High-mobility electron transport on cylindrical surfaces

Klaus-Jürgen Friedland
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- **Concept to create high mobility electron gases on free standing semiconductor heterostructures -Experiment**

- **Electronic effects on a cylinder surface**

- Adiabatic transport transport – nontrivial trajectories
Landauer Büttiker using open orbits
- Quantum Hall effect on a cylinder surface
Landauer Büttiker fails – take self- consistent screening
- Commensurable resistance oscillations for tangentially directed magnetic fields
Calculation of ‘skipping orbits’

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Contributions

- A. Riedel,
- R. Hey,
- H. Kostial[†] PDI
- U. Jahn,
- M. Höricke,
- A. Siddiki University Mugla, Turkey
- D. K. Maude. HMFL CNRS, France

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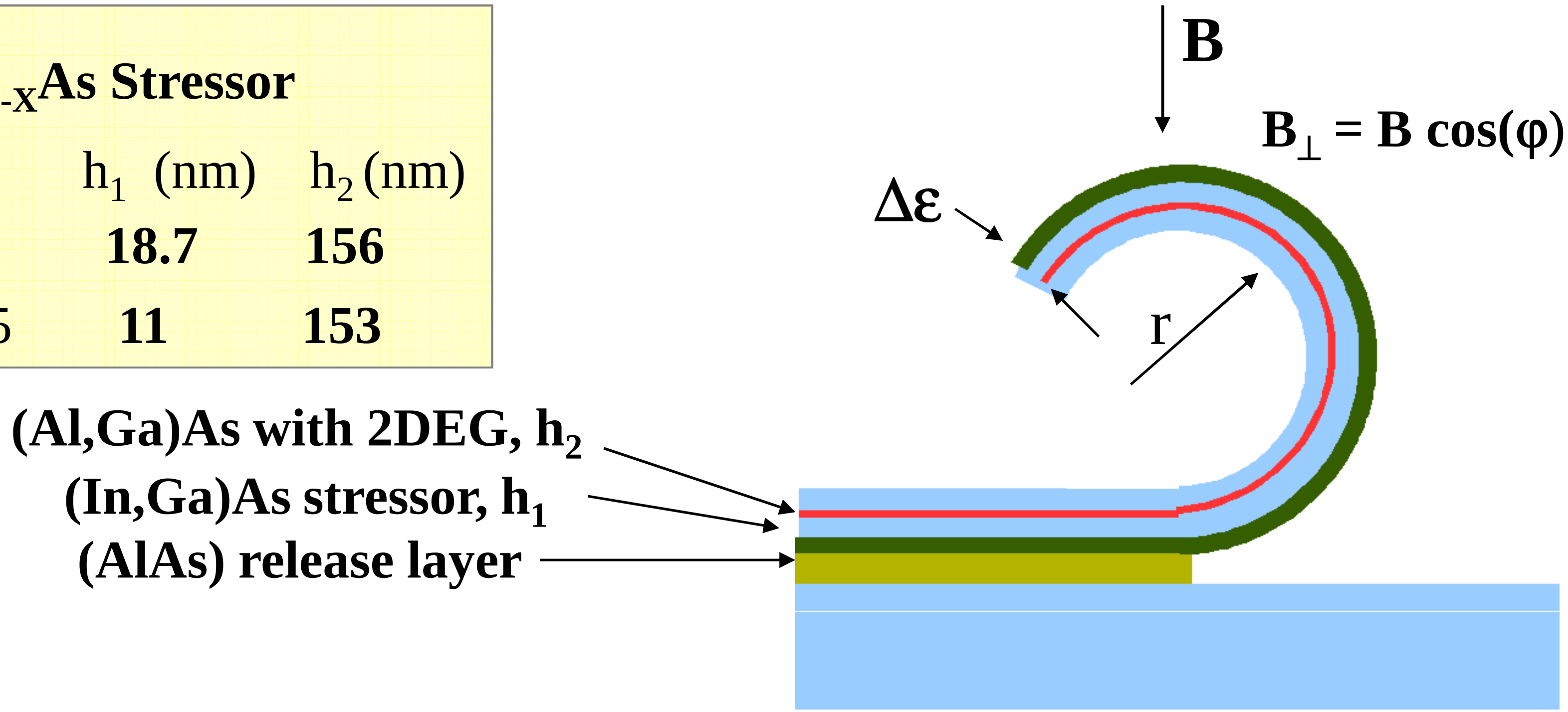
Rolling-up a Heterostructure

	$In_XGa_{1-X}As$ Stressor		
	x_{In}	h_1 (nm)	h_2 (nm)
#A	0.13	**18.7**	**156**
#B	0.195	**11**	**153**



$$r = \frac{h_1^4 + 4\chi h_1^3 h_2 + 6\chi h_1^2 h_2^2 + 4\chi h_1 h_2^3 + \chi^2 h_2^4}{6\varepsilon\chi(1+\upsilon) h_1 h_2 (h_1 + h_2)}$$

ratio of Young's moduli χ, Poisson ratio ν and strain ε

- **Strain gradient Δε ≈ 1 %**
- **Magnetic field gradient ≈ 1T/µm**

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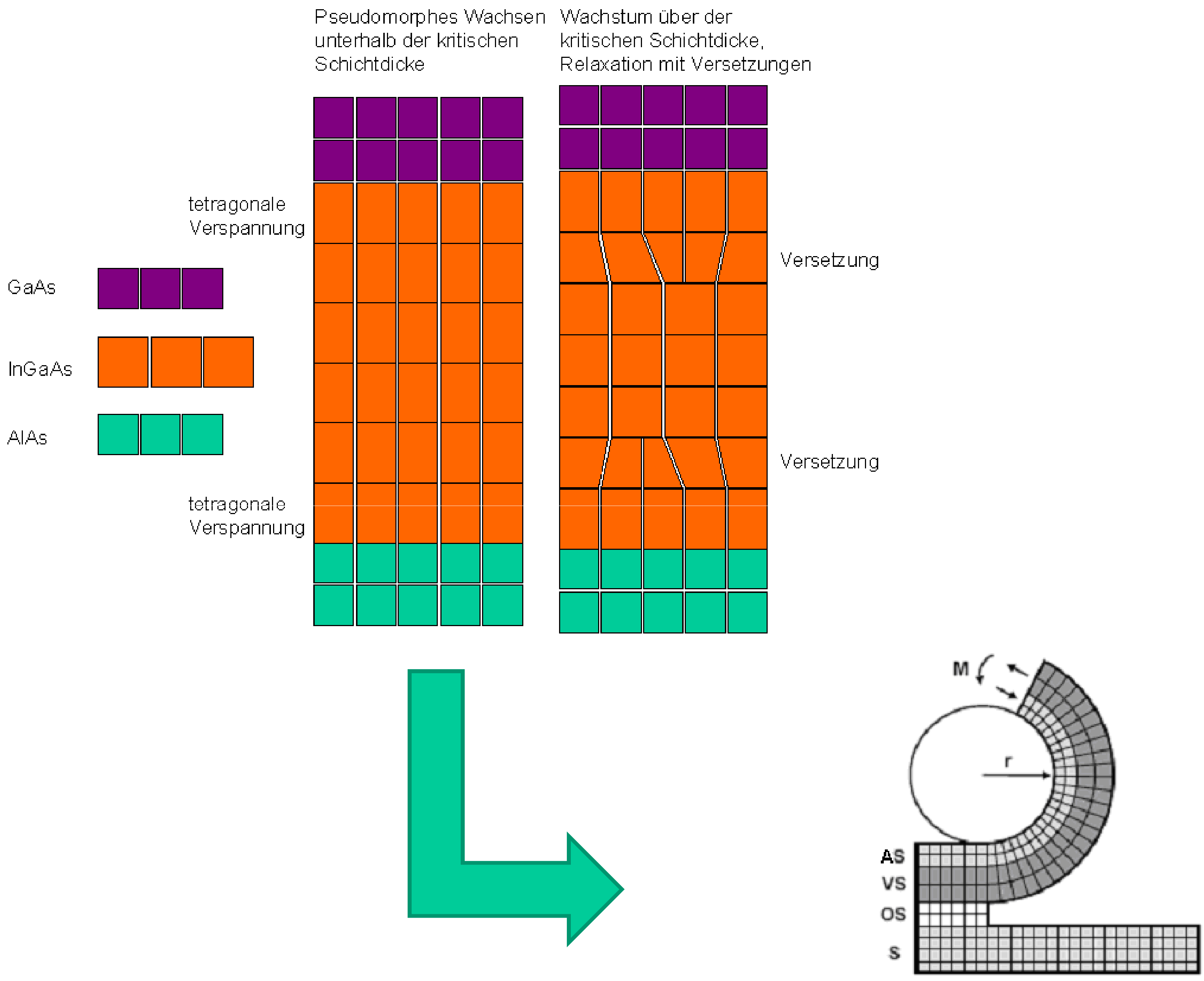
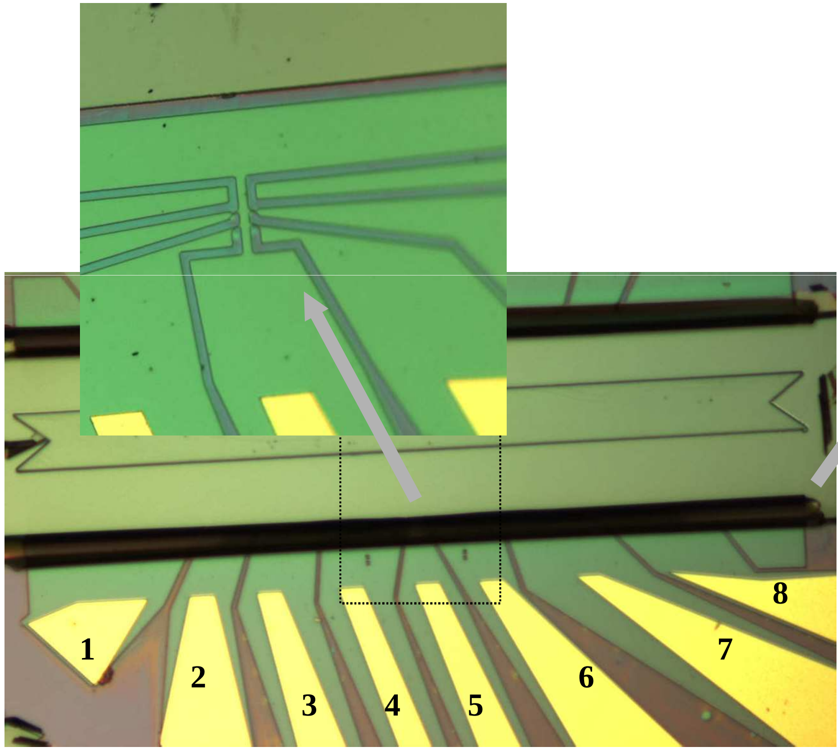

Pseudomorphes Wachsen unterhalb der kritischen Schichtdicke
Wachstum über der kritischen Schichtdicke, Relaxation mit Versetzungen
tetragonale Verspannung
GaAs
InGaAs
AlAs
Versetzung
Versetzung
tetragonale Verspannung
M
r
AS
VS
OS
S

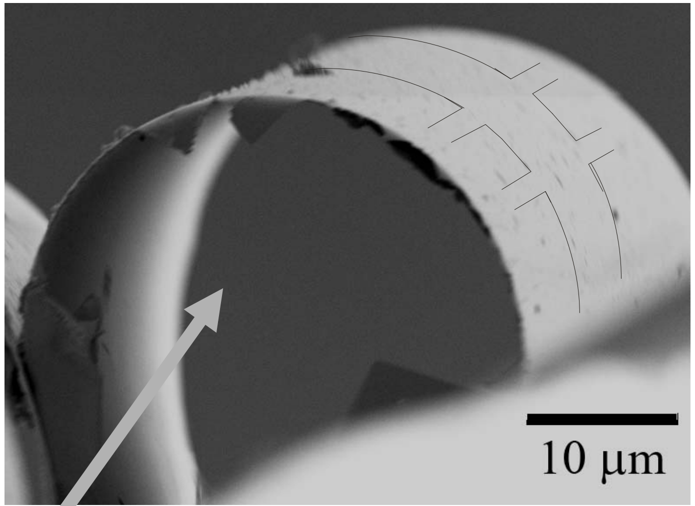
Heterostucture tubes containing a Hall bar

-Shallow mesa- etching and
 Ohmic contacts on the flat surface
-Etching from a starting line,
 rolling-up a tube $r \cong 20\ \mu m$



