

Justification of the primitive equations

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We aim at completing the justification of the primitive equations (PE) diagram, as follows.

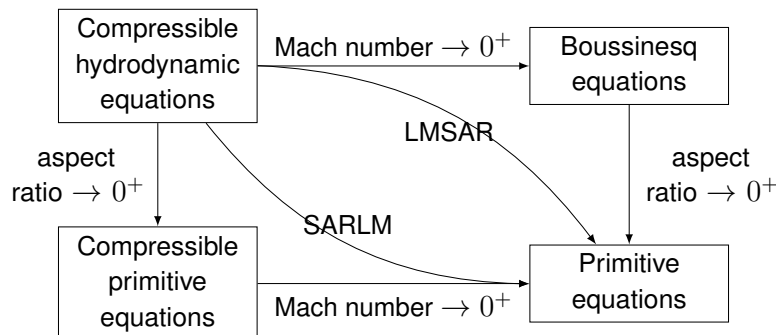


FIGURE 1. The PE diagram

In various settings, the justifications along the path LMSAR in figure 1 are available. See, e.g., Klainerman-Majda (CPAM, 1981), Li–Titi (JMPA, 2018). Our goal is to establish the path SARLM.

Our theorems, below, are concerning the well-posedness and the small Mach number limit of the compressible primitive equations.

Theorem 1. *Given appropriate initial data, there exists a unique local strong solution to the isentropic compressible primitive equations with constant viscosities in the following two cases: with gravity but without vacuum; with vacuum but without gravity.*

Theorem 2. *Given appropriate initial data, there exist global weak solutions to the isentropic compressible primitive equations with density-dependent viscosities.*

Theorem 3. *With well-prepared initial data, solutions of the compressible primitive equations with constant viscosities converge to a solution of the incompressible primitive equations, as the Mach number goes to zero.*

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