Chemotaxis–consumption model with realistic boundary conditions

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The talk we discuss the behavior of the concentration of some bacteria n swimming in water (for example of the species *Bacillus subtilis*), whose otherwise random motion is partially directed towards higher concentrations c of a signaling substance (oxygen) they consume. After a transition phase, the system can be described using the stationary chemotaxis model

$$\begin{cases} \partial_t n + u \cdot \nabla n = \Delta n - \nabla \cdot (n \nabla c) \\ \partial_t c + u \cdot \nabla c = \Delta c - nc \\ \partial_t u - u \cdot \nabla u = \Delta u + \nabla P - n \nabla \varphi \\ \nabla \cdot u = 0, \end{cases}$$

on a bounded domain Ω. Previous studies of chemotaxis models with consumption of the chemoattractant (with or without fluid) have not been successful in explaining pattern formation even in the simplest form of concentration near the boundary, which had been experimentally observed.

Following the suggestions that the main reason for that is usage of inappropriate boundary conditions, this talk considers no-flux boundary conditions for n and the physically meaningful condition

$$\partial_\nu c = 1 - c$$

for the signaling substance c and Dirichlet boundary conditions for the flow u.

In the talk, we study the existence of a global weak solution and its uniqueness in dimension two. Moreover, for $\phi = const.$ we discuss how to show that there exists a unique stationary solution for any given mass $\int_\Omega n > 0$. This solution is non-constant and in the special case of Ω being a ball, the solutions n and c are strictly convex.

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References

[1] M. Braukhoff and J. Lankeit. Stationary solutions to a chemotaxis–consumption model with realistic boundary conditions. arXiv:1902.01167. To appear in Math. Models Meth. Appl. Sci., 2019.