A. B. Vorob'ev et al., Physica E, 2004.

S. Mendach, et a., Physica E, 2004.



Drawback for (Al,Ga)As System:
Surface states at the new surface

→→ depletion of carriers,

decrease mobility ☹

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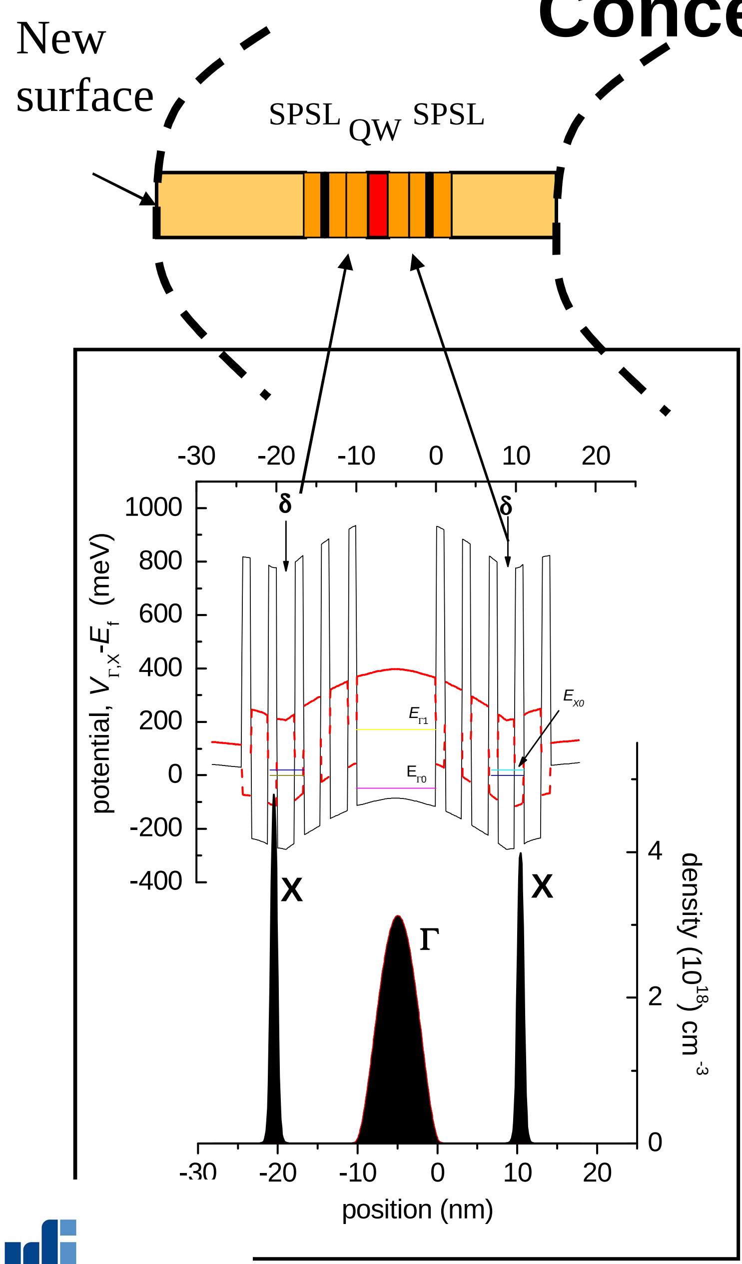
Concept for high-mobility 2DEGs on cylindrical surfaces



Concept for freestanding heterostructures:

Screening of potential fluctuations also from surface charges of the new surface

→→

2DEGs on tubes:

Mobility of up to 100 m^2/Vs at $n_S \approx 0.7 \times 10^{16}\ m^{-2}$

K.-J. Friedland et al., Phys. Rev.Lett. 1996. ☺

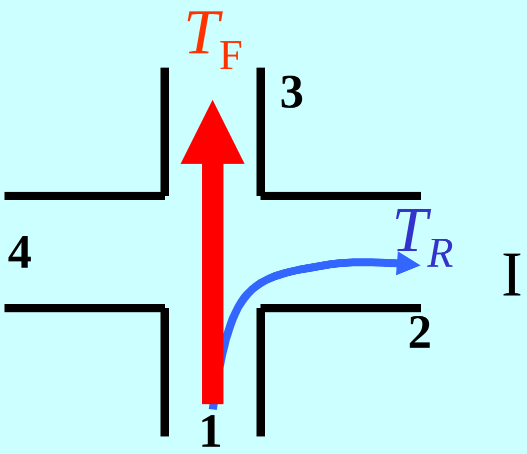
Adiabatic transport on cylindrical surface

Mean free path compares with the rolling radius: $l_{mfp} \cong 20\ \mu m \cong r$

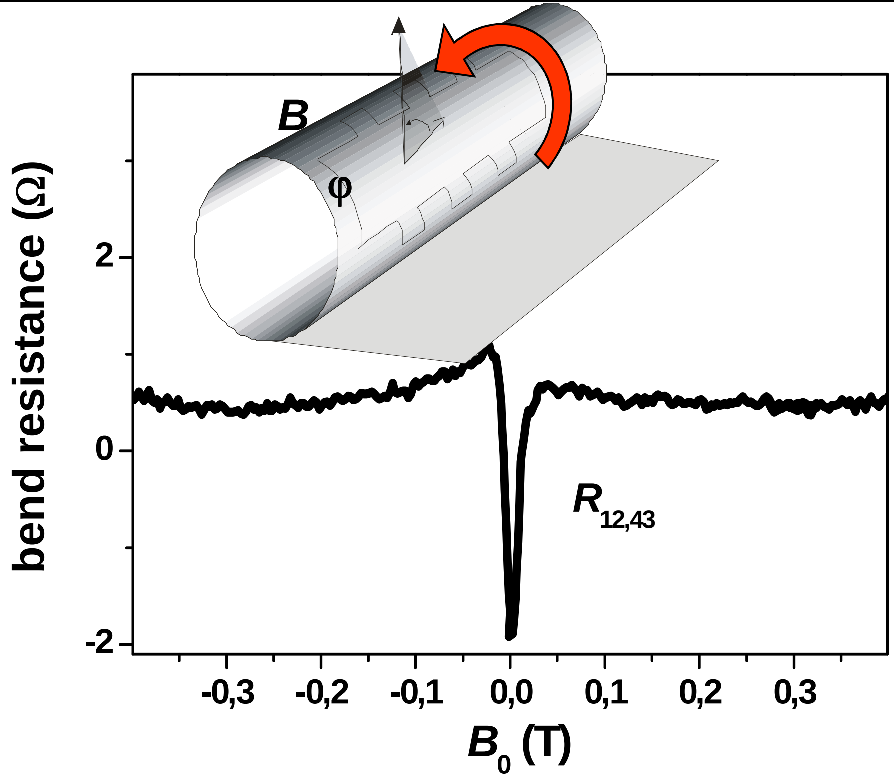
$\varphi = 0°$ $\Rightarrow$ **low gradient**

- **Negative bend resistance in a wide cross junction**

  at zero magnetic field



$$R_B = R_{12,43} = \frac{h}{2e^2}\frac{T_R T_L - T_F^2}{\widehat{D}} < 0,$$



$$T_F^2 > T_L T_R$$

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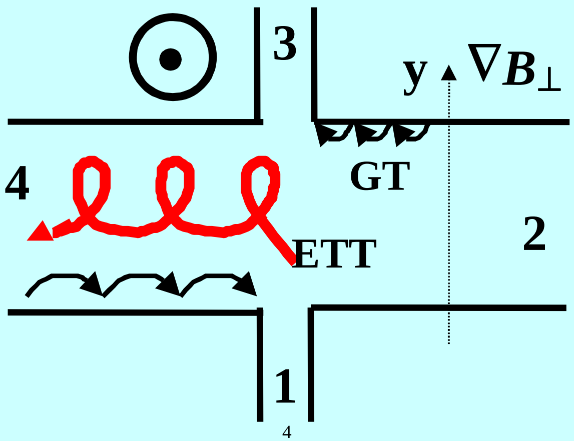
K.-J. Friedland et al., Phys. Rev.B 2007.

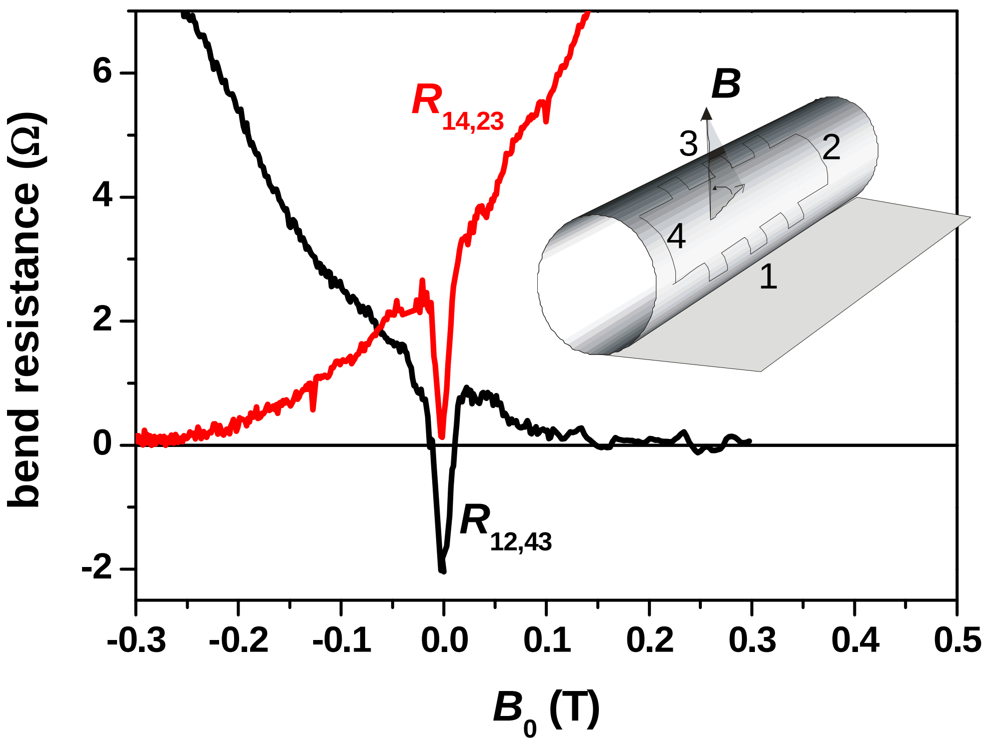
Ballistic transport on cylindrical surface II

Mean free path compares with the rolling radius: $l_{mfp} \cong r$,
$\varphi = 29° \Rightarrow \delta B_\perp / B_\perp \cong 300\%$

- **Extended trochoid-like trajectories (ETT)** move oppositely to
- **guided trajectories (GT)**



$$R_B = R_{12,43} = \frac{h}{2e^2} \frac{T_R^{GT} T_L^{ETT}}{\widehat{D}},$$



$$B < 0, \quad T_R^{GT} > 0, \quad T_R^{GT} T_L^{ETT} > 0,$$
$$B > 0, \quad T_R^{GT} = 0, \quad T_R^{GT} T_L^{ETT} = 0$$

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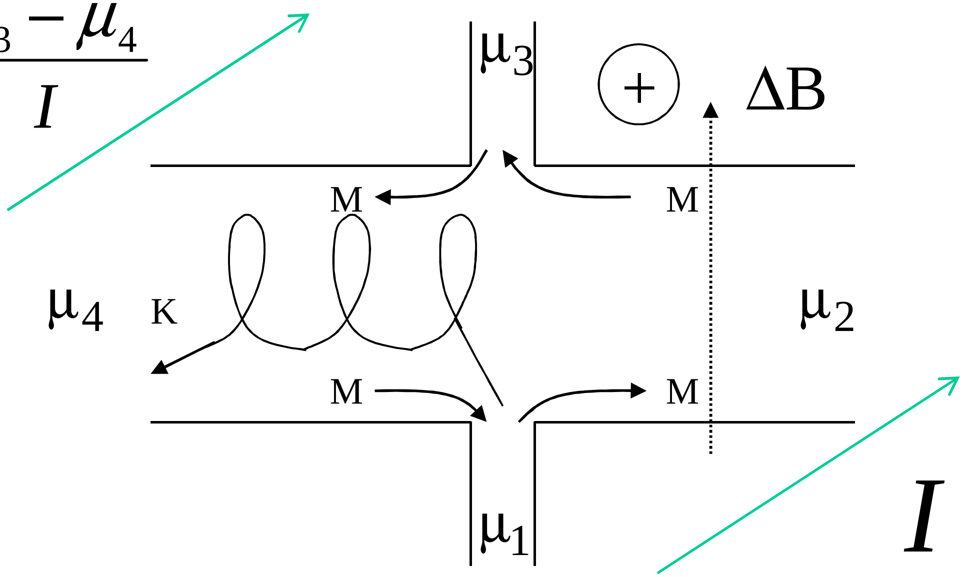
K.-J.Friedland et al., Phys. Rev.B 2007

$$R = \frac{\mu_3 - \mu_4}{I}$$

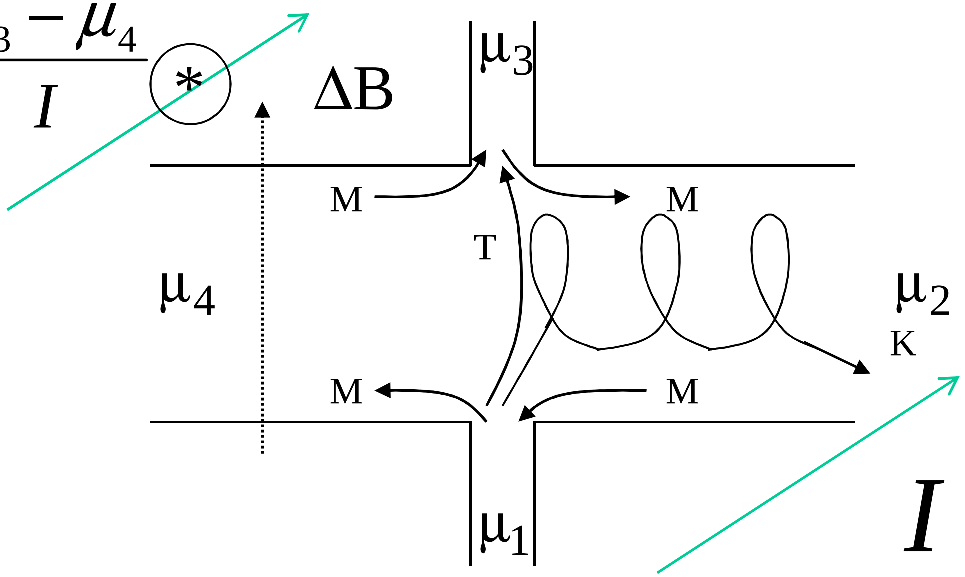


$$\begin{pmatrix} I \\ -I \\ 0 \\ 0 \end{pmatrix} = \frac{h}{2e^2} \begin{pmatrix} -M-K & 0 & 0 & M+K \\ M & -M & 0 & 0 \\ 0 & M & -M & 0 \\ K & 0 & M & -M-K \end{pmatrix} \begin{pmatrix} \mu_1 \\ \mu_2 \\ \mu_3 \\ \mu_4 \end{pmatrix}$$

$$\frac{\mu_3 - \mu_4}{I} = \frac{h}{2e^2} \frac{K}{M(M+K)},$$

$$\frac{\mu_2 - \mu_1}{I} = \frac{h}{2e^2} \frac{1}{M}, \qquad \mu_2 = \mu_3$$

$$R = \frac{\mu_3 - \mu_4}{I}$$



$$\begin{pmatrix} I \\ -I \\ 0 \\ 0 \end{pmatrix} = \frac{h}{2e^2} \begin{pmatrix} -M-K & M+K & 0 & 0 \\ K & -M-K & M & 0 \\ T & 0 & -M & M \\ M & 0 & 0 & -M \end{pmatrix} \begin{pmatrix} \mu_1 \\ \mu_2 \\ \mu_3 \\ \mu_4 \end{pmatrix}$$

