# Non-interpenetration in thin films

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Many objects in mechanics or our daily life appear in the thin-film geometry. While non-interpenetration is well understood from the point of view of modeling in bulks, and corresponds to injectivity of the deformation, this seems not to be the case in the geometry of planar rods or thin films. In fact, only scattered results have appeared in the literature, so far (see e.g. [1, 2]).

Within this talk, we concentrate for simplicity on the case of planar rods and their deformations contained in the class

$$\mathcal{X}_p((a,b);\mathbb{R}^2) = \Big\{ y \in W^{2,p}((a,b);\mathbb{R}^2) : y'(x) \neq 0 \text{ a.e. in } (a,b) \text{ and } |y'|^{-\alpha} \in L^1((a,b)) \text{ with } \alpha > \frac{p}{p-1} \Big\}.$$

Such deformations are already locally injective, but interpenetration may or may not have appeared. We will introduce a definition of non-interpenetration corresponding, roughly speaking, to the idea that non-penetrating deformations can be uniformly approximated by injective ones.

We will prove that any non-penetrating deformation can be approximated by injective ones, no only uniformly, but also strongly in $W^{2,p}$ by mappings in the class $\mathcal{X}_p$, given some technical conditions. This will allow us to characterize $\Gamma$- limits of bulk energies enforcing non-interpenetration upon dimension reduction in the membrane regime.

## References

[1] O. Pantz: The modeling of deformable bodies with frictionless (self-) contacts. *Arch. Rat. Mech. Anal.*, **188.2** (2008): 183–212.

[2] H. Olbermann, E. Runa: Interpenetration of matter in plate theories obtained as Gamma-limits. *ESAIM COCV* **23** (2017), 119–136.