Interaction of mean curvature flow and diffusion

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We consider a hypersurface Γ embedded in $\mathbb{R}^d$ with some quantity on it whose concentration c varies along the surface Γ. The energy of the system is given by

$$E(\Gamma, c) := \int_\Gamma G(c)$$

with a sufficiently smooth energy density $G : \mathbb{R} \to \mathbb{R}$. We are interested in how the system evolves to decrease the energy most efficiently while conserving the total mass of the quantity.

Mathematical formulation. Formally, a (L^2, H^{-1})-gradient flow of the energy functional above leads to the following equations

$$\text{(1)} \qquad \begin{cases} V_{\Gamma(t)} &= g\big(c(t)\big) H_{\Gamma(t)} \\ \partial^\circ c(t) &= \Delta_{\Gamma(t)} G'(c)(t) + g\big(c(t)\big) H^2_{\Gamma(t)} c(t) \end{cases} \qquad \text{on } \Gamma(t) \text{ for all } t \in [0, T]$$

where $V_{\Gamma(t)}$ is the normal velocity of the evolving hypersurface $\Gamma(t)$ and $H_{\Gamma(t)}$ its mean curvature. Further, the normal time derivative is denoted by ∂° and $\Delta_{\Gamma(t)}$ is the Laplace-Beltrami-Operator on $\Gamma(t)$. For short notation we use $g(c) := G(c) - G'(c)c$.

Results. We state a short time existence result of (1) and discuss the (non-) preserving of convexity and mean convexity of the hypersurface.

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