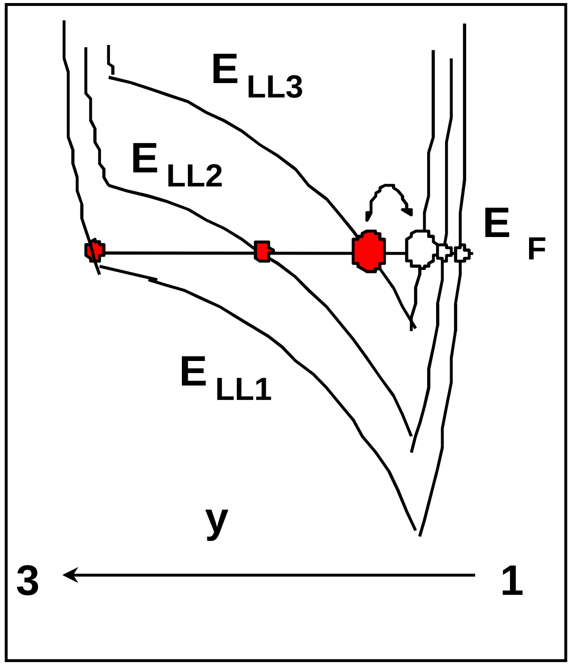
$$\mu_1 = \mu_3 = \mu_4,$$

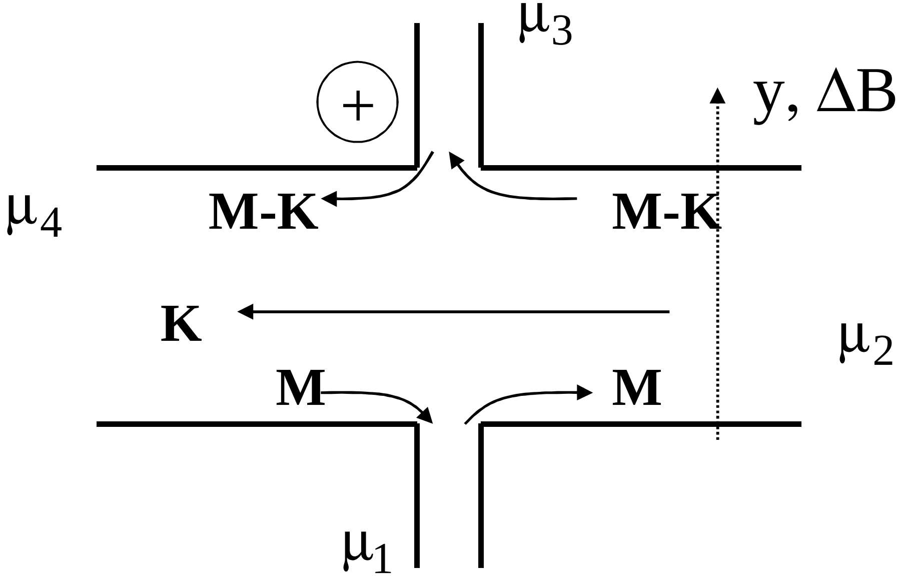
$$\frac{\mu_2 - \mu_1}{I} = \frac{h}{2e^2} \frac{1}{K+M},$$

B=0: Negative bend resistance

$$\frac{\mu_3 - \mu_4}{I} = -\frac{h}{2e^2} \frac{1}{T},$$

Quantum Hall effect – one dimensional Landau states





Quantum Hall effect dominated by

- one-dimensional Landau states (1DLS) at the low magnetic field side with

- maximum number of states

Landauer Büttiker approach:

$$\begin{pmatrix} I \\ -I \\ 0 \\ 0 \end{pmatrix} = \frac{h}{2e^2} \begin{pmatrix} -M & 0 & 0 & M \\ M & -(M-K)-K & 0 & 0 \\ 0 & M-K & -(M-K) & 0 \\ 0 & K & M-K & -M \end{pmatrix} \begin{pmatrix} \mu_1 \\ \mu_2 \\ \mu_3 \\ \mu_4 \end{pmatrix}$$

$$\frac{\mu_2 - \mu_1}{I} = \frac{\mu_3 - \mu_1}{I} = \frac{h}{2e^2}\frac{1}{M}, \quad \mu_2 = \mu_3 = \mu_4$$

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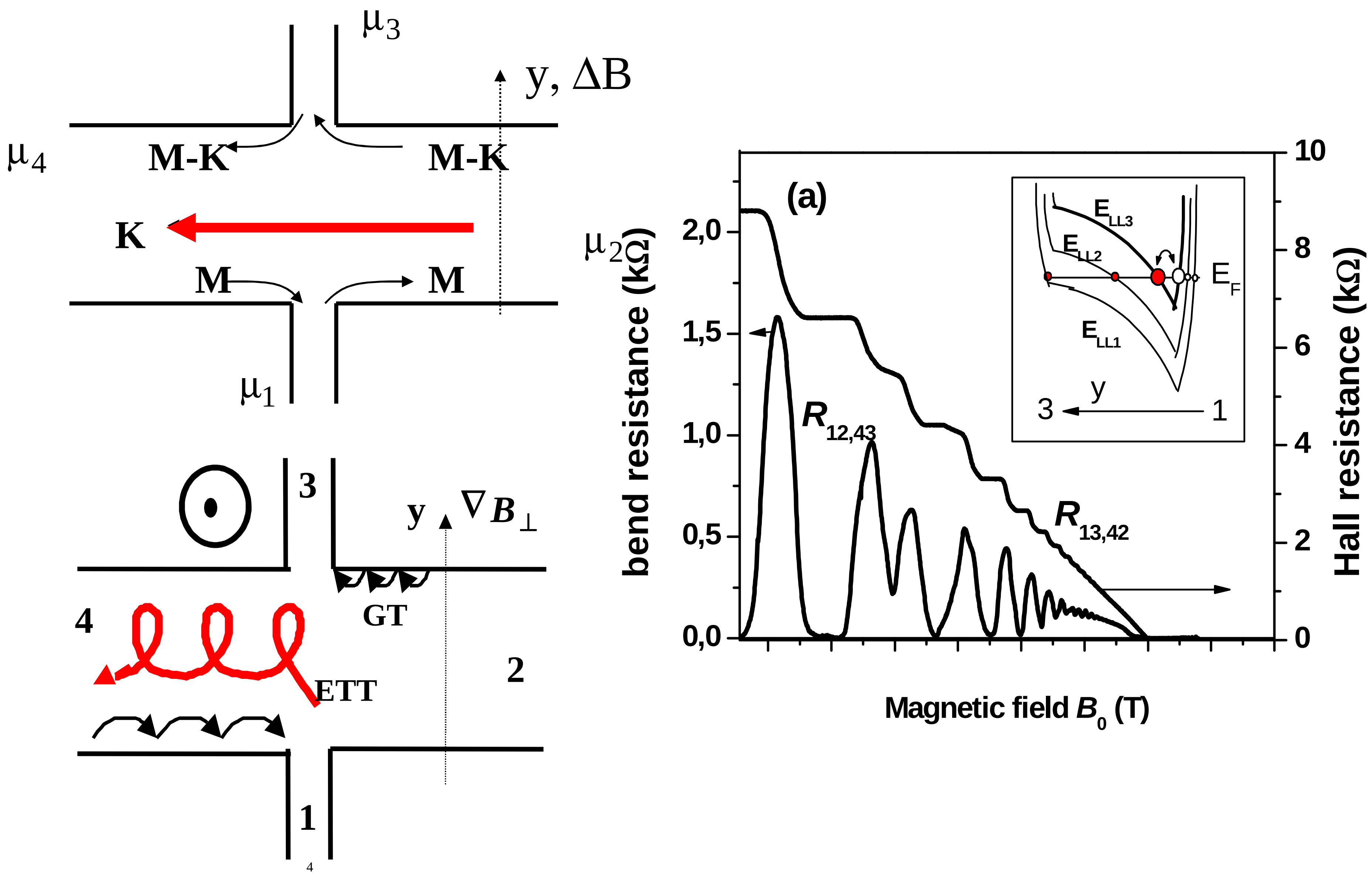
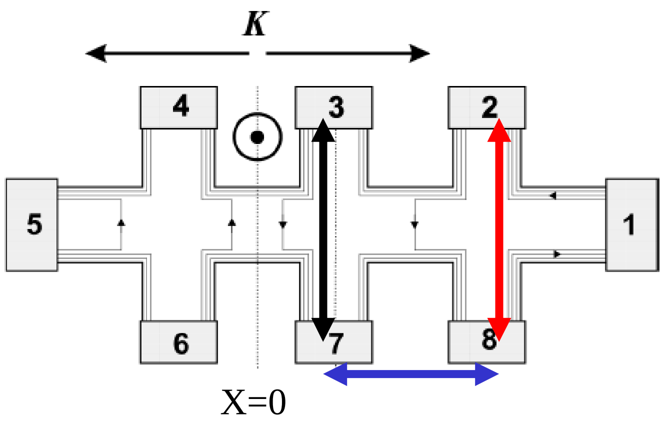
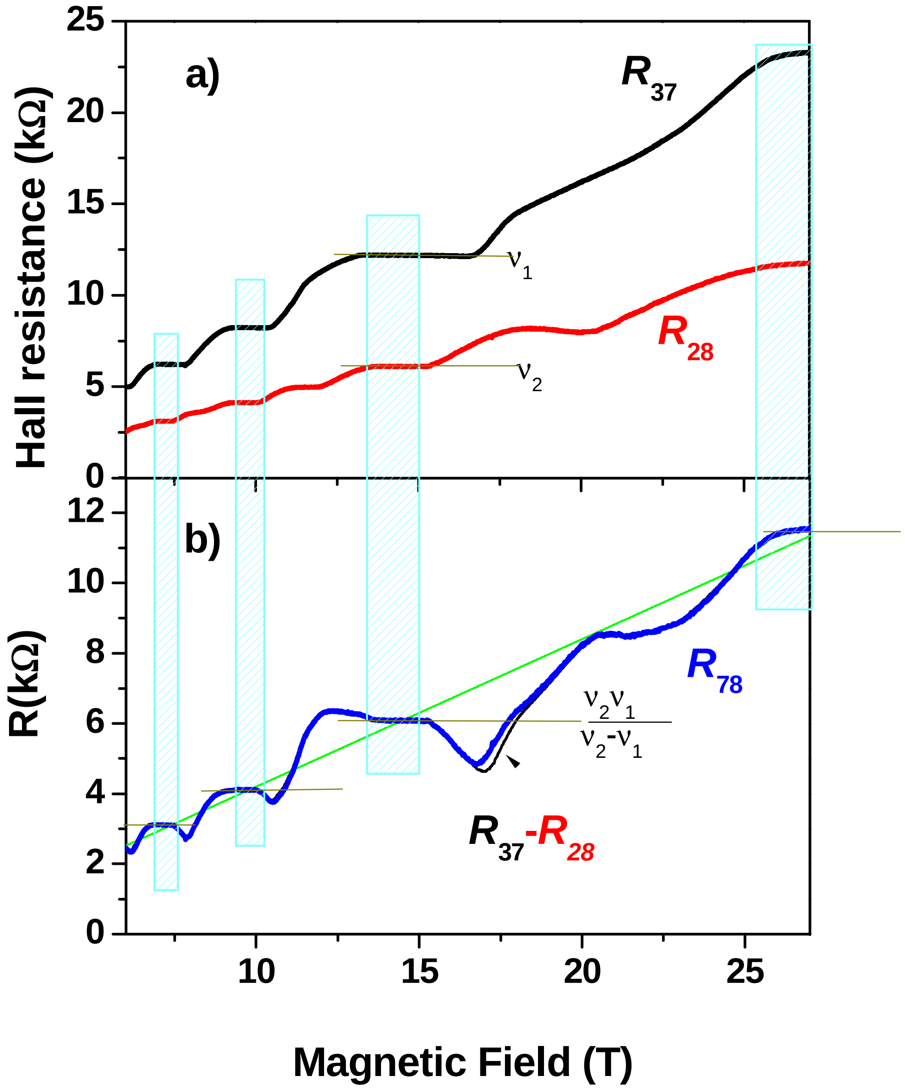
ETT ←→ QHE


μ3
y, ΔB
μ4
M-K
M-K
K
μ2
M
M
μ1
3
y
∇B⊥
GT
4
ETT
2
1
4
(a)
2,0
1,5
1,0
0,5
0,0
10
8
6
4
2
0
bend resistance (kΩ)
Hall resistance (kΩ)
R12,43
R13,42
E LL3
E LL2
E F
E LL1
y
3
1
Magnetic field B0 (T)

Quantum Hall effect





Landauer-Büttiker approach

- ***One-dimensional Landau states***
- ***Bulk → insulator***

$$R_{43} = \frac{\mu_4 - \mu_3}{I_{51}} = \frac{h}{2e^2}\left(\frac{1}{\nu_0} - \frac{1}{\nu_{(46)}}\right) = R_0 - R_{46}$$

$$R_{67} = \frac{\mu_6 - \mu_7}{I_{51}} = \frac{h}{2e^2}\left(\frac{1}{\nu_0} - \frac{1}{\nu_{(37)}}\right) = R_0 - R_{37}$$

$$R_{32} = \frac{\mu_3 - \mu_2}{I_{51}} = 0$$

$$R_{78} = \frac{\mu_7 - \mu_8}{I_{51}} = \frac{h}{2e^2}\left(\frac{1}{\nu_{(37)}} - \frac{1}{\nu_{(28)}}\right) = R_{37} - R_{28}.$$

