathbb{R}^n \supset \Omega \to \mathbb{R}^N$. Clearly minimizers are solutions of the non-uniformly elliptic system

$$\operatorname{div}\{DF(\nabla u)\} = 0$$

with non-smooth coefficients. The best known sufficient bound for regularity of solutions is $q < p\,\frac{n+2}{n}$. On the other hand, if we allow an additional x-dependence of the density we have the much weaker result $q < p\,\frac{n+1}{n}$. If one additionally imposes the local boundedness of the minimizer, then these bounds can be improved to $q < p + 2$ and $q < p + 1$. We give natural assumptions for F closing the gap between the autonomous and non-autonomous situation.

Furthermore, we discuss regularity results concerning local minimizers $u : \mathbb{R}^n \supset \Omega \to \mathbb{R}^n$ of variational integrals like

$$\int_\Omega \{F(\cdot, \epsilon(w)) - f \cdot w\}\, dx$$

defined on energy classes of solenoidal fields. For the potential F we assume a (p, q)-elliptic growth condition. In the situation without x-dependence it is known that minimizers are of class $C^{1,\alpha}$ on an open subset Ω_0 of Ω with full measure if $q < p\,\frac{n+2}{n}$ (for $n = 2$ we have $\Omega_0 = \Omega$). In this article we extend this to the case of non-autonomous integrands. Of course our result extends to weak solutions of the corresponding nonlinear Stokes type system

$$\operatorname{div}\{D_\epsilon F(\cdot, \epsilon(u))\} = \nabla\pi - f,$$

where $\pi : \Omega \to \mathbb{R}$ is the unknown pressure function.

Lecture

Discontinuous Galerkin Methods on Graded Meshes

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It is well-known that optimal order convergence in the presence of corner singularities can be achieved for conforming finite element methods if properly graded meshes are used. In this talk we will discuss the extension of this approach to discontinuous Galerkin methods and also the convergence results of multigrid algorithms for the resulting discrete problems.

Lecture

Analytic regularity on polyhedra

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Solutions of linear elliptic boundary value problems with analytic data on polyhedra are analytic away from the corners and edges and are globally in some spaces of weighted analytic functions that have been defined by Babuska and Guo in 1993. This result is needed if one wants to show exponential convergence of the hp version of the finite element method. The result is also part of folklore, but a proof has only now been given.

In the talk I will focus on the following points:

- Definition of the right families spaces of analytic functions with homogeneous and inhomogeneous norms
- Anisotropic weighted regularity near edges (proof using nested open sets)
- The elements of the full proof for the polyhedral case by a combination of dyadic partition techniques as in the 2D case, results for smooth domains, and results for neighborhoods of edges

Coauthors: Monique Dauge, Serge Nicaise

References

[1] M. Costabel, M. Dauge, S. Nicaise *Analytic regularity for linear elliptic systems in polygons and polyhedra.* HAL: hal-00454133 ; arXiv : 1002.1772.

Lecture

Weighted analytic regularity in corner domains: The two-dimensional case

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We present a new and systematic proof of a result already proved by Babuska and Guo in papers published between 1988 and 1993. Our proof consists of three main steps:

1) We revisit the proof of analytic regularity of solutions of elliptic boundary problems, providing a family of finite estimates with analytic control on the length of derivatives.
2) We use a dyadic partition of the polygonal domain near each of its corners: The estimates obtained in step 1 can thus be transported to each neighborhood and become estimates in weighted spaces with homogeneous norms of Kondrat'ev type (so-called K-spaces).
3) The previous step allows to conclude in the Dirichlet case, or more generally when the variational space can be optimally embedded in a space with homogeneous norm (namely if u/r is square integrable - with r the distance to corners). In the Neumann case, u/r is not square integrable in general. Nevertheless, we have developed a refinement of the dyadic partition technique in order to obtain analytic estimates in weighted spaces with non-homogeneous norms (so-called J-spaces): The trick is to start from a priori estimates in the smooth case between semi-norms, instead of full norms. These results can be successfully extended to three-dimensional polyhedra (see M. Costabel's talk).

