Brandenburgische Technische Universität Cottbus

Fano effect in open quantum systems

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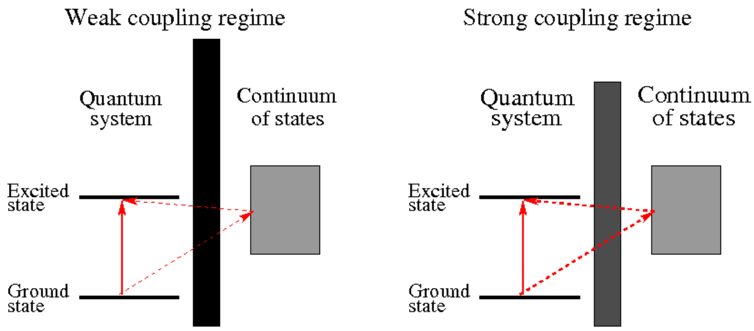
University of Bucharest

1. Fano effect

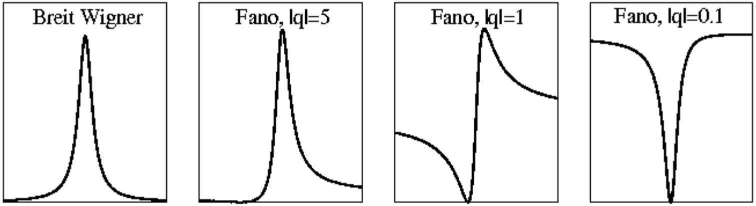


Fingerprints of the Fano effect

Fano function:

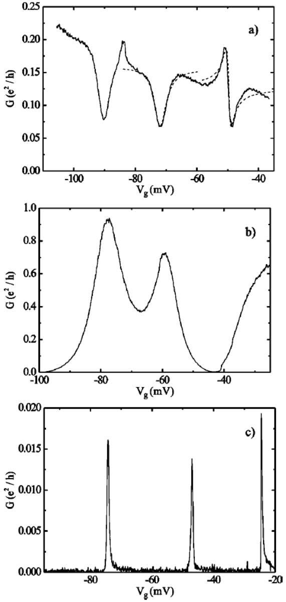
$$f(e) \sim \left| \frac{1}{e+i} + \frac{1}{q} \right|^2$$

q = Fano asymmetry parameter



2. Motivation

Conductance through a quantum dot



a) Fano-regime
strong coupling
between dot and
contacts

b) Kondo-regime
intermediate coupling

c) Coulomb blockade
weak coupling

J. Göres et al, Phys. Rev. B, 2188 (2000)

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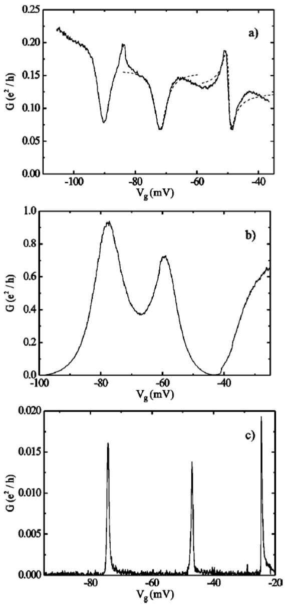

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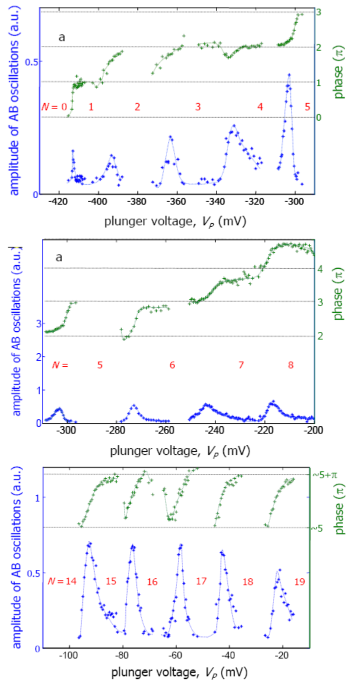
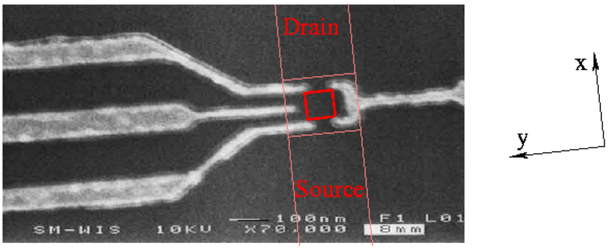
Transmission phases