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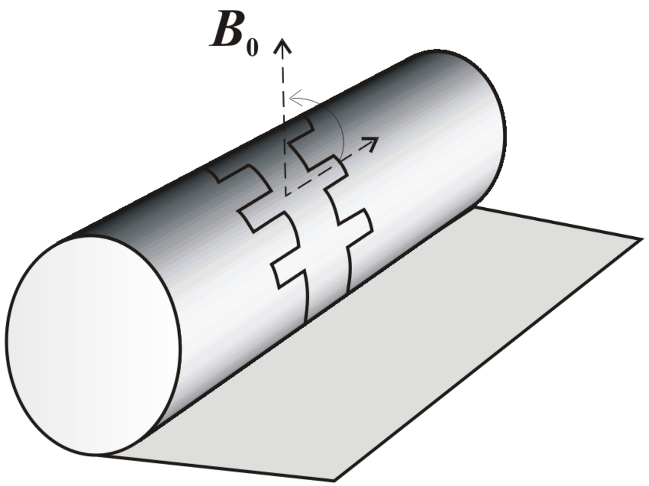
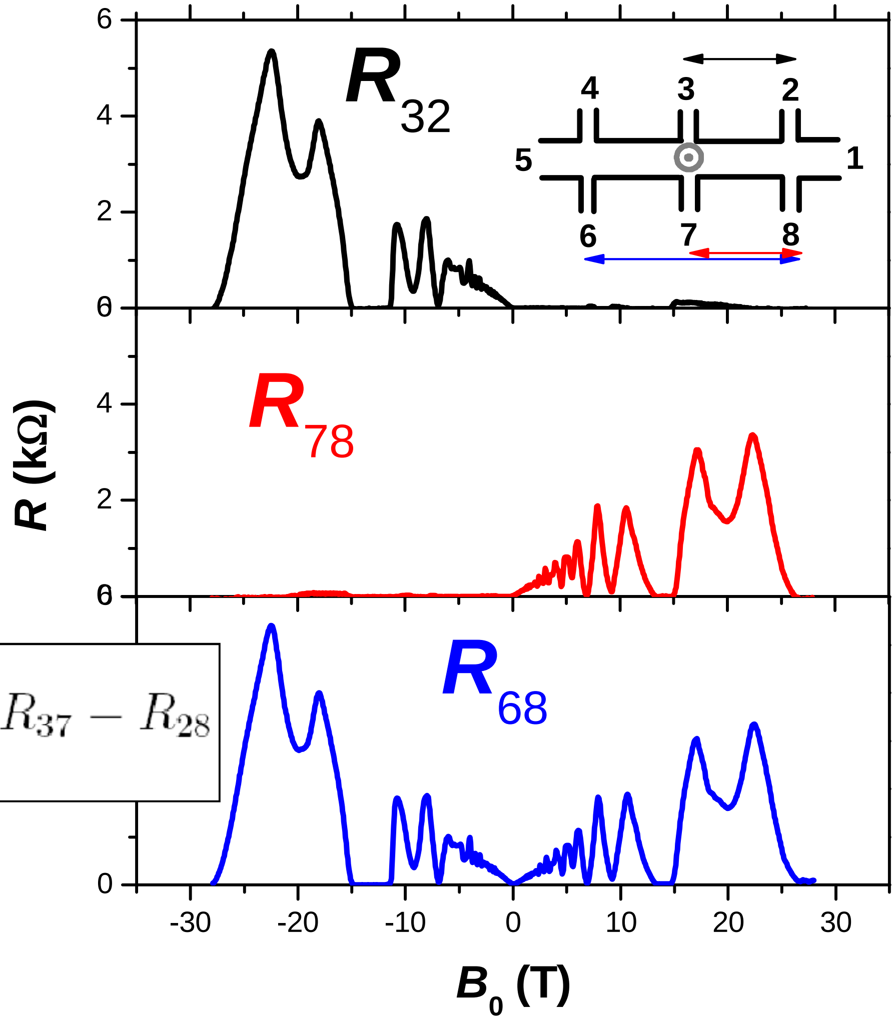
1D Landau states





Landauer Büttiker

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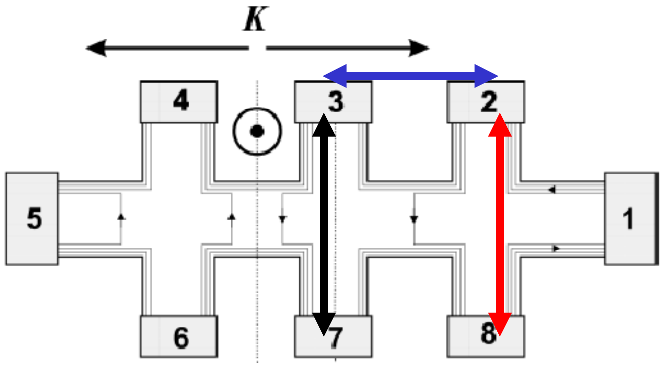
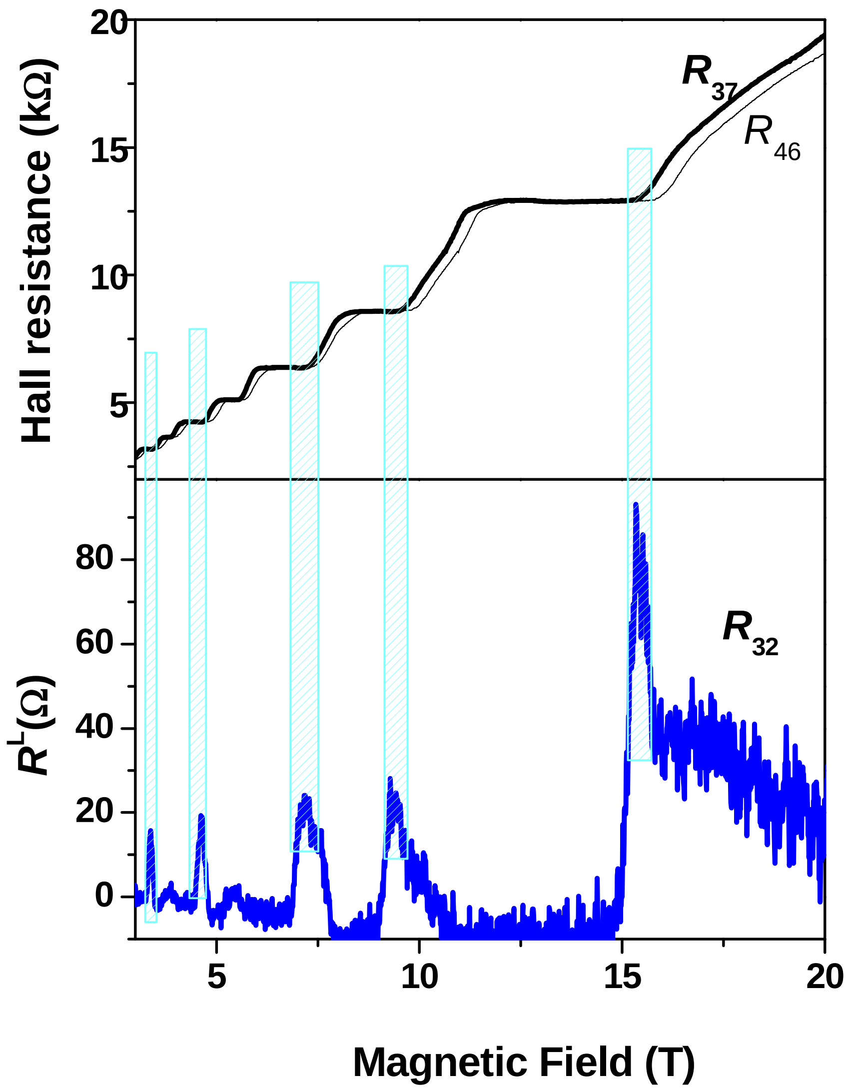
Quantum Hall effect





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$$R_{32} = \frac{\mu_3 - \mu_2}{I_{51}} = 0$$

?

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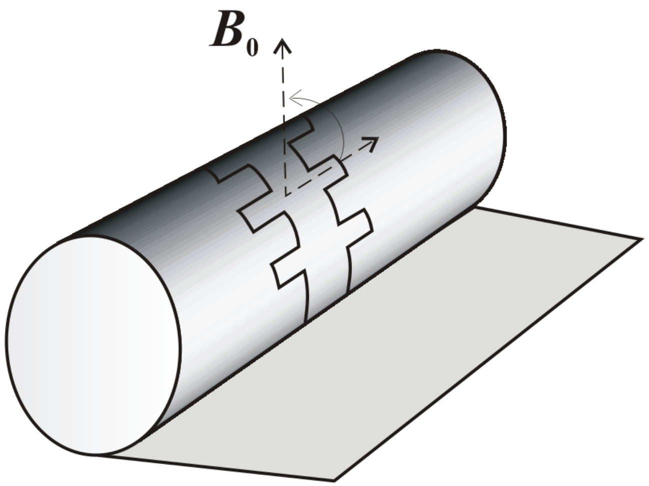
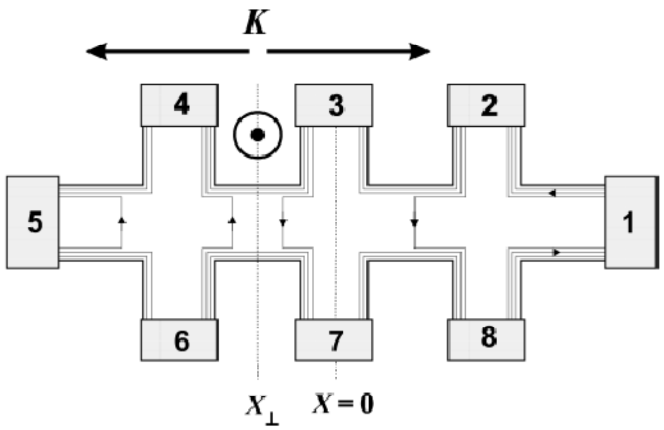
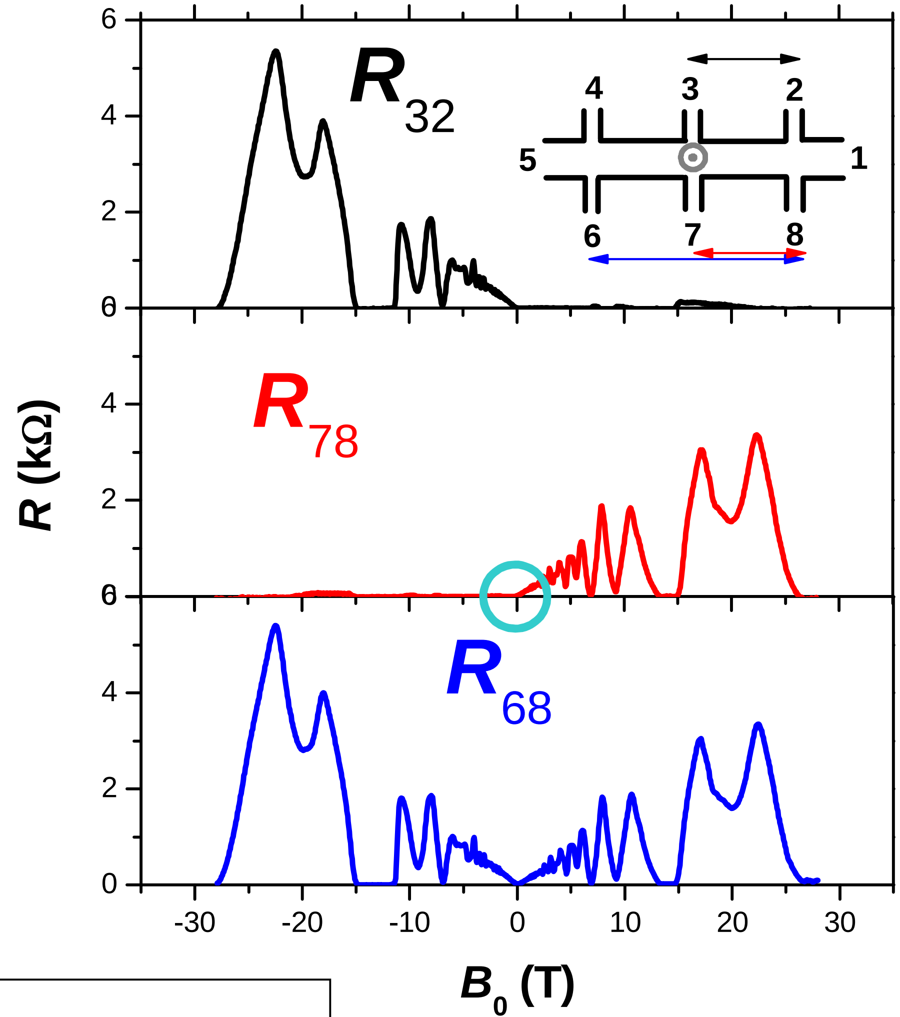
1D Landau states







Landauer Büttiker

$$R_{32} = \frac{\mu_3 - \mu_2}{I_{51}} = 0$$

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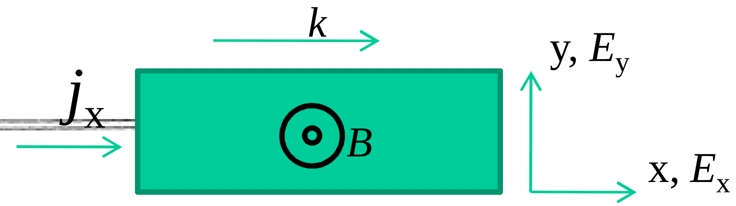
JETP Letters, Vol. 72, No. 10, 2000, pp. 503–505. From Pis'ma v Zhurnal Éksperimental'noĭ i Teoreticheskoĭ Fiziki, Vol. 72, No. 10, 2000, pp. 723–726.



Some Exact Solutions for the Classical Hall Effect in an Inhomogeneous Magnetic Field[1]

A. V. Chaplik

Institute of Semiconductor Physics, Siberian Division, Russian Academy of Sciences, pr. Akademika Lavrent'eva 13, Novosibirsk, 630090 Russia

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Received October 4, 2000

The classical Hall effect in inhomogeneous systems is considered for the case of one-dimensional inhomogeneity. For a certain geometry of the problem and for the magnetic field linearly depending on the coordinate, the distribution of current density corresponds to the skin-effect. © 2000 MAIK "Nauka/Interperiodica".