From the joint work "Analytic Regularity for Linear Elliptic Systems in Polygons and Polyhedra" by Martin Costabel, Monique Dauge, Serge Nicaise. arXiv:1002.1772

Lecture

Singular behavior of the solution of the heat equation in weighted L^p-Sobolev spaces

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We consider the heat equation on a polygonal domain Ω of the plane in weighted L^p-Sobolev spaces

$$\begin{aligned} \partial_t u - \Delta u &= h(x,t), && \text{in } \Omega \times]0, 2\pi[, \\ u &= 0, && \text{on } \partial\Omega \times [0, 2\pi], \\ u(\cdot, 0) &= u(\cdot, 2\pi), && \text{in } \Omega. \end{aligned} \tag{1}$$

Here h belongs to $L^p(0, 2\pi; L^p_\mu(\Omega)) = \{v \in L^p_{loc}([0, 2\pi]\times\Omega) : r^\mu v \in L^p([0, 2\pi]\times\Omega)\}$, with a real parameter μ and $r(x)$ the distance from x to the set of corners of Ω.

We give sufficient conditions on μ, p and Ω that guarantee that problem (1) has a unique solution $u \in L^p(0, 2\pi; L^p_\mu(\Omega))$ that admits a decomposition into a regular part in weighted L^p-Sobolev spaces and an explicit singular part.

The classical Fourier transform techniques do not allow to handle such a general case. Hence we use the theory of sums of operators.

Poster

Approximate Models for Wave Propagation Across Thin Periodic Interfaces

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This work is dedicated to the study of asymptotic models associated with electromagnetic waves scattering from a complex periodic ring. This structure is made of a dielectric ring which contains two layers of wires winding around it. We are interested in situations where the thickness of the ring and the distance between two consecutive wires are very small compared to the wavelength of the incident wave and the diameter of the ring. One easily understands that in those cases, numerical computation of the solution would become prohibitive as the small scale (denoted by δ) goes to 0, since the used mesh needs to accurately follow the geometry of the heterogeneities. In order to overcome this difficulty, we shall derive approximate models where the periodic ring is replaced by effective transmission conditions. The numerical discretization of approximate problems is expected to be much less expensive than the exact one, since the used mesh has no longer to be constrained by the small scale. From a technical point of view, these approximate models are derived from the asymptotic expansion of the solution with respect to the small parameter δ.

Lecture

New regularity results for nonlinear elliptic diagonal systems

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We consider Bellman systems to stochastic differential games with respect to Stackelberg equilibriums. In this setting, in contrast to Nash equilibrium, there is a hierarchical order of players where certain players are aware of the strategy of certain other competitors. This leads to Bellman systems

$$-\Delta u_\nu + \lambda u_\nu = H_\nu(x.u, Du)$$

where the right hand side may grow quadratically and carries new difficulties compared to Nash games since the known regularity theory for elliptic nonlinear systems is not applicable.
The systems in consideration satisfy certain positivity conditions for the Hamiltonians H_ν. They are sufficient to guarantee the existence of C^α-regularity and $W^{2,p}$-solutions.

Poster

Optimal elliptic regularity for some polygonal spatio–material constellations in three–dimensional real space

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We present optimal regularity of elliptic div–grad operators

$$-\nabla\cdot\mu\nabla : \ W^{1,p}_{\Gamma}(\Omega) \to W^{-1,p}_{\Gamma}(\Omega)$$

for several spatio–material model constellations in three–dimensional real space. The elliptic coefficient function μ on a polyhedron Ω takes its values in the set of real, symmetric, positive definite 3×3 matrices and $W^{1,p}_{\Gamma}(\Omega)$ is the Sobolev space with a homogeneous Dirichlet condition on $\partial\Omega \setminus \Gamma$. The following phenomena — alone or in combination — can cause singularities in the solution of the elliptic equation:

- Edges and vertices of the spatial domain.
- Edges where Dirichlet boundary conditions meet Neumann boundary conditions.
- Material interfaces.

We investigate for instance the interplay of changing boundary conditions with material heterogeneities, multimaterial edges, or what happens when a geometric edge on the Neumann boundary meets a material interface.

Lecture

Overlapping domain problems with cracks and rigid inclusions

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We consider boundary value problems in non-smooth domains describing cracks and rigid inclusions. Non-penetration conditions of inequality type are imposed at the crack (inclusion) faces which lead to nonlinear problem formulations. Solution existence is proved, and qualitative properties of solutions are analyzed.

