Homogenization and dimension reduction in textiles

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In this work, we investigate periodic structures made of fibers or yarns, like textiles and derive their macroscopic properties via simultaneous homogenization and dimension reduction. As reference domain we consider a canvas structure, which we assume to consist of periodically oscillating and isotropic beams with periodicity ε and radius r. Furthermore, the beams are in contact and thereby the elasticity problem is restricted on a cone fulfilling non-penetration and gap conditions. To obtain different compactness results for all components of the displacement, we apply the decomposition of displacements for beams, [1], yielding an elementary and a warping displacement. The derived estimates depend on the small parameters, the elastic energy, [2], and the contact. Moreover, we introduce an adapted unfolding operator, [3] with an incorporated dimension reduction from three to two dimensions. The properties of the unfolding operator together with the compactness results leads to its weak convergence, equivalent to the two-scale convergence. Consequently, the unfolded limits of the displacements, the strain tensor and contact condition yield homogenized 2D-model for a textile.

References

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