PACS numbers: 72.15.Gd

$$j_x = \sigma_{xx}E_x + \sigma_{xy}E_y \qquad (1)$$
$$j_y = -\sigma_{xy}E_x + \sigma_{xx}E_y$$

$$j_y = 0, \quad E_y = -\frac{\sigma_{xy}}{\sigma_{xx}}E_x = -\mu B E_x \qquad (2)$$

$div\,\mathbf{j} = 0$

$$div\,\mathbf{j} = \frac{\delta j_x}{\delta x} = (\sigma_{xx} + \frac{\sigma_{xy}^2}{\sigma_{xx}})\frac{\delta E_x}{\delta x} = -(\sigma_{xx} + \frac{\sigma_{xy}^2}{\sigma_{xx}})\frac{\delta^2 \Phi}{\delta x^2} = 0$$

General solution $\Phi = C_1(y)x + C_0(y)$ where $\mathbf{E} = -\nabla\Phi(x, y)$

From (2): $$\frac{\delta\Phi}{\delta y} = -\mu B\frac{\delta\Phi}{\delta x} \qquad \frac{\delta C_1(y)}{\delta y}x + \frac{\delta C_0(y)}{\delta y} = -\mu B C_1(y)$$

Field gradient along the current : $B(x) = B_0 kx \qquad C_1 = Ce^{-\mu B_0 ky}, \quad C_0 = 0$

$$E_y = C\mu B_0 kye^{-\mu B_0 ky}, \quad E_x = -Ce^{-\mu B_0 ky} \quad j_x = -\sigma_{xx}Ce^{-\mu B_0 ky}$$

Asymmetry:

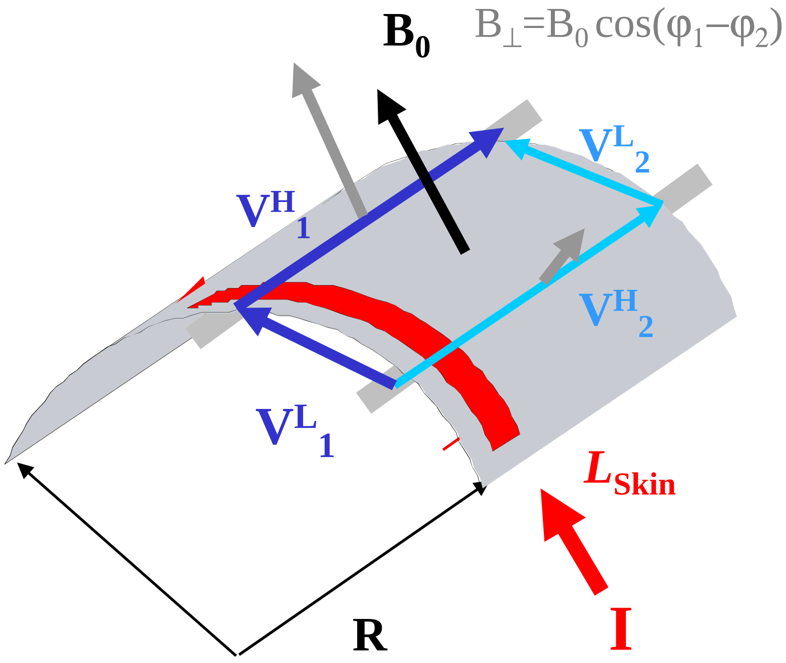
$V^H{}_2 \neq V^H{}_1 \rightarrow V^L{}_1 - V^L{}_2 = V^H{}_1 - V^H{}_2$

'Skin channel for the current:

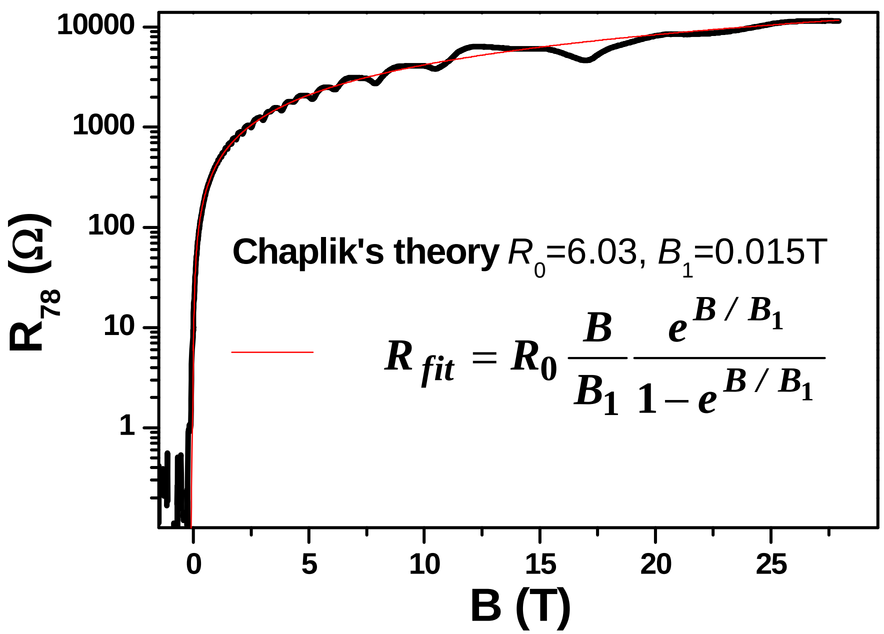
$$L_{Skin} = \frac{1}{\mu \nabla B},$$

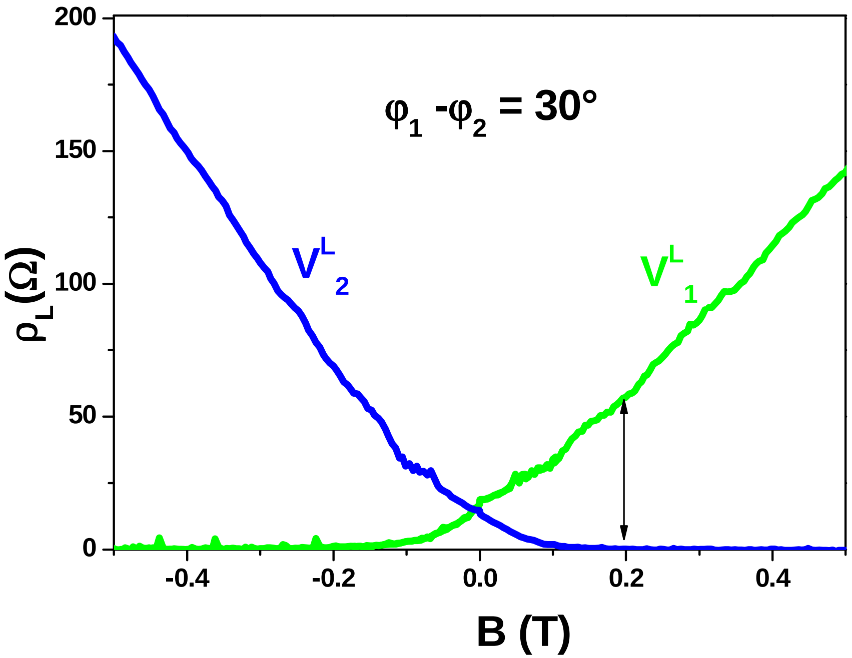
A.V. Chaplik, JETP Lett. (2000)

($L_{Skin} \cong$ 1 µm at B=0,2T)



'Static Skin effect'





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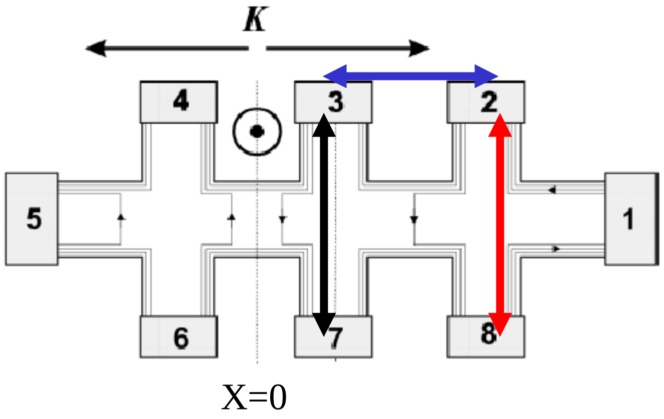
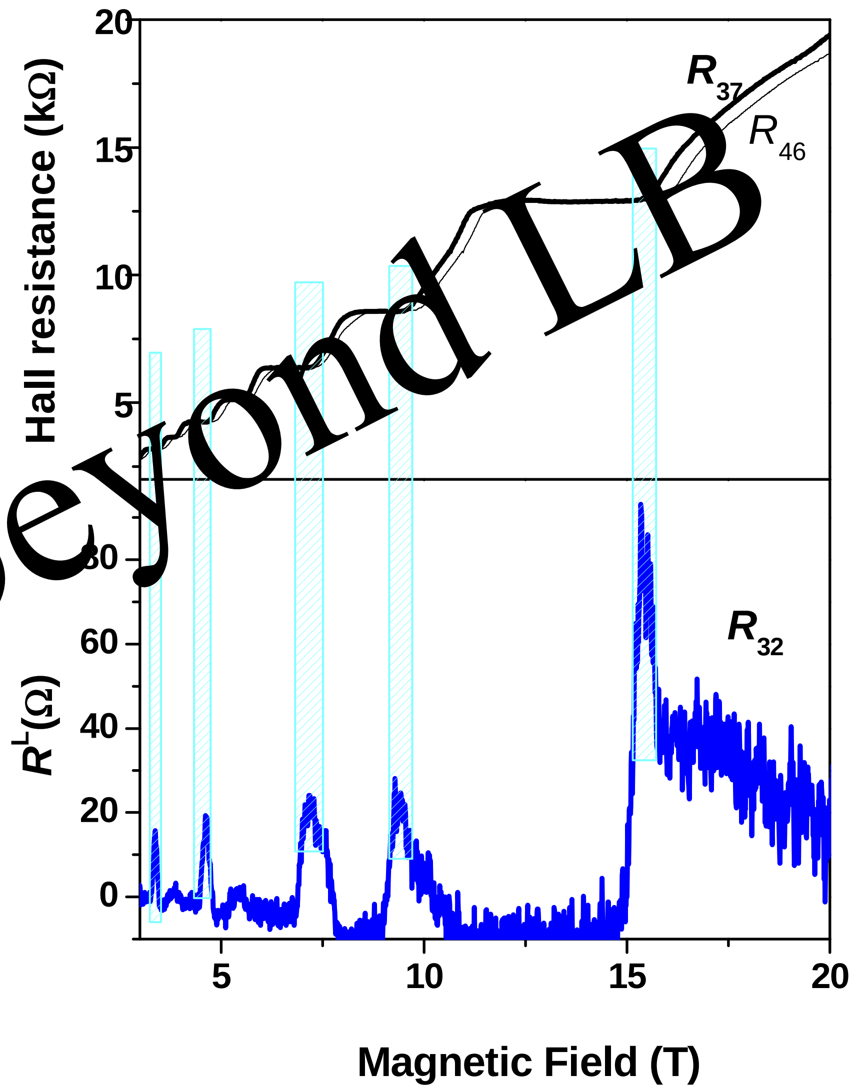
Quantum Hall effect





Landauer-Büttiker approach

- ***One-dimensional Landau states***
- ***Bulk → insulator***

$$R_{43} = \frac{\mu_4 - \mu_3}{I_{51}} = \frac{h}{2e^2}\left(\frac{1}{\nu_0} - \frac{1}{\nu_{(46)}}\right) = R_0 - R_{46}$$

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$$R_{32} = \frac{\mu_3 - \mu_2}{I_{51}} = 0$$

Solution beyond LB

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Self-consistent calculation of the density and current distribution

- **Total electrostatic potential energy** $V_{tot}(x,y) = V_{bg}(x,y) + V_{ext}(x,y) + V_H(x,y)$

$V_{bg}(x,y)$ *background potential generated by the donors*

$V_{ext}(x,y)$ *external potential from the gates (which will be used to simulate the filling factor gradient)*

$V_H(x,y)$ *Hartree potential to describe the mutual electron-electron interaction*

- **Electron density** $n_{el}(x,y) = \int D(E,x,y) f(E + V_{tot}(x,y) - \mu^*) dE$

$D(E,x,y)$ *(local) density of states*

$f(E) = 1/[\exp(E/k_b T)) + 1]$ *Fermi function*

μ^* *electro*chemical potential

- **Hartree potential explicitly depends on the electron density via**

$$V_H(x,y) = \frac{2e}{\kappa} \int_A K(x,y,x',y') n_{el}(x',y') dx' dy'$$

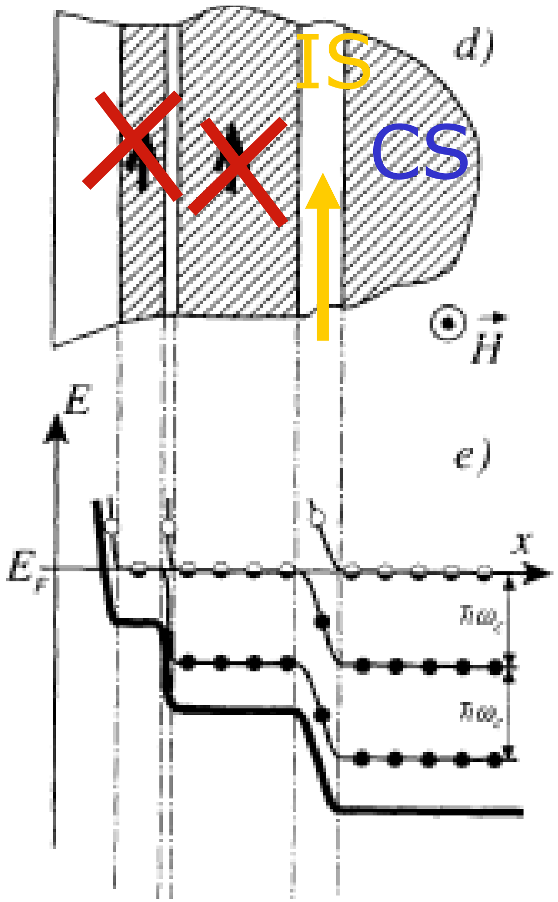
$K(x,y,x',y')$ *solution of the 2D Poisson* equation *satisfying the periodic boundary conditions,*

