Rigorous derivation of reduced models in thin fluid–thin structure interaction problems

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We analyze a linear fluid-structure interaction (FSI) problem between a thin layer of a viscous fluid and a thin elastic plate with the aim of deriving a simplified reduced model [1]. The FSI problem is a coupled system, where the fluid is described by the Stokes equations and the structure by the linear elasticity equations, both in three spatial dimensions. We first derive an energy-energy dissipation inequality quantified in terms of the relative fluid thickness ε, which is assumed to be a small parameter. Lamé constants of the elastic structure are assumed to be large and behave like $h^{-\kappa}$ for some $\kappa > 0$, where $h > 0$ denotes the relative thickness of the elastic structure, which is also assumed to be small and related to ε through a power law $\varepsilon = h^\gamma$ for some $\gamma > 0$.

Based on the energy-energy dissipation inequality we derive the key a priori estimates and identify the right time scale which eventually, on the limit as $h \downarrow 0$, give rise to a reduced scalar two dimensional model. The reduced model is given by a sixth-order evolution equation for the out-of-plane displacement of the bending plate. It is a consequence of simultaneous dimension reduction in the structure and in the fluid. In the structure, the linear elasticity equations reduce to the linear bending plate model with the limit fluid pressure acting as a normal force, and in the fluid, the so called lubrication approximation is performed. Finally, utilizing the reduced model we construct approximate solutions to the original FSI problem and derive basic error estimates which provide even strong convergence results.

Acknowledgments: This work has been supported in part by Croatian Science Foundation under projects 7249 (MANDphy) and 3706 (FSIApp).

References

[1] M. Bukal and B. Muha, Convergence analysis of a thin fluid - thin structure interaction problem – the linear case, *in preparation* (2019).