Detecting the location of the boundary layers in singular perturbation problems with general linear non-local boundary conditions

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Singular perturbation problems have been studied by many mathematicians, scientists and engineers. In fact, these problems are the mathematical models of physical problems and natural phenomena [1, 2, 3]. The development of small parameter methods led to the efficient use of the boundary–layer theory in various fields of applied mathematics. For instance, fluid mechanics, fluid dynamics, elasticity, quantum mechanics, plasticity, chemical–reactor theory, aerodynamics, plasma dynamics, magneto hydrodynamics, rarefied-gas dynamics, oceanography, meteorology, diffraction theory, reaction–diffusion processes, no equilibrium and radiating flows and other domains of the great world of fluid motion.

Because of the approximate solutions of these problems are as the sum of internal Solution (boundary layer area) and external ones, therefore the formation or non-formation of boundary layers should be specified [4, 5, 6, 7].

In this paper, we investigated this subject for a singular perturbation problem including a first order differential equation with general non-local boundary condition. It is necessary to say that for local boundary conditions it is simple and there is no difficulty. However, for non-local case the formation of boundary layers is not as straight forward as local case. To tackle this problem we make use of generalized solution of differential equations and some necessary conditions. At the end, some unsolved problems were presented for second and fourth order differential equations and matrix form of first order differential equations with general linear non-local boundary conditions [8].

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