

*Time-dependent transport through a nanostructure  
coupled to an electromagnetic field in a cavity*

Viðar Guðmundsson

Science Institute, University of Iceland, Iceland

`vidar@raunvis.hi.is`

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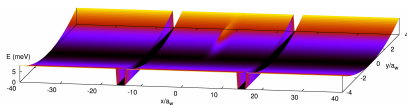
# Background - Motivation

## Presently

- Time-dependent transport
- System - leads, bias
- GME many-electron formalism
- Coulomb interaction
- Finite quantum wire
- Quasi-1D leads
- Magnetic field
- Embedded subsystems
- Coulomb blocking, correlations, transients, steady state

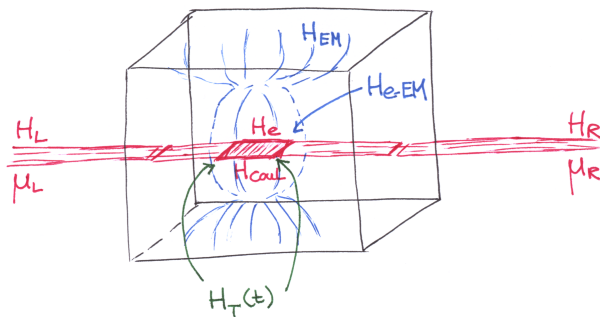
## Future

- External photon field
- Full EM-coupling
- Cavity modes
- Polarization



# Approach

$$\begin{aligned}
 H(t) = & \underbrace{H_e + H_{\text{Coul}}}_{\text{diagonalize+cut}} + H_{\text{EM}} + H_{e\text{-EM}} + H_{\text{L.R}} + H_{\text{T}}(t) \\
 & \underbrace{\hspace{10em}}_{\text{diagonalize+cut} \rightarrow H_S} \\
 & \underbrace{\hspace{15em}}_{\text{project on system}(H_S) \rightarrow \text{GME}}
 \end{aligned}$$



# e-EM coupling

## Full electron-photon coupling

$$\begin{aligned} & \int d\mathbf{r} \psi^\dagger \left\{ \frac{1}{2m^*} \left( -i\hbar\nabla - \frac{e}{c} [\mathbf{A} + \mathbf{A}_{\text{ext}}] \right)^2 \right\} \psi \\ &= \int d\mathbf{r} \psi^\dagger \left\{ \frac{1}{2m^*} \left( -i\hbar\nabla - \frac{e}{c} \mathbf{A} \right)^2 \right\} \psi \\ &+ \frac{1}{c} \int d\mathbf{r} \mathbf{j} \cdot \mathbf{A}_{\text{ext}} + \frac{e^2}{2m^*c} \int d\mathbf{r} \rho A_{\text{ext}}^2 \\ &\approx \int d\mathbf{r} \psi^\dagger \left\{ \frac{\pi^2}{2m^*} \right\} \psi + \frac{1}{c} \int d\mathbf{r} \mathbf{j} \cdot \mathbf{A}_{\text{ext}} \\ &= H_e + H_{e\text{-EM}} \end{aligned}$$

## Single cavity mode

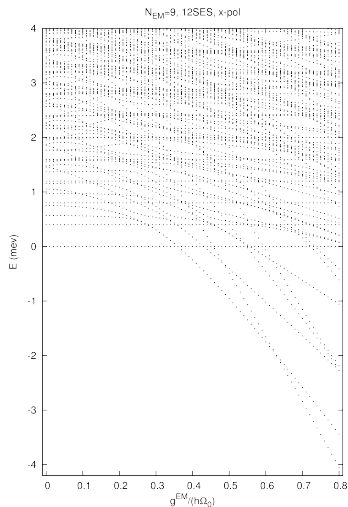
$$H_S = \sum_a E_a c_a^\dagger c_a + \hbar \nu a^\dagger a + \frac{1}{2} \sum_{abcd} \langle ab | V_{\text{Coul}} | cd \rangle c_a^\dagger c_b^\dagger c_d c_c \\ + \sum_{ab} c_a^\dagger c_b \left\{ g_{ab} a + g_{ab}^* a^\dagger \right\}$$

$$H_S = \sum_{\mu} |\mu\rangle \tilde{E}_{\mu} \langle \mu| \\ + \sum_{\mu\nu ab} |\mu\rangle \langle \mu | \mathcal{V}^\dagger c_a^\dagger c_b \mathcal{V} | \nu \rangle \langle \nu | \left\{ g_{ab} a + g_{ab}^* a^\dagger \right\}$$