M. Avinun-Kalish et al, Nature 436, 529 (2005)

3. Scattering Problem

Scattering potential



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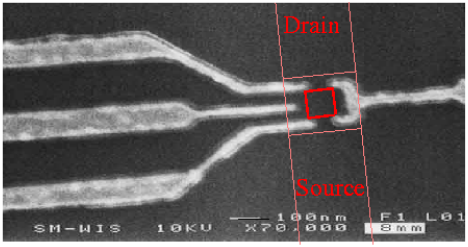
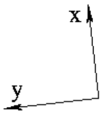
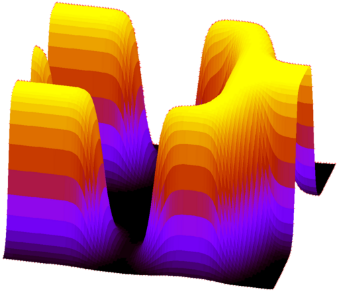




Quantum dot



Afif Siddiki, unpublished

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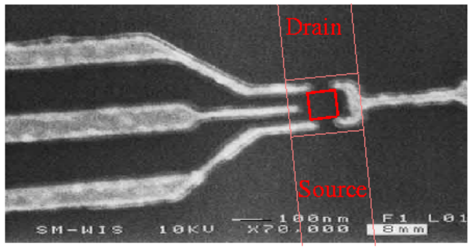
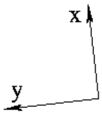
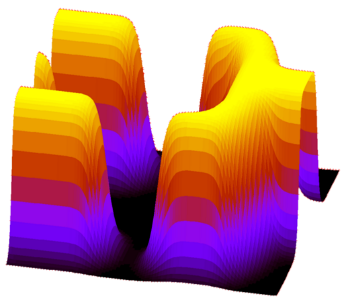


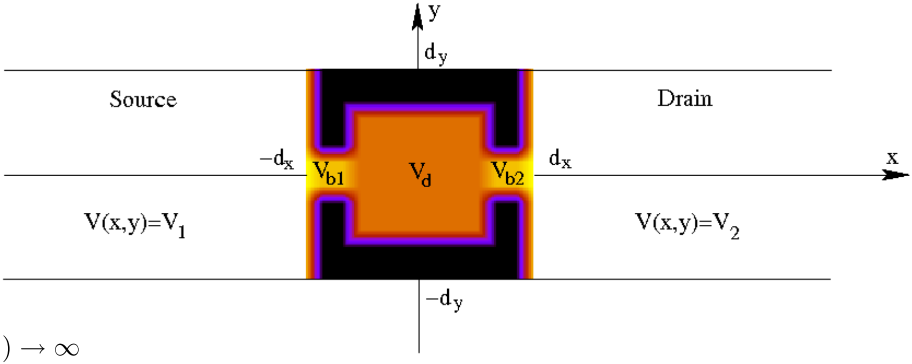


Source	Quantum dot	Drain
$x < -d_x$		$x > d_x$
$V(x, \mid y \mid < d_y) = V_1$		$V(x, \mid y \mid < d_y) = V_2$