Poster

Asymptotic of capacity of a system of closely placed bodies

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The problem of capacity of many bodies arose simultaneously with the notion of capacity and it is presented in many textbooks on electrostatics. Solution of the problem strongly depends on the shapes, positions and number of the bodies. In recent decades, a significant progress in the analysis of electrostatic problems and computation of capacity of systems of bodies was related to the progress in numerical methods and the increasing power of computers. Nevertheless, for disordered three-dimensional systems of many bodies, the electrostatics problem cannot be solved with standard computational software even by using a powerful modern computer. Additional problems arise when the bodies are placed closely one to other. At the same time, the closeness can be used to apply asymptotic method to the analysis of the problem. Analyzing the problem of capacity of many closely placed bodies, the authors found that it is closely related to the effect described by the Soviet physicist, Nobel Prize Laureate I.E. Tamm in 1927. We call this effect Tamm shielding effect. We found that Tamm shielding effect is the keystone phenomenon for transport processes in structures with piecewise material characteristics, i.e., for systems of bodies, particle filled composites, powders, etc. We found that Tamm shielding effect is not a universal one, but takes place only under condition that capacity of neighbored bodies computed in whole R^n tends to infinity when the distance between the bodies tends to zero. This condition is valid for bodies with smooth boundary and may be not satisfied for bodies with cone-like boundary. This research is supported by a Marie Curie IIF within the 7th EC FP.

A.A.Kolpakov and A.G.Kolpakov *Capacity and Transport in Contrast Composite Structures: Asymptotic Analysis and Applications.* Taylor&Francis / CRC Press, Boca Raton, FL, 2010.

Lecture

The Dirichlet problem for non-divergence parabolic equations with discontinuous in time coefficients in a wedge

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The Dirichlet problem for non-divergence parabolic equations with discontinuous in t coefficients in a wedge is considered. Coercive estimates in weighted anisotropic Sobolev spaces are proved. One cannot freeze coefficients in time in such problems and the choice of the weights in this situation will be discussed. This is a joint work with Alexander Nazarov of St Petersburg State University.

Lecture

Inhomogeneous and anisotropic phase-field quantities in the sharp interface limit

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Phase transition phenomena are usually described by two types of models, i. e. *sharp interface* and *diffuse phase field* models.

In a sharp interface model, interfaces, separating coexisting phases or structural domains, are modeled as hypersurfaces at which certain quantities fulfill jump conditions. Interfacial energy can be accounted for by integrating a surface energy density over the hypersurface. In general, the surface energy density will be inhomogeneous and anisotropic, i. e. the density will depend on the position in space and on the local orientation of the interface.

In a diffuse interface model, interfacial energy is modeled within the context of the van der Waals–Cahn–Hilliard theory of phase transitions. The diffuse phase-field model describes the interface between different phases as a thin transition region, where the order parameter, representing the phases, changes its state smoothly.

In this talk we consider situations in which the energy of the system consists of an inhomogeneous anisotropic interfacial energy and an elastic energy resulting from stresses caused by different elastic properties of the phases. We study the sharp interface limit for an inhomogeneous anisotropic interfacial energy supplemented by elastic energy contributions in a situation where we prescribe the total mass. We derive an anisotropic, inhomogeneous and elastically modified Gibbs–Thomson law as the singular limit of the phase field model.

This is a joint work with Harald Garcke, University of Regensburg.

Lecture

Stationary solutions to the Vlasov-Poisson System in Singular Geometries

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We consider the Vlasov-Poisson system which governs the motion of charged particles under the influence of an electrostatic field. The particles are described by their distribution function $f(t, \boldsymbol{x}, \boldsymbol{v})$, where $\boldsymbol{x} \in \Omega \subset \mathbb{R}^d$ is the position variable and $\boldsymbol{v} \in \mathbb{R}^d$ is the velocity variable. The Vlasov equation reads:

$$\partial_t f + \boldsymbol{v} \cdot \nabla_{\boldsymbol{x}} f + \boldsymbol{E} \cdot \nabla_{\boldsymbol{v}} f = 0 \quad \text{in } (0,T) \times \Omega \times \mathbb{R}^d, \tag{1}$$

where the field $\boldsymbol{E}(t, \boldsymbol{x})$ is given by the equations in $(0,T) \times \Omega$:

$$\boldsymbol{E} = -\nabla_{\boldsymbol{x}}(\phi[f] - \phi_e), \quad -\Delta\phi[f] = \int f(t, \boldsymbol{x}, \boldsymbol{v})\, d\boldsymbol{v} := \rho[f], \quad -\Delta\phi_e = \rho_e. \tag{2}$$

Above, ϕ_e is an external potential and ρ_e is the density of a neutralising background (fixed charges).

