Nonlinear interfacial instabilities in viscous multilayer flows

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This talk is concerned with the nonlinear dynamics of viscous flows when immiscible fluids are involved. The flows are caused by shearing, gravity or a pressure gradient (or all three of the above). Mathematically we need to deal with moving boundary problems with interfaces separating fluids that can encounter singular events such as touching solid walls, pinching or even cusp formation in the case of external effects such as electric fields. This presentation will concentrate on shear induced motions of multi fluid flows and utilize a combination of asymptotics, PDE analysis and computations as well as direct numerical simulations to explore the dynamics.

Simple shear flows such as plane Couette flows are linearly stable for all Reynolds numbers. In the presence of more than one immiscible fluid, an interfacial instability can emerge to produce traveling waves or spatiotemporal chaos. The instabilities require inertia and have been reported in experiments. We will describe these using multiscale asymptotic analysis finding good agreement with both direct numerical simulations and experiments.

When multiple layers are present (applications include coating flows) there are now at least two free interfaces. Asymptotic solutions will be presented that yield a system of coupled PDEs for the interfacial positions. The PDEs are parabolic with fourth (surface tension) or second (no surface tension but different densities) order diffusion. The equations generically support instabilities even at zero Reynolds numbers. These emerge physically from an interaction between the interfaces and manifest themselves mathematically through hyperbolic to elliptic transitions of the fluxes of the equations. We use the theory of 2x2 systems of conservation laws to derive a nonlinear stability criterion that can tell us whether a system which is linearly stable (i.e. the initial conditions are in the hyperbolic region of the flux function) can (i) become nonlinearly unstable, i.e. a large enough initial condition produces a large time nonlinear response, or (ii) remains nonlinearly stable, i.e. the solution decays to zero irrespective of the initial amplitude of the perturbation.

Having described weakly nonlinear solutions we will consider fully nonlinear deformations in the large surface tension limit to derive coupled Benney type equations that are now of degenerate parabolic type. Their fluxes also support hyperbolic-elliptic transitions and numerical solutions will be described giving rise to intricate nonlinear traveling waves. Transitional flows are harder to find than in the weakly nonlinear models, but examples will be given where linearly stable initial conditions transition into elliptic regions to sustain energy growth and saturation to nonlinear states.

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