Quantitative immersability of Riemann metrics and the infinite hierarchy of prestrained shell models

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We propose results that relate the following two contexts:

(i) Given a Riemann metric G on a thin plate, we study the question of what is the infimum of the averaged deficit of an immersion from being an orientation-preserving isometric immersion of G over all weakly regular immersions. This deficit is measured by the non-Euclidean energies E^h (where h is the plate's thickness) which can be seen as modifications of the classical nonlinear three-dimensional elasticity.

(ii) We perform the full scaling analysis of E^h, in the context of dimension reduction as h goes to 0, and the derivation of Gamma-limits of $h^{-2n}E^h$ for all n. We show the energy quantization, in the sense that the even powers $2n$ of h are indeed the only possible ones; all of them are also attained.

For each n, we identify conditions for the validity of the scaling h^{2n}, in terms of the vanishing of Riemann curvatures of G up to appropriate orders, and in terms of the matched isometry expansions. We also establish the singular asymptotic behaviour of the minimizing immersions.

The problems that we discuss arise from the description of elastic materials displaying heterogeneous incompatibilities of strains that may be associated with growth, swelling, shrinkage, plasticity, etc. Our results and methods display the interaction of nonlinear pdes, geometry and mechanics of materials in the prediction of patterns and shape formation.

Acknowledgments: Support by the NSF grant DMS-1613153 is acknowledged.