So far, the existence and uniqueness of solutions to the system (1)–(2) in an arbitrary domain is an open problem. In this work, we examine the equilibria ($\partial_t f = 0$) of the system. They satisfy:

$$f(\boldsymbol{x}, \boldsymbol{v}) = \gamma(\tfrac{1}{2}\, |\boldsymbol{v}|^2 + \phi[f](\boldsymbol{x}) - \phi_e(\boldsymbol{x}) - \beta) \quad \text{and} \quad \iint f(t, \boldsymbol{x}, \boldsymbol{v})\, d\boldsymbol{x}\, d\boldsymbol{v} = M, \tag{3}$$

where γ is an arbitrary function, β is a constant to be determined so that the second condition (the mass constraint) holds. The existence and uniqueness of the solution to (3) in a general bounded domain follows from an optimisation argument. Nevertheless, this problem is equivalent to a nonlinear elliptic problem, called the *Boltzmann problem*:

$$-\Delta\phi = G(\phi - \phi_e - \beta) := \rho, \quad \text{with} \quad \int \rho(\boldsymbol{x})\, d\boldsymbol{x} = M,$$

where the function $G(s) := C_d \int_0^\infty \gamma(s + r)\, dr$.

In the case where the domain Ω is a polygon with (at least) a re-entrant corner, we study the regularity properties of the solution, examine its behaviour near the re-entrant corner(s), and establish various asymptotic estimates.

Lecture

A logarithmic singularity for the end of bonded joints

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The joints (adhesive bonding, brazing, welding) used to assemble structures are generally very thin and thus lead to strongly refined meshes in computational mechanics. The matched asymptotics approach provides a method to consider separately two scales: the structure where the joint is “forgotten” (it is invisible at this scale) and the joint itself. Within the linear elastic framework, matching these two scales makes possible to single out two corrections to the simplified macroscopic model, that take into account the boundary effects in case of butt joints. The first one (a second order phenomenon) corresponds to a point force acting at the point (at the macro scale) where the joint breaks the free surface, the second one (third order) is a kind of pinching at the same point. Both rely on the Poisson effect, under tension, the joint tends to shrink and acts on the surrounding substrates. The shrinkage perpendicular to the edge gives rise to the first correction and parallel to the edge to the second one. Results are illustrated on two examples and their consequences in term of fracture: the brazing of SiC structures embarked on satellites, the flexion of layered sediments in the soil.

Lecture

Approximation of dynamic boundary conditions

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In this talk we discuss the approximation of dynamic boundary conditions for the Allen–Cahn equation. We show how different dynamics on the boundary can be obtained by solving a system of parabolic equations in the bulk and in a boundary layer whose thickness tends to zero. Different scalings of the coefficients give rise to the different dynamics on the boundary.

Poster

Internal layers for transmission problems in thin shell theory: Rigid junction case

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In this note we study two-dimensional transmission problems for the linear Koiter's model of a multi-structure made of two thin shells. Those problems are depending on a small parameter $\varepsilon \ll 1$. The formal limit problem fails to give a solution satisfying all boundary and transmission conditions; it gives only the outer solution. Both in the case of regular or singular loadings, we derive a limit problem which allows us to determine the inner expansion explicitly.

Lecture

Singular behavior of the solution of the Helmholtz equation in weighted L^p-Sobolev spaces

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We study the Helmholtz equation

$$-\Delta u + zu = g \text{ in } \Omega \tag{1}$$

with Dirichlet boundary conditions in a polygonal domain Ω of the plane, where z is a complex number such that $\Re z \geq 0$. Here g belongs to

$$L^p_\mu(\Omega) = \{v \in L^p_{loc}(\Omega) : r^\mu v \in L^p(\Omega)\},$$

with a real parameter μ and $r(x)$ the distance from x to the set of corners of Ω. We give sufficient conditions on μ, p and Ω that guarantee that problem (1) has a unique solution $u \in H^1_0(\Omega)$ that admits a decomposition into a regular part in weighted L^p-Sobolev spaces and an explicit singular part. We further obtain some estimates where the explicit dependence on $|z|$ is given. An application of these results to the heat equation will be given in the lecture of C. De Coster.

