Variational models of phase-change with Eulerian interfacial energies

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The description of solid-solid phase transitions classically requires to model interfacial energy contributions between different phases. Such contributions are often consisting of measures of the interface surfaces, possibly modulated as a function of specific orientations of the stresses. Moving from the evidence that measures and orientations vary from the reference to the actual configuration via the deformation map, different theories aiming at considering interfaces in Eulerian variables have been set forth. Among these, the proposition by M. Šilhavý [1, 2] allows, at least in part, a satisfactory mathematical treatment. I will report on this theory and present existence results for equilibrium, both in the two-phase and in the multiphase case. Eventually, I will present a phase-field approach and show its convergence to a sharp-interface limit as the relaxation parameter goes to zero. This corresponds to a sort of Modica-Mortola argument, but in Eulerian variables.

This is a joint work with Diego Grandi (Ferrara), Martin Kruzik (Prague), and Edoardo Mainini (Genova) [3].

Acknowledgments: This work was partially supported by the FWF-GAČR project I2375-16-34894L and by the OeAD-MŠMT project CZ 17/2016-7AMB16AT015.

References

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