Screening theory in the QHE

R. Gerhardts, K. Güven A. Siddiki … (2003-2007)

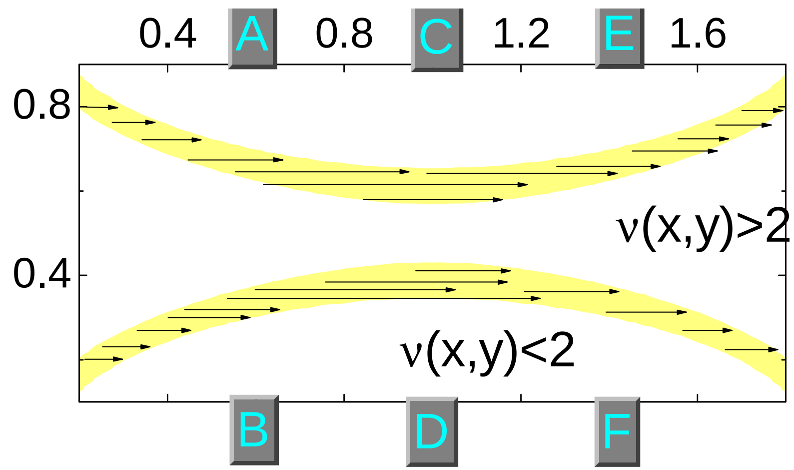
Classical and quantum mechanical drift velocities, $\vec{v}_D = c\frac{\vec{E}\times\vec{B}}{B^2}$ $v_y = -\frac{eE_x}{m\omega_c}$

→ **Current flows along the Incompressible Stripes**



- Wave Guide on a Cylinder Surface

- Self consistent calculation of carrier and current distribution



-quantized conductance

at average filling factors $\nu > 2$

Calculations A. Siddiki

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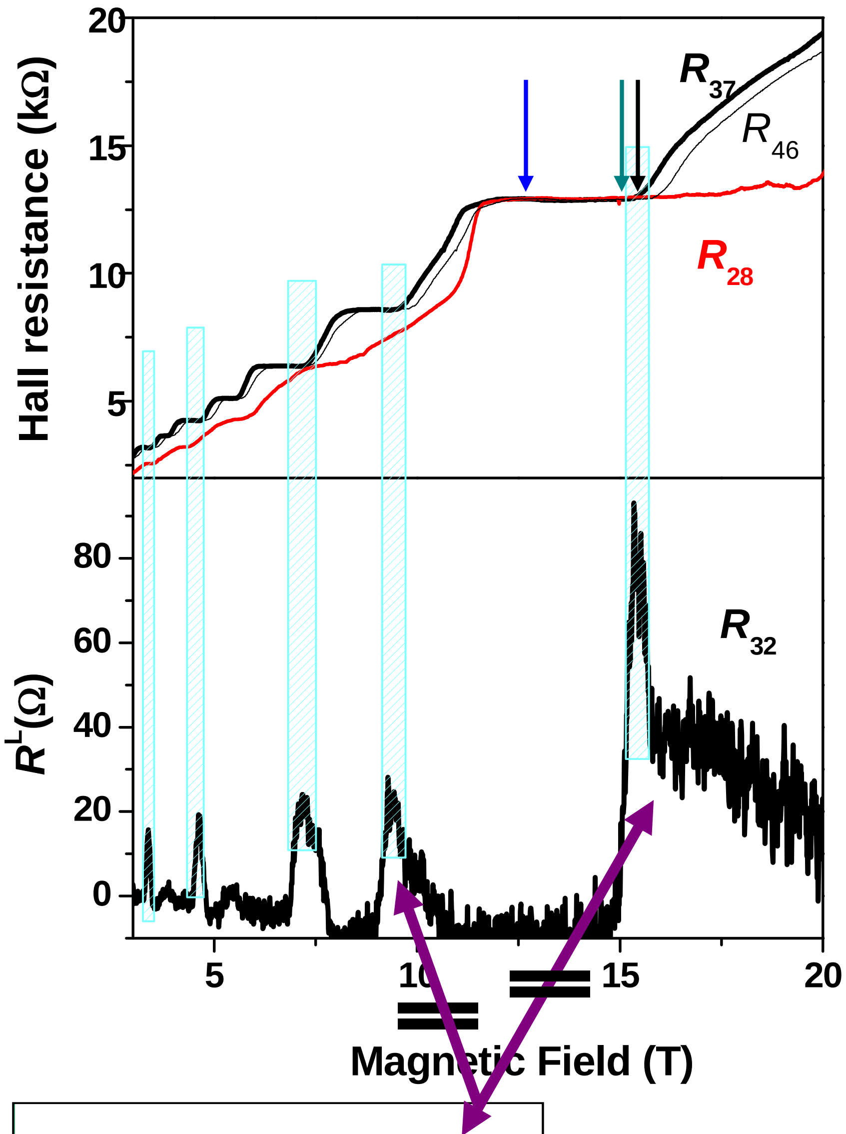
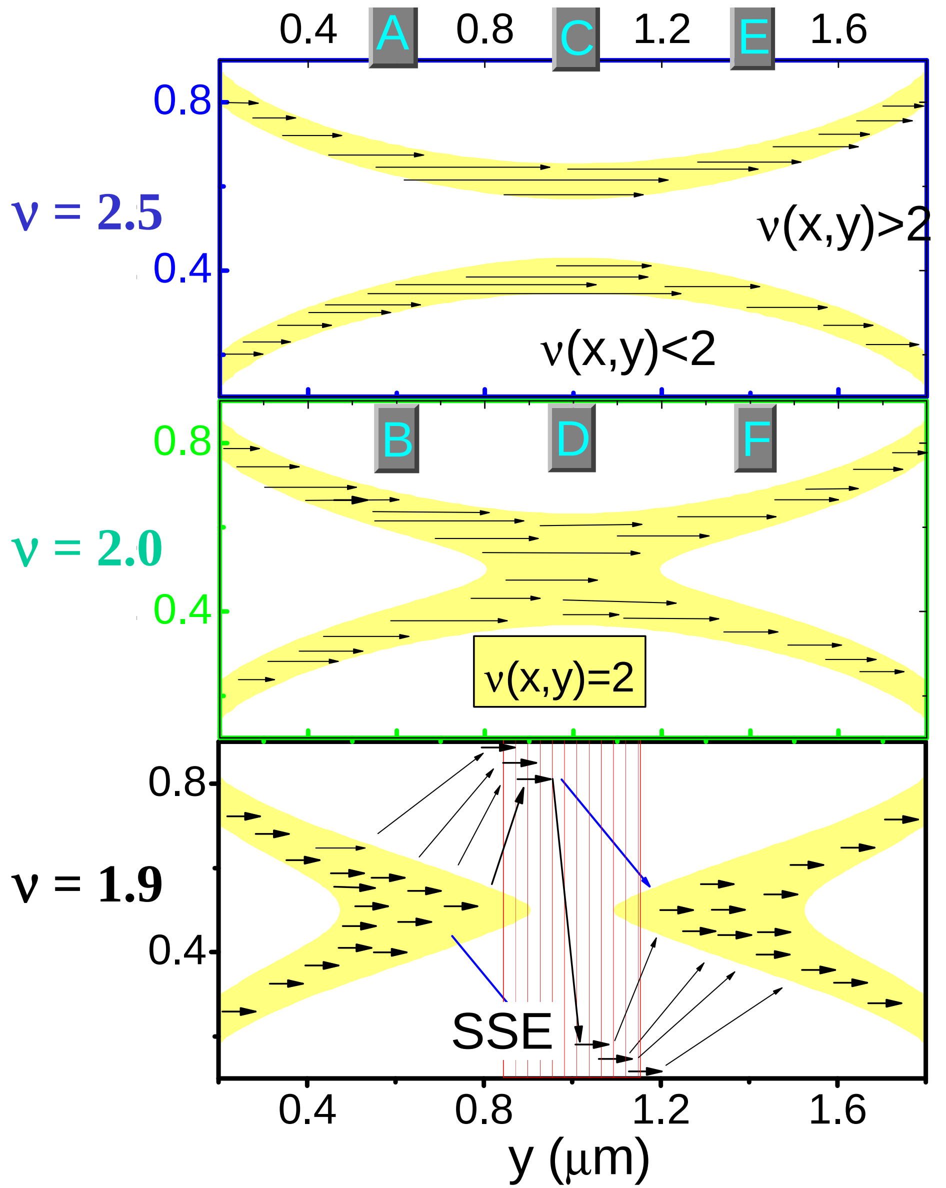
Quantum Hall effect II

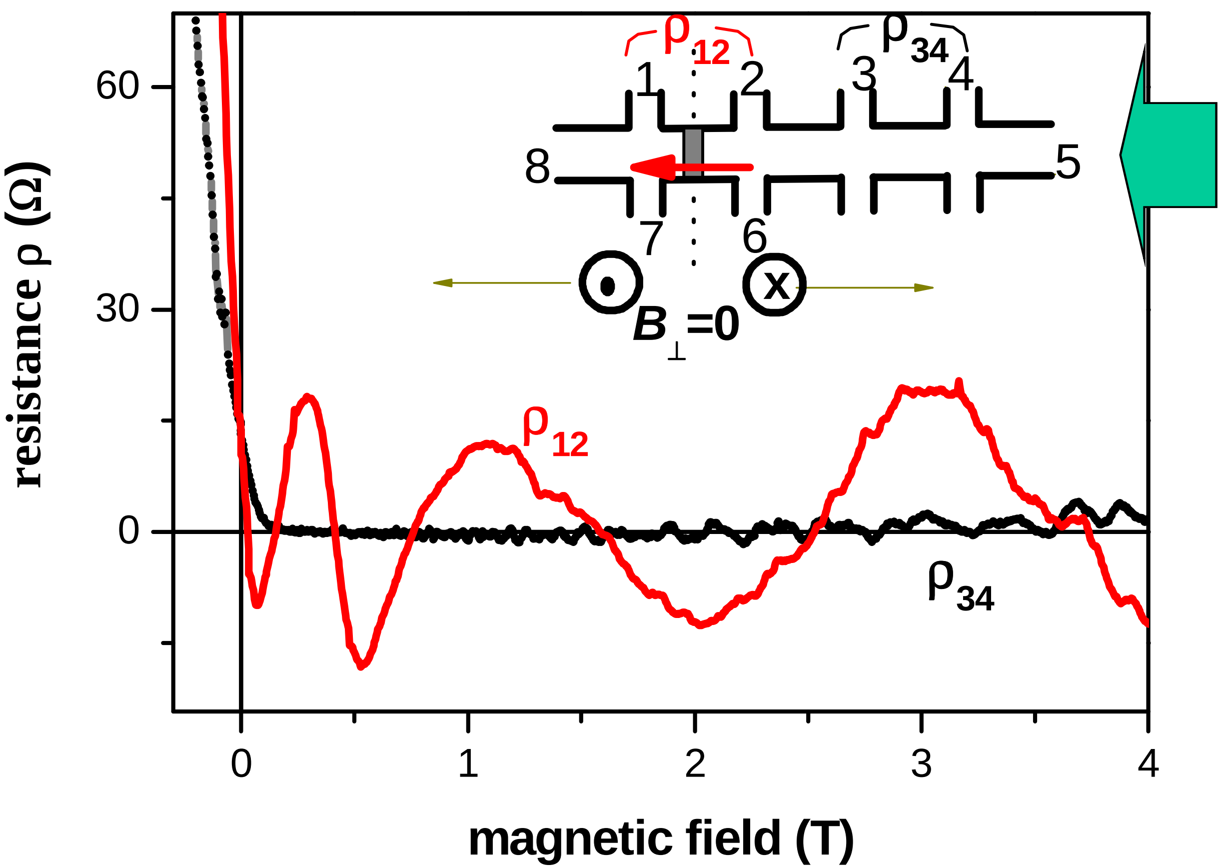


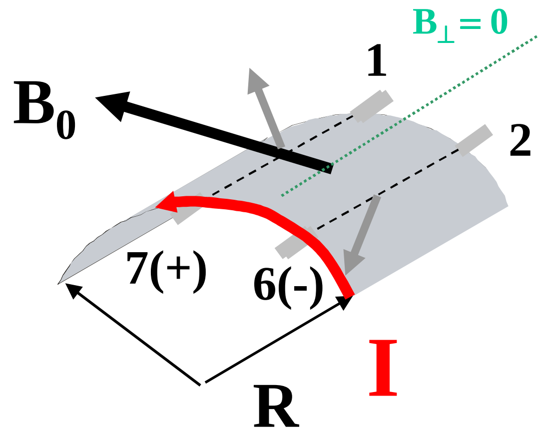


$$R_{32} = \frac{\mu_3 - \mu_2}{I_{51}} = 0$$

K.-J. Friedland et al., Phys. Rev.B 2009.