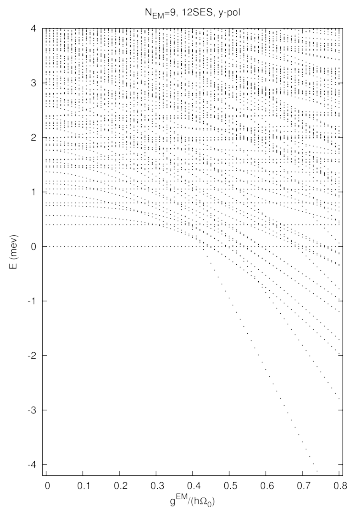
$$\underbrace{|\mu\rangle \otimes |N_{\text{ph}}\rangle \longrightarrow |\mu\rangle_{\text{e-EM}}}_{\text{diagonalization}}$$

# Energy spectra

x-polarization,  $B = 0.1$  T

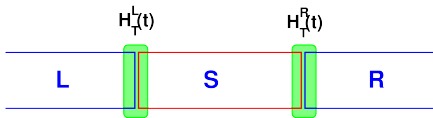


y-polarization,  $B = 0.1$  T



# GME

- Weak coupling to leads
- Non-Markovian
- Memory effects
- $\mathcal{P} = \rho_L \rho_R \text{Tr}_{LR}$
- $H_T^l(t) = \chi^l(t) \sum_{q,a} \{ T_{qa}^l d_{ql}^\dagger c_a + (T_{qa}^l)^* c_a^\dagger d_{ql} \}$
- Reduced statistical operator  
 $\rho_S(t) = \mathcal{P}\{W(t)\}$



Liouville-von Neumann equation

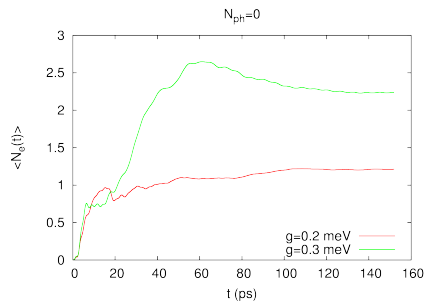
$$\dot{W}(t) = -\frac{i}{\hbar} [H(t), W(t)] = -i\mathcal{L}W(t)$$

$$\langle A(t) \rangle = \text{Tr}\{W(t)A\} = \text{Tr}_S\{\rho_S(t)A\}$$

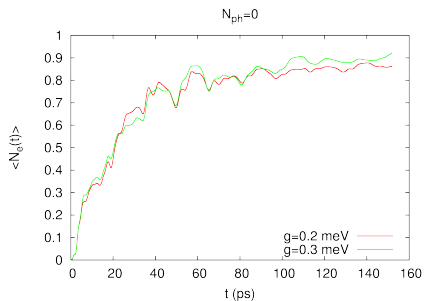
$$i\hbar\dot{\rho}_S(t) \approx \mathcal{L}_S\rho_S(t) + \frac{1}{i\hbar}\text{Tr}_{LR} \left\{ \mathcal{L}_T(t) \int_0^t ds e^{-i(t-s)\mathcal{L}_0} \mathcal{L}_T(s) \rho_L \rho_R \rho_S(s) \right\}$$