Lecture

Asymptotic expansion of the solution of a transmission problem in electromagnetism with a singular interface

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We investigate a scalar interface problem originating from a skin effect model in electromagnetism. The problem is posed in a domain made of two materials, dielectric and highly conducting. We focus on the case of a corner interface between the two subdomains. In this context, we present a multi-scale expansion for the solution of the transmission problem at high conductivity. For a smooth interface, boundary layers are present close to the surface of the conductor. For a corner interface, singularities appear in the dielectric part, and generate corner layers in the conducting part. Numerical simulations will be presented. This work is a collaboration with M. Dauge and C. Poignard.

Lecture

Optimal elliptic regularity near 3-dimensional, heterogeneous Neumann vertices

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We investigate the regularity of solutions v to the elliptic equation $-\nabla \cdot \mu \nabla v = f$, including Neumann conditions, on a 3-dimensional, polyhedral domain Π. The main result states, that, under suitable assumptions on the right hand side and on the coefficient function μ, for every vertex of Π there is a neighbourhood $\mathcal{U}$ such that $\nabla v \in L^p(\Pi \cap \mathcal{U})$ for a $p > 3$.

Lecture

Wave-crack interaction in finite elastic bodies

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This paper continues studies in [4], [3] on crack propagation in a bounded linear elastic body under the influence of incident waves. There are considered shear waves, whereas we discuss the influence of plane elastic waves to a running crack in this paper. Actually, the time dependent problem is formulated in a two-dimensional current cracked configuration by a system of linear elasto-dynamic equations. In order to describe the behaviour of the elastic fields near the straight crack tip, we transform these equations to a reference configuration and derive the dynamic stress singularities. Furthermore, we assume that an energy balance law is valid, compare [2],[1]. Exploiting the knowledge on the singular behaviour of the crack fields, we derive from the energy balance law a dynamic energy release rate. Comparing this energy release rate with an experimentally given fracture toughness we get an ordinary differential equation for the crack tip motion. We present first numerical simulations for a mode-I crack propagation.

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Lecture

A mathematical point of view in Electrowetting

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Electrowetting is a technique allowing to control the spreading of a conducting liquid on a substrate with an applied electric field. Numerous industrial applications of this principle can be found: liquid lenses, lab on a chip ...

In this talk we would like to focus on a mathematical study of this phenomenon. Some observations didn't find any clear explanations yet and mathematical modelling raises interesting problems. This especially leads to solve a partial differential equation on a non-smooth domain. We consider a shape optimization model based on energy minimization. Working on both theoretical and numerical aspects, we try to improve the understanding of Electrowetting. We mainly focus on what happens near the triple line (liquid-solid-gas interface) where, due to the geometry of the domain considered, the electrostatic field is singular.

Lecture

Viscous Acoustic Equations in periodically perforated chamber - A Modelling by Surface Homogenisation and Matched Asymptotic Expansions

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Viscous fluids exhibit a boundary layer close to rigid walls whose thickness depends on the variation in time. We regard acoustic waves transported in a viscous fluid in perforated chamber. The size and the distance of the holes and the thickness of the boundary layer in comparison to the wave-length motivates a two-scale asymptotic expansion. By matched asymptotic expansion and surface homogenisation the near field solution is given by cell problems around the wall and the far field solution by problems on the limit domain.

Poster

Propagation of acoustic waves in junction of thin slots

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On this poster, we present the theory of matched asymptotics applied to the propagation of a time domain acoustic wave in a junction of thin slots. This allows us to propose improved Kirchoff conditions for the 1D limit problem. These conditions are analyzed and validated numerically.

Poster

Extracting Generalized Flux/Stress Intensity Functions along circular singular edges for the Laplace equation and the elasticity system in 3-D domains

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We herein follow the methods presented in [1] to explicitly determine the solution to the Laplace equation and the elasticity system in the vicinity of a circular singular edge in a general 3-D domain. It is shown that these are given in the form of an asymptotic series involving primal functions and two levels of shadow functions as follows:

$$\tau = \sum_{\ell=0}^{\infty} \sum_{k=0} \partial_\theta^\ell A_k(\theta) \rho^{\alpha_k} \sum_{i=0} \left(\frac{\rho}{R}\right)^{i+\ell} \phi_{\ell,k,i}(\varphi)$$

where R is the distance of the singular point from the center of the edge, ρ and φ are "polar" coordinates from the edge, and θ is the position along the edge. For the elasticity system, a similar series is obtained:

$$\begin{pmatrix} u_\rho \\ u_\varphi \\ u_\theta \end{pmatrix} = \sum_{\ell=0} \sum_{k=0} \partial_\theta^\ell A_k(\theta) \rho^{\alpha_k} \sum_{i=0} \left(\frac{\rho}{R}\right)^{i+\ell} \begin{pmatrix} \phi_\rho(\varphi) \\ \phi_\varphi(\varphi) \\ \phi_\theta(\varphi) \end{pmatrix}_{\ell,k,i}$$