Resistance oscillations for tangentially directed magnetic fields





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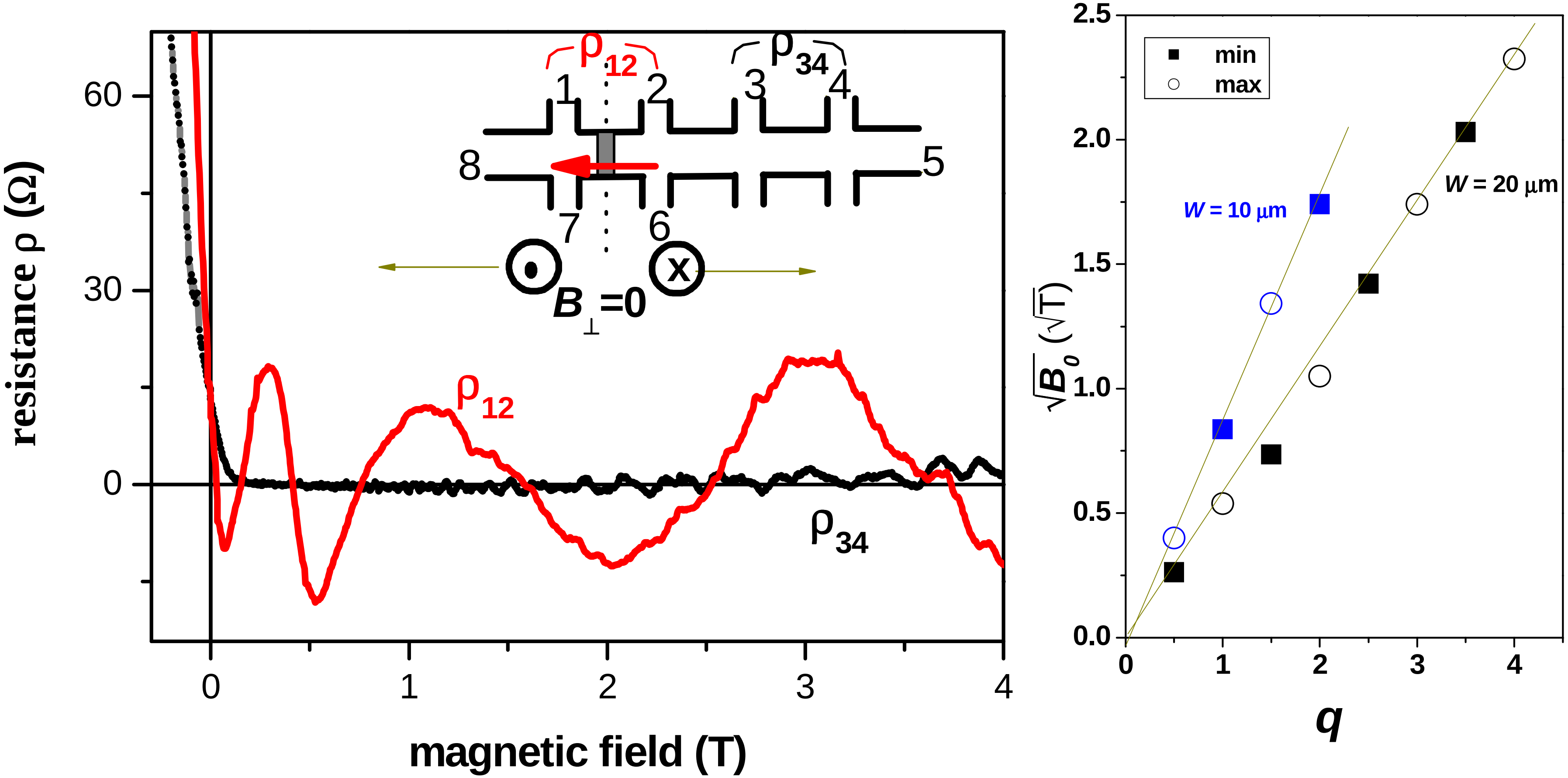
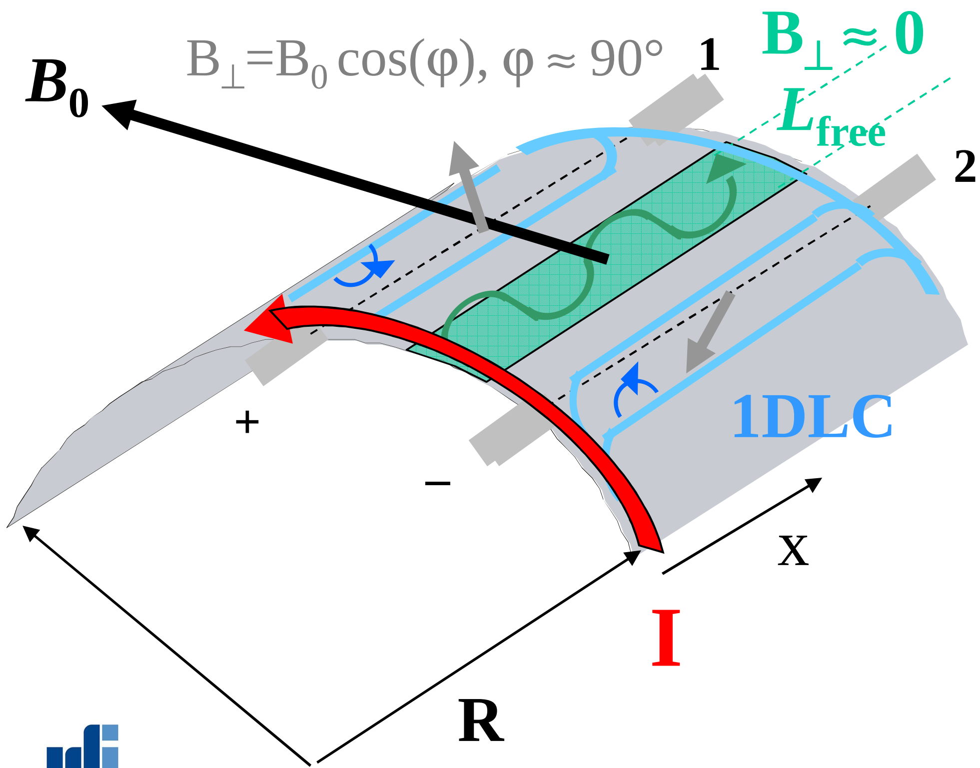
Resistance oscillations
for tangentially directed magnetic fields



Oscillations with 'free electron states'

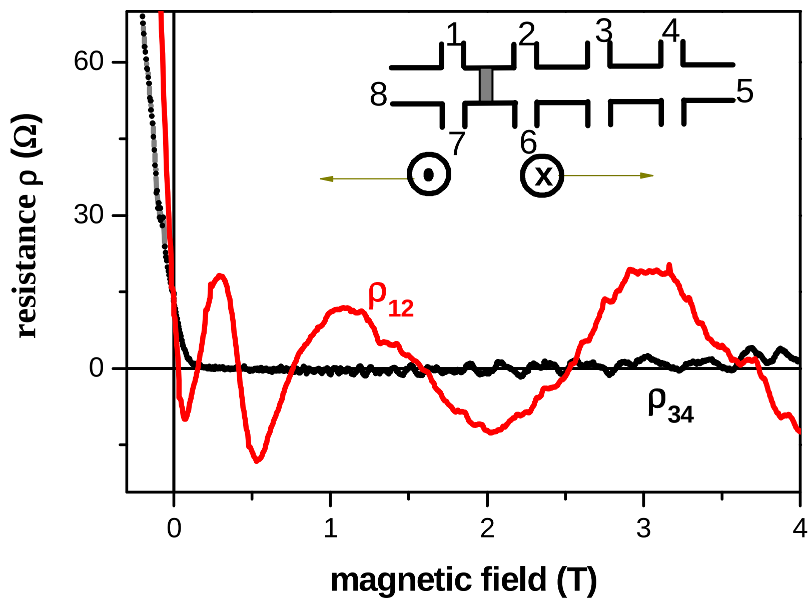


Calculation of the effective potential:

-Free electron states , classically
'Snake-like orbits - SLO
- stripe of width L_{free}

$$E_F = \frac{1}{2m}\left[\hbar k_X - \frac{e\nabla B}{2}\left(\frac{L_{free}}{2}\right)^2\right]^2$$

J. E. Müller, Phys. Rev. Lett. (1992)



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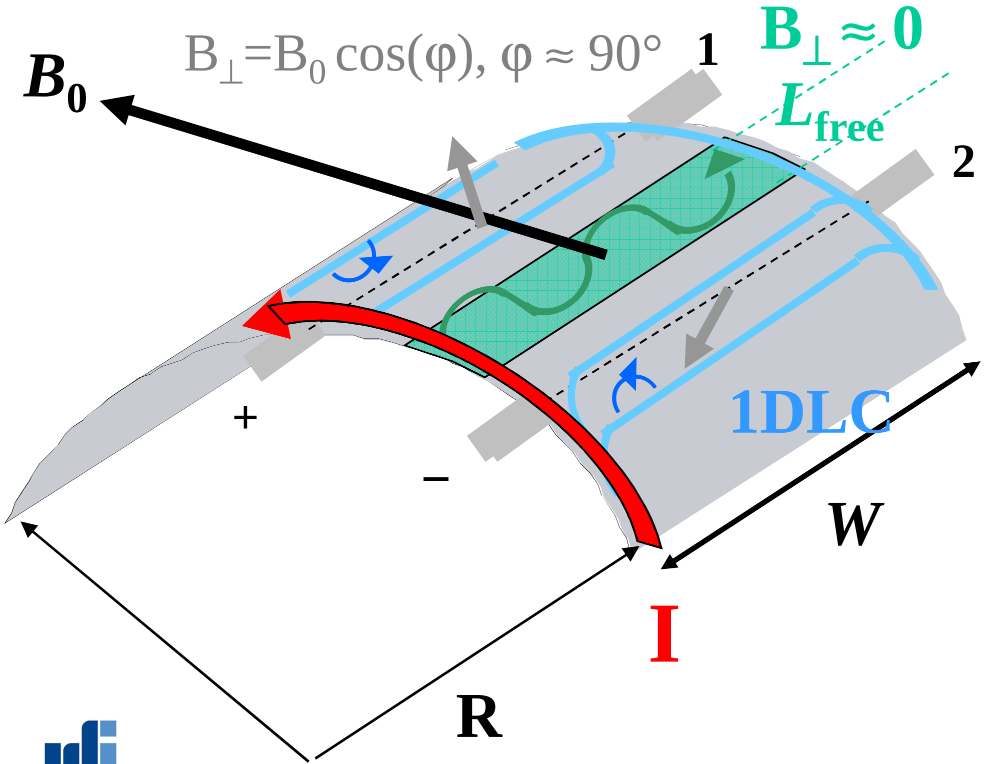
Oscillations with 'free electron states'



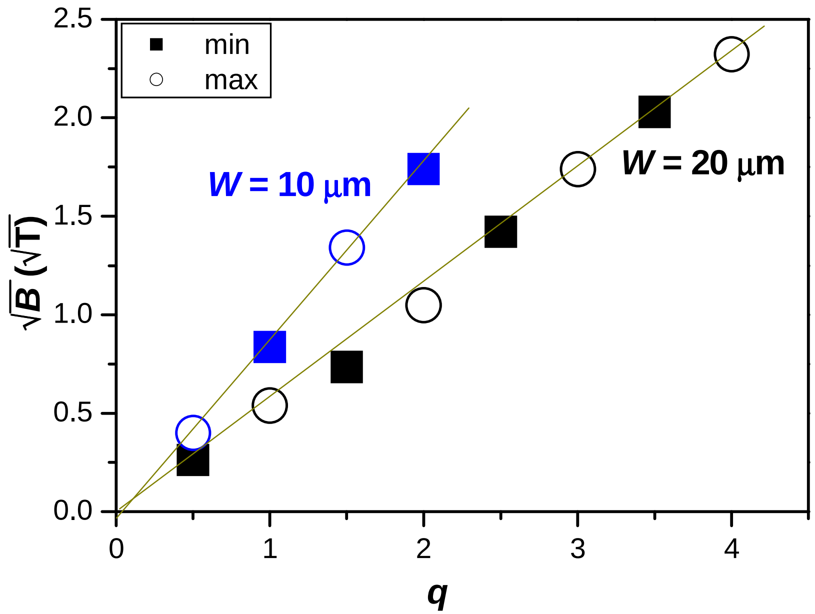
Commensurability

$$L_{free}^2 = 2\sqrt{2}\,\frac{\hbar(k_F - k_X)}{eB_0}R$$

SLO period

← →wave-guide width *W*

$$\frac{W}{L_{free}} = C \times \sqrt{B_0} = 2\pi q,$$



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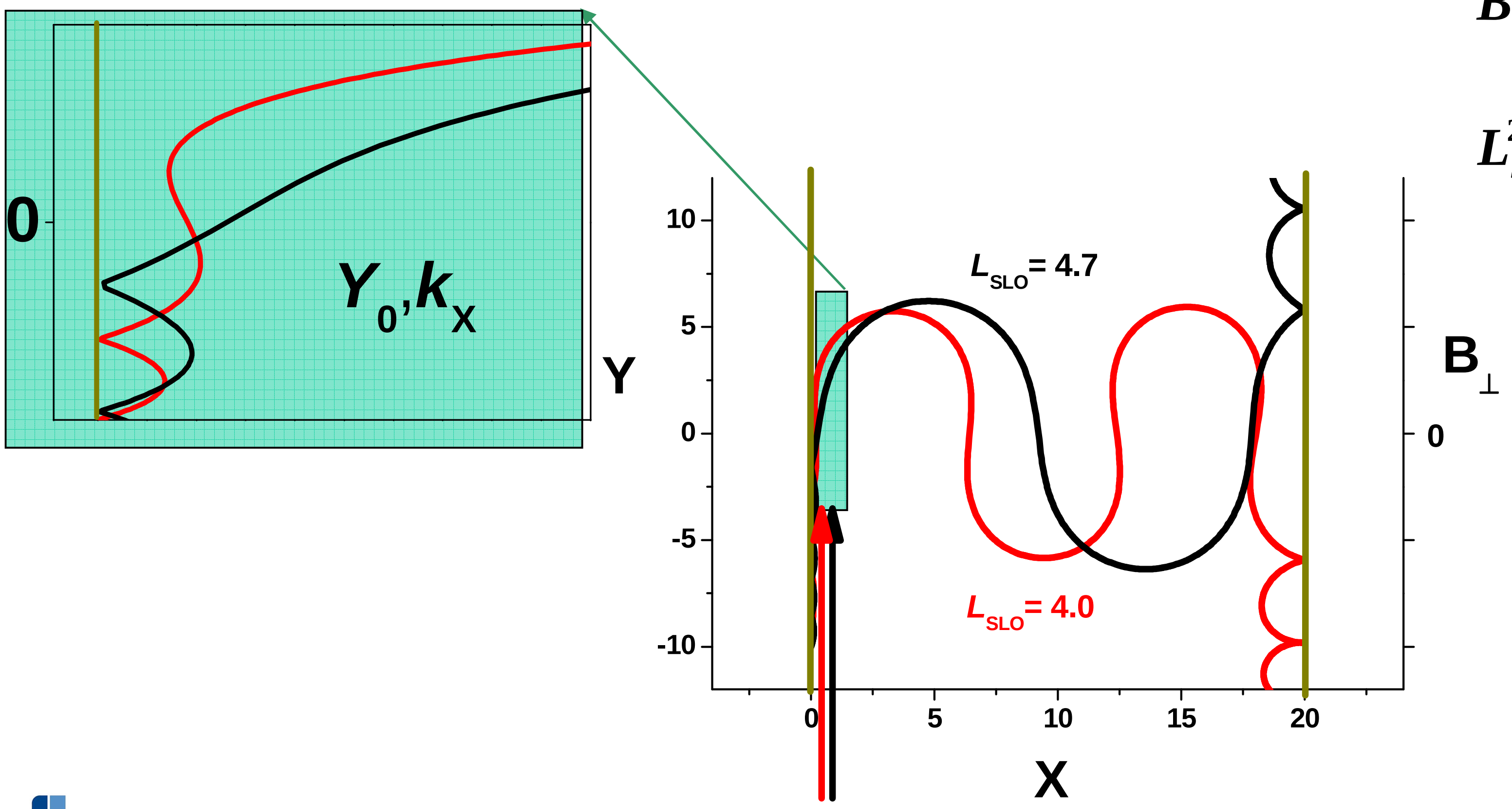
Calculation – adiabatic skipping orbits
(‘Snake’-like orbits)

