Pinning of interfaces by localized dry friction

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We consider the differential inclusion

$$\begin{aligned} u_t + \partial R(u_t)\varphi(x, u(x)) - \Delta u \quad &\ni \quad F \quad \text{in } \mathbb{R}^n \times (0,\infty), \\ u(\cdot, 0) \quad &= \quad 0 \quad \text{on } \mathbb{R} \times \{0\}. \end{aligned} \tag{1}$$

with ∂R the subdifferential of $R(a) = |a|$, i.e., R is a 1-homogeneous dissipation potential modeling dry friction. This differential inclusion models the propagation of an interface, e.g., a phase boundary, in an environment with obstacles: the graph of the function $u\colon \mathbb{R}^n \times [0,\infty) \to \mathbb{R}$, $n \geq 1$ represents the interface propagating in $\mathbb{R}^{n+1}$ according to a local driving force, which is comprised of a regularizing term (the Laplacian as an approximation of the line tension), a constant driving force F, and the interaction of the interface with the obstacles captured in φ, e.g.

$$\varphi(x, y) = \sum_{k\in\mathbb{N}} f_k \varphi_0(x - x_k, y - y_k).$$

Here, φ_0 describes the shape of the obstacles, (x_k, y_k) the center of an obstacle and f_k its strength. See [1] for further references regarding modeling.

The model implies that energy has to be expended to pass across an obstacle. Hence, the interface becomes arrested until enough curvature is accumulated such that it is energetically more favorable to pass across the obstacle.

The treatment of (1) in the context of pinning and depinning requires a comparison principle. We prove this property and hence the existence of viscosity solutions. Moreover, under reasonable assumptions, they are equivalent to weak solutions.

Our main result thus states that, in the case of obstacles with centers distributed according to a Poisson point process, there exists a deterministic lower bound $F^* > 0$ for the critical force such that for every $F \geq 0$ with $F \leq F^*$ there exists a stationary supersolution $\overline{u} \geq 0$ to (1), i.e., every solution u satisfies $u(x,t) \leq \overline{u}(x)$. This effect is called pinning.

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References

[1] Nicolas Dirr, Patrick Dondl, and Michael Scheutzow. Pinning of interfaces in random media. *Interfaces and Free Boundaries*, pages 411–421, 2011.