# Total charge

x-polarization,  $B = 0.1$  T

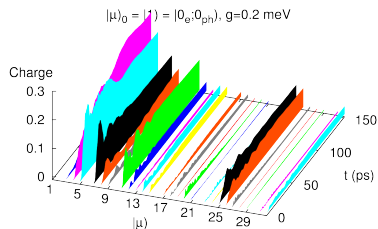


y-polarization,  $B = 0.1$  T

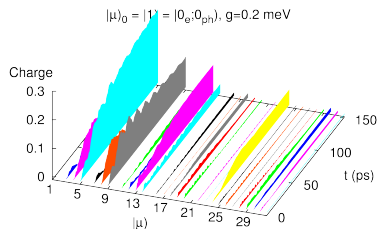


# Charge per MES $|\mu\rangle$

x-polarization,  $B = 0.1$  T

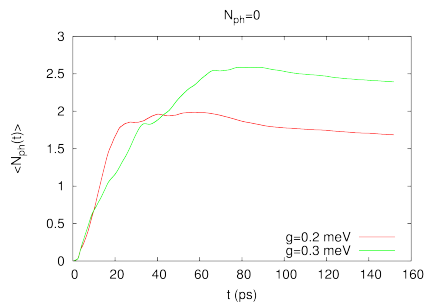


y-polarization,  $B = 0.1$  T

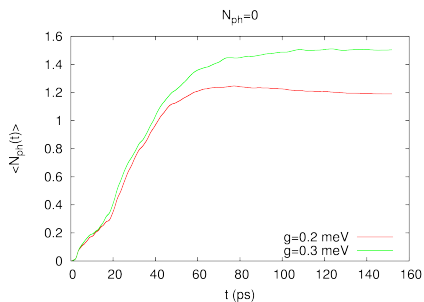


# Total number of photons

x-polarization,  $B = 0.1$  T

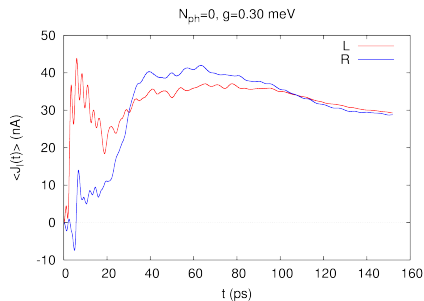


y-polarization,  $B = 0.1$  T

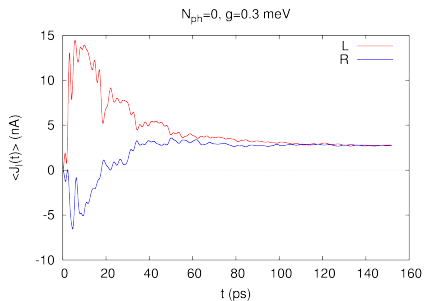


# Left and Right total current

x-polarization,  $B = 0.1$  T

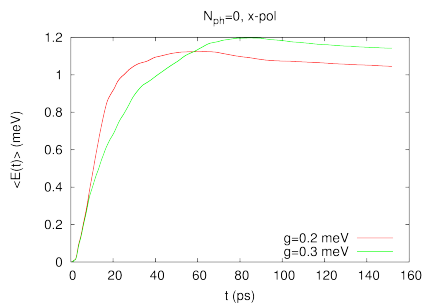


y-polarization,  $B = 0.1$  T

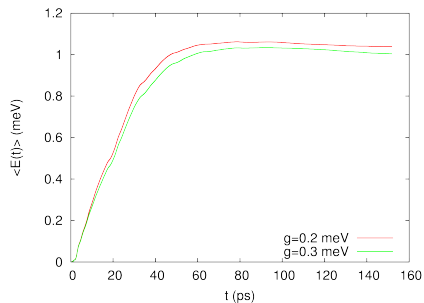


# Mean total energy

x-polarization,  $B = 0.1$  T



y-polarization,  $B = 0.1$  T



# Summary

- Collaboration with, Andrei Manolescu, Valeriu Moldoveanu, Chi-Shung Tang
  - Preliminary results, unpublished
  - Coulomb Interaction
  - Photon coupling
  - Many -electron, -photon model
  - Truncation Scheme
  - Extensive testing needed
- 
- [New Journal of Physics 11, 073019 \(2009\), and 113007 \(2009\)](#)
  - [Phys. Rev. B81, 155442 \(2010\), and 205319 \(2010\)](#)