Optimal convergence rates in stochastic homogenization of nonlinear uniformly elliptic PDEs

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We derive optimal-order homogenization rates for random nonlinear elliptic PDEs with monotone nonlinearity in the uniformly elliptic case. More precisely, for a random monotone operator on $\mathbb{R}^d$ with stationary law (i. e. spatially homogeneous statistics) and fast decay of correlations on scales larger than the microscale $\varepsilon > 0$, we establish homogenization error estimates of the order ε in case $d \geq 3$, respectively of the order $\varepsilon |\log \varepsilon|^{1/2}$ in case $d = 2$. Previous results in nonlinear stochastic homogenization have been limited to a small algebraic rate of convergence ε^δ. We also establish error estimates for the approximation of the homogenized operator by the method of representative volumes of the order $(L/\varepsilon)^{-d/2}$ for a representative volume of size L. Our results also hold in the case of systems for which a (small-scale) $C^{1,\alpha}$ regularity theory is available.