Explicit expressions for the primal and shadow eigen-pairs are provided in case of a penny-shaped crack for an axisymmetric and a non axisymmetric situation. This explicit solution is then exploited, in conjunction with a variation of the quasi-dual-function method [1, 2] to extract the series coefficients $A_k(\theta)$, known as the generalized flux/stress intensity functions. Numerical results will be provided for various example problems.

Coauthors: Zohar Yosibash, Monique Dauge, Martin Costabel

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Lecture

Modeling and Shape Optimization for Compressible Navier-Stokes Equations - Singular Limits in Compressible Flows and Applications

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Modeling and shape optimization problems for a class of compressible flows are considered in [1]-[8]. The approximate solutions for the model in non stationary case can be obtained by parabolic regularization of the governing equations proposed by P.L. Lions and E. Feireisel. Such a regularization includes a small parameter $\varepsilon > 0$,

$$\partial_t(\varrho\,\mathbf{u}) + \operatorname{div}\Big((\varrho\,\mathbf{u} - \varepsilon\nabla\varrho)\otimes\mathbf{u}\Big) + \nabla p + \mathbb{C}\,\mathbf{u} = \operatorname{div}\mathbb{S}(\mathbf{u}) + \varrho\,\mathbf{f} + \mathbf{h},$$

$$\partial_t\varrho + \operatorname{div}(\varrho\mathbf{u}) = \varepsilon\Delta\varrho.$$

The equations are supplemented with the nonhomegeneous boundary and initial conditions in a bounded domain, and the passage to the limit $\varepsilon \to 0$ is performed to obtain the existence of weak solutions to the model.
The modeling of compressible flows in bounded domains with nonhomogeneous boundary conditions seems to be appropriate for numerical analysis. The well posedness of such a model in stationary cases, as well as of the associated shape optimization problem of drag minimization is established in [1]-[8]. In the non stationary case with nonhomogeneous boundary conditions in bounded domains, the limit passage for $\varepsilon \to 0$ is performed [9]. Shape differentiability of the drag or of the work functionals can be established by a new singular limit procedure. Some numerical results for drag minimization are also included.

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Lecture

Modeling of nonlinear effects at the tip zones for a crack onset

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Consider a plane anisotropic homogeneous elasticity problem in a domain with an interior crack. Here a mathematical frame is developed where nonlinear effects in the tip zones like crack kinking or plastic zones can be modeled in an enlarged state space with the help of additional conditions at the crack tips. Using generalized Green's formulae the solutions to these problems turn out to minimize energy functionals which contain terms additional to the classical elastic energy and work of external forces. They can be interpreted as performed work and energy stored in the crack tips.

Lecture

Singularities at the Tip of a Crack in Anisotropic Composites

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One of the problems in fracture mechanics is the prediction of quasi-static crack propagation in anisotropic brittle structures, composed of different materials. This can be done by energy criteria for plane problems. According to GRIFFITH, a crack is growing in such a way that the total energy always is minimal. If the crack approaches a material interface, different scenarios of crack growth are possible, depending on the toughness of the interface and the elastic properties of the composite materials itself. Problems of this kind were investigated by many authors for structures composed of linear elastic, isotropic materials, e.g. HE and HUTCHINSON [HH89]. For modeling quasi-static crack growth using the energy principle (see e.g. [AN02, Ste09]) the asymptotic expansion of the displacement field at the crack tip is essential. In general, the structure of this expansion reads as follows [NS94a, NS94b]:

$$u \sim \sum_{\nu=1}^{N} r^{\Lambda_\nu} \sum_{j=1}^{J_\nu} \sum_{k=0}^{\kappa_{j,\nu}-1} K_{\nu,j,k} \sum_{q=0}^{k} \frac{1}{q!} \big(\ln(r)\big)^q \Phi^{\nu,j,k-q}(\varphi) + \ldots, \qquad r \to 0. \tag{1}$$

