# Balanced viscosity solutions to multi-rate systems

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Several mechanical systems are modeled by the static momentum balance for the displacement $u$ coupled with a rate-independent flow rule for some internal variable $z$. Over the last 15 years, the vanishing-viscosity approach to the mathematical modeling of this kind of processes has been developed as a method to select solutions with a mechanically feasible behavior at jumps.

We aim to extend this approach, regularizing *both* the static equation and the rate-independent flow rule by adding viscous dissipation terms modulated by coefficients that vanish to zero with different rates. Thus, the displacement and the internal variable relax to elastic equilibrium and to rate-independent evolution, respectively, with different relaxation rates.

The vanishing-viscosity analysis leads to a notion of Balanced Viscosity solution to the original rate-independent system that provides an accurate description of the system behavior at jumps. We illustrate this solution concept both in the frame of an abstract rate-independent system, and for a concrete rate-independent model for damage and plasticity.

## References

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