Local cyclotron radius: $$R_{cycl}(Y, B_0) = \frac{\hbar k_F}{eB_\perp} = \frac{L_{SLO}^2}{Y}$$

$$B_\perp = B_0 \frac{Y}{R}$$

$$L_{SLO}^2 = R\frac{\hbar k_F}{eB_0}$$



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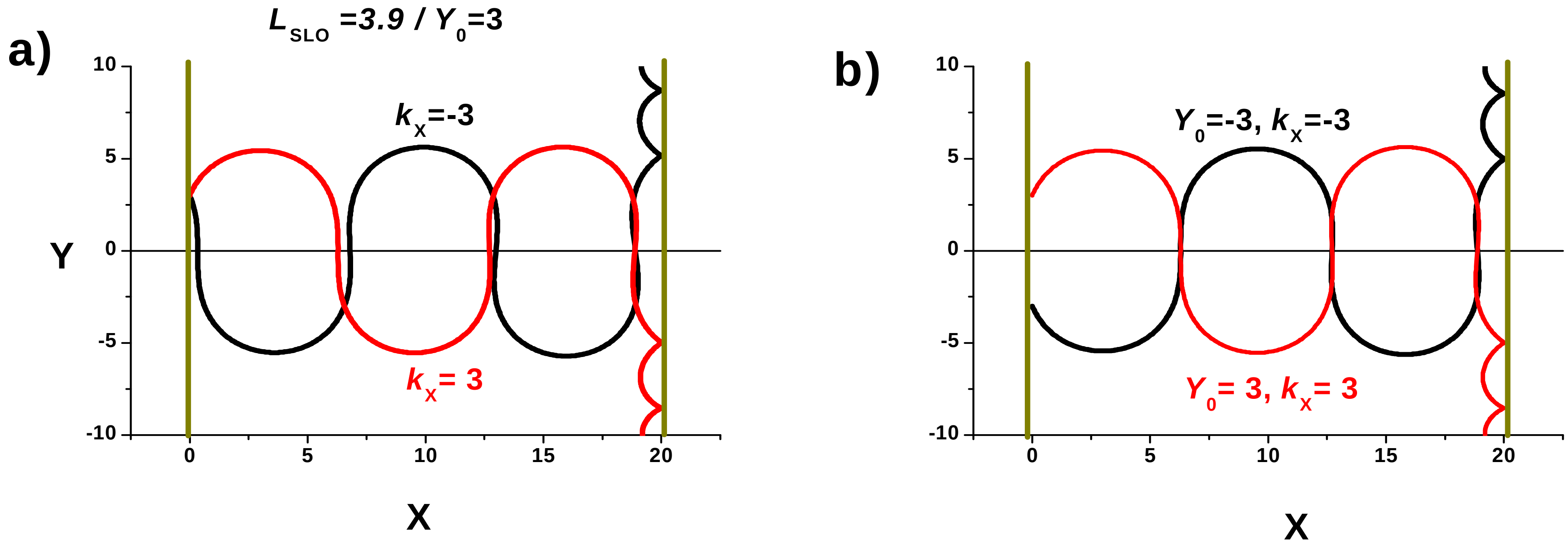
Calculation – rough boundary scattering



Compensation of skipping orbits with statistical scattering at the rough wave-guide boundary

Need a preferential momentum directions → k_y k_x pre-selection

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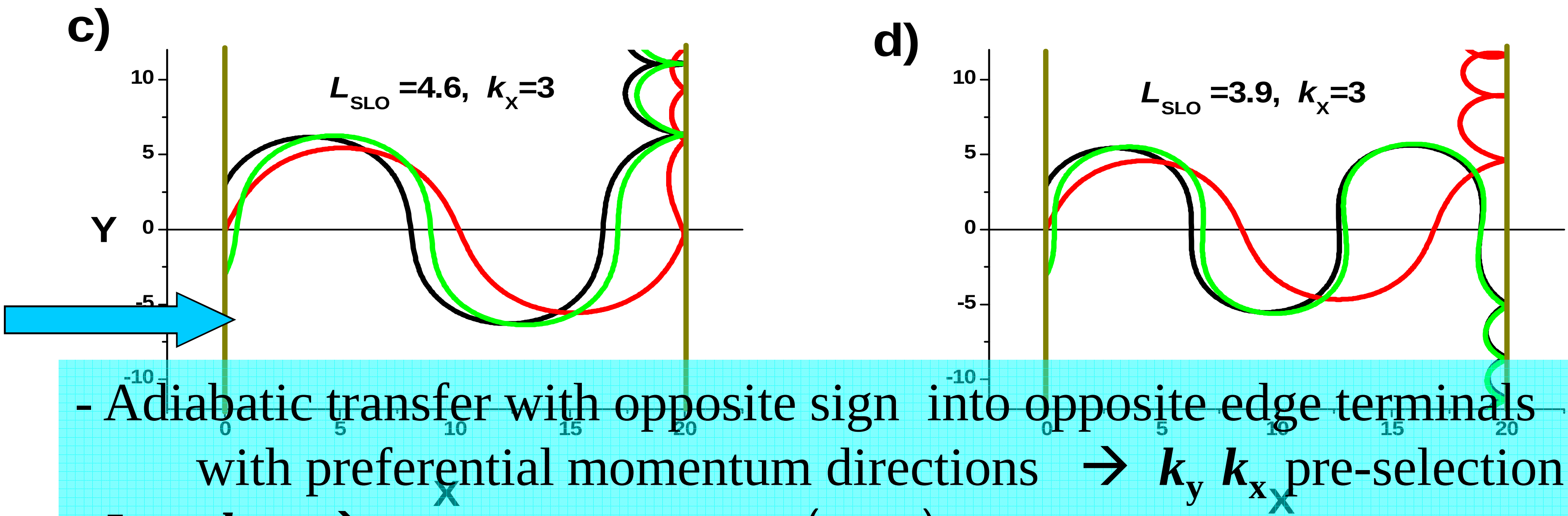
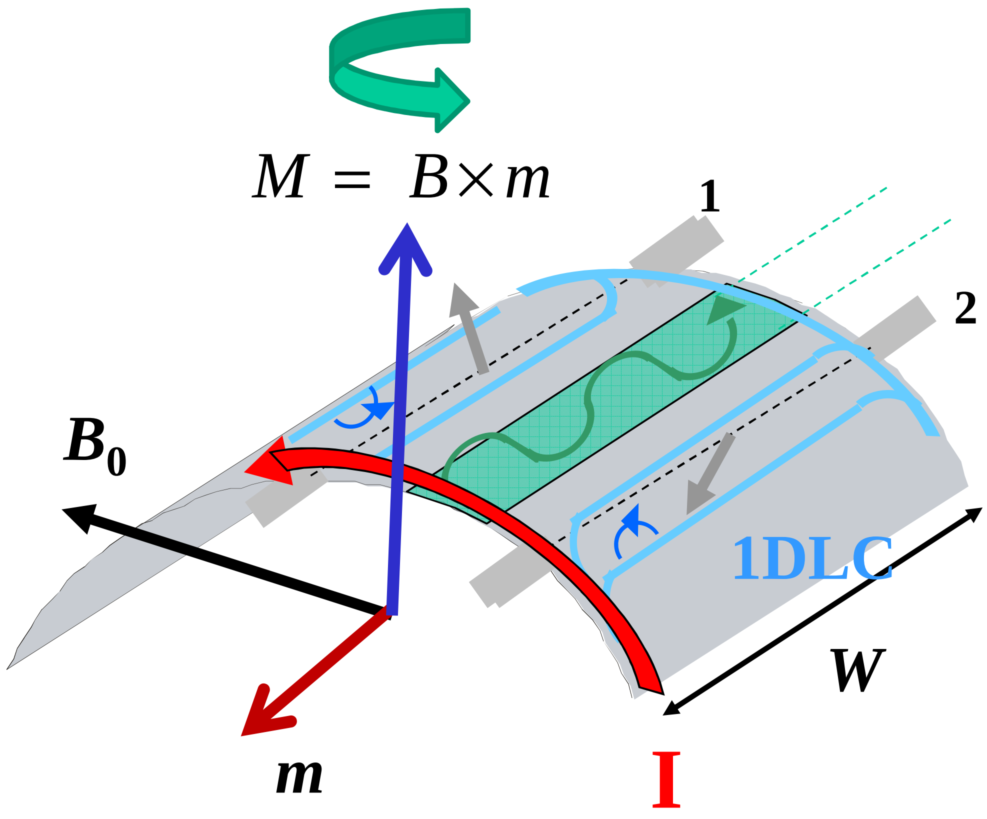
Calculation - k_y k_x pre-selection



- Adiabatic transfer with opposite sign into opposite edge terminals with preferential momentum directions → k_y k_x pre-selection
- Low k_X →

$$L_{SLO} \cong L_{free} / \left(2\sqrt{2}\right)$$

Transverse force due to torque from induced magnetic moment



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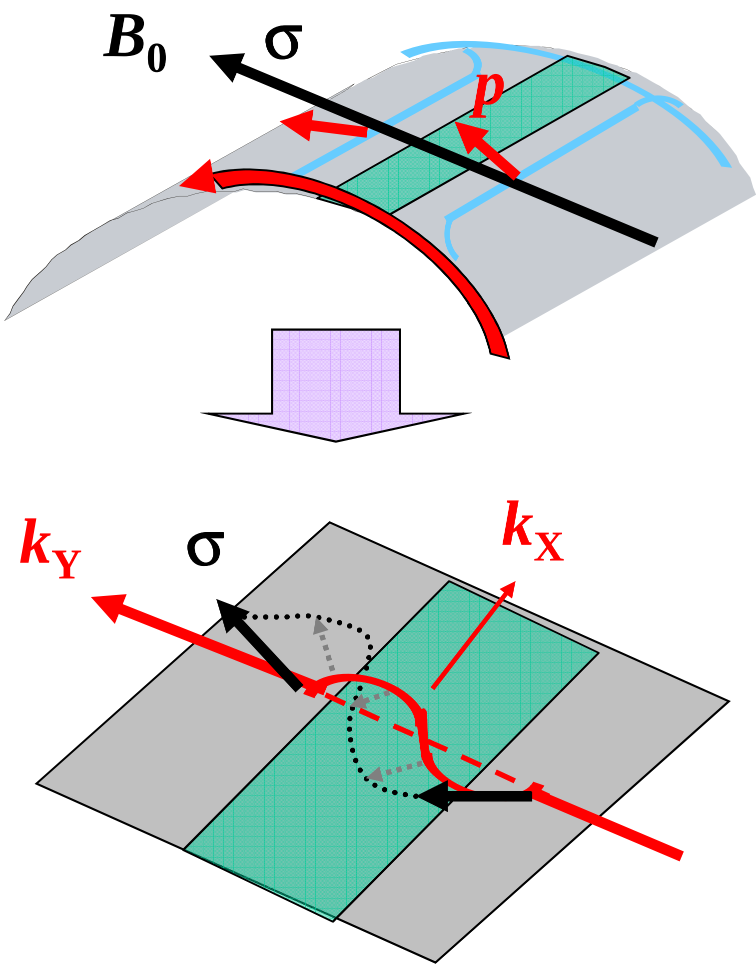
‘zitterbewegung’ of a 1D ‘Skin’-channel



$$H_{so} \propto \left[\vec{k}\ \vec{\sigma}\right]$$

‘zitterbewegung’ due to spin precession

J. Schliemann Phys.Rev.B (2006)

!! **All electrons along L_{free} have the chance to acquire an orthogonal momentum p_x**

Spin de-phasing length

$$L_{so} = \frac{\hbar^2}{2m\beta} \cong 2.6\mu m$$

using Dresselhaus term β for 13 nm wide GaAs QW

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K.-J. Friedland et al., phys. stat. sol., 2008

Summary

QHE :

- **Transport theory beyond Landauer Büttiker**

  - Sequential transport along incompressible/compressible regions

  - Screening theory in nontrivial geometries to model lateral position of incompressible stripes → quantized in R_l and R_H

Oscillations in tangentially oriented fields:

- **Commensurability of free-electron length L_{free} with the wave guide width W**

  → 'Snake'-like trajectories compensate by scattering at rough boundaries

  - Torque from induced magnetic moment and/or

  - Spin precession in a one-dimensional skin-channel allow to acquire the necessary orthogonal momentum k_X

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