Here, N is some integer, the number J_ν is called the multiplicity of the eigenvalue Λ_ν and $\kappa_{j,\nu}$ is the partial multiplicity of Λ_ν. The set $\{\Phi^{\nu,j,0}, \ldots, \Phi^{\nu,j,\kappa_{j,\nu}-1} : j = 1, \ldots, J_\nu\}$ is a system of JORDAN chains corresponding to Λ_ν, (r, φ) denote polar coordinates centered at the crack tip. In the case of homogeneous materials, (1) simplifies to the well-known form

$$u \sim r^{1/2} \Big(K_1 \Phi^1(\varphi) + K_2 \Phi^2(\varphi) \Big) + \ldots, \qquad r \to 0.$$

The coefficients K_j are the stress intensity factors and in this case, the angular parts Φ^j are known explicitly for isotropic, and some classes of anisotropic materials [SNS08].

Also in the case of a solid composed of two different isotropic materials with the crack tip at the material interface, (1) does not contain logarithmic terms. The eigenvalue $\Lambda \in (0, 1)$ can be found (numerically) as a root of a transcendental equation (see e.g. [Gup73]). In contrast to this results, the singular behavior of the displacement field is not known for general anisotropic composite solids and depends on the mismatch of the elastic properties of the different materials as well as on the geometry of the interface itself.

This contribution presents ideas how the asymptotic expansion of the displacement field can be calculated, if the crack tip contacts a material interface between anisotropic, linear elastic solids.

To get a deeper understanding of the asymptotic structure of the displacement field at the crack tip, a small parameter δ is introduced in the material properties as follows: If A^0 denotes the HOOKE tensor of the material on one side of the interface, the HOOKE tensor on the other side of the interface is assumed to be $A^\delta := A^0 + \delta B$. For small δ, this is only a perturbation of the elastic properties and the asymptotic behavior of the displacement field should not differ a lot from the homogeneous case. From this consideration, an ansatz for the asymptotic terms of the following type is natural:

$$r^{\Lambda_j}\Phi^j(\varphi) = r^{1/2+\alpha_j\delta+\dots}\Big(\Phi^{j,0}(\varphi) + \delta\Phi^{j,1}(\varphi) + \dots\Big), \qquad j = 1, 2.$$

A method to calculate the values α_j and the angular parts for small parameters δ is shown. It turns out, that in general there can be two eigenvalues $\Lambda_1 \neq \Lambda_2$ with multiplicity one and if there exists only a single eigenvalue Λ, there can be logarithmic terms in the asymptotic decomposition (1). Finally, some effects of this results on the modeling of quasi-static crack propagation near material interfaces are shown.

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Lecture

From Damage to Delamination in Nonlinearly Elastic Materials

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Brittle Griffith-type delamination of compounds is deduced by means of Γ-convergence from the partial, isotropic damage of sandwich-structures consisting of three constituents by flattening the middle component to thickness 0. Both processes are assumed to be rate-independent and they are treated in their so-called energetic formulation. This approach relies on a stability condition and an energy balance for an energy functional and a dissipation potential.

The limit passage is performed via a double limit: First, we gain a delamination model involving the gradient of the delamination variable, which is essential to overcome the lack of uniform coercivity arising from the passage from partial damage to (complete) delamination. In a second limit the delamination-gradient is suppressed. Both limit models contain noninterpenetration and transmission conditions along the interface.

This is a joint work with A. Mielke and T. Roubíček.

Lecture

Circular edge singularities for the Laplace equation and the elasticity system in 3-D domains

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The solution to the Laplace operator in the vicinity of a circular singular edge in a three-dimensional domain is derived and provided in an explicit form. It is an asymptotic solution represented by a family of eigen-functions with their shadows, and the associated edge flux intensity functions (EFIFs), which are functions along the circular edge. We provide explicitly the solution for a penny-shaped crack for an axisymmetric case as well as a case in which the domain or loading is non-axisymmetric. The mathematical machinery developed in the framework of the Laplace operator is extended to derive the asymptotic solution (three displacements vector) for the elasticity system in the vicinity of a circular edge in a three-dimensional domain. As a particular case we present explicitly the series expansion for a traction free or clamped penny-shaped crack in an axisymmetric or a non-axisymmetric situation.

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