Afif Siddiki, unpublished

3. Scattering Problem

Scattering potential



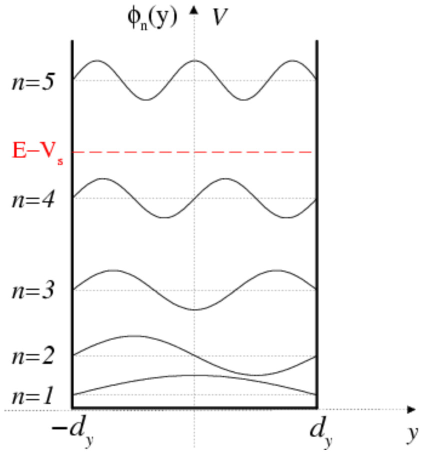
$V(x, \mid y \mid > d_y) \to \infty$

2D-Schrödinger Equation

$$\left[-\frac{\hbar^2}{2m^*}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)+V(x,y)\right]\Psi(x,y)=E\Psi(x,y)$$

3. Scattering Problem

2D-Schrödinger Equation in Contacts



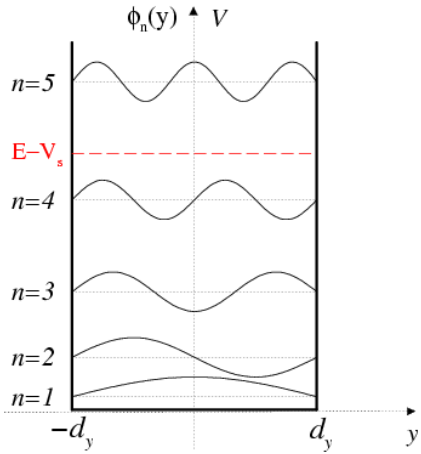
$$\phi_n(y) = \frac{1}{\sqrt{d_y}} \sin\left[\frac{n\pi}{2d_y}(y + d_y)\right]$$

$$E_{yn} = \frac{\hbar^2}{2m^*}\left(\frac{\pi}{2d_y}\right)^2 n^2, \quad n \geq 1$$

Every n defines a scattering energy channel.

3. Scattering Problem

2D-Schrödinger Equation in Contacts



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Every n defines a scattering energy channel.

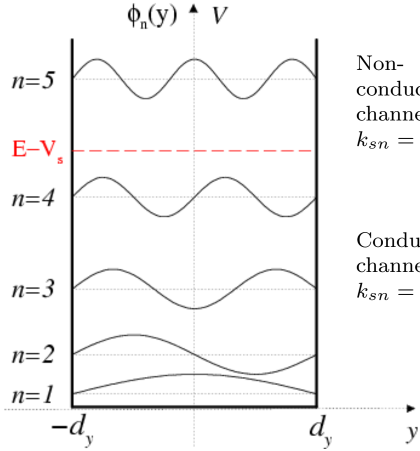
Scattering States in Contacts

$$\Psi(E; x, y) \sim e^{\pm i k_{sn} x} \phi_n(y)$$

$$k_{sn} = \sqrt{\frac{2m^*}{\hbar^2}(E - E_{yn} - V_s)}, \quad s = 1, 2$$

3. Scattering Problem

2D-Schrödinger Equation in Contacts



Non-conducting channels $k_{sn} = i\|k_{sn}\|$

Conducting channels $k_{sn} = \|k_{sn}\|$

$$\phi_n(y) = \frac{1}{\sqrt{d_y}} \sin\left[\frac{n\pi}{2d_y}(y + d_y)\right]$$

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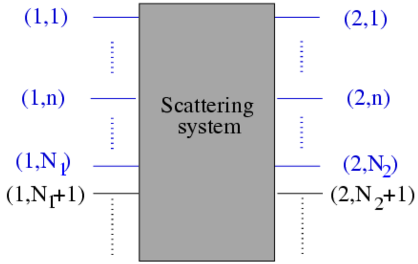
3. Scattering Problem

Conducting channels
$n \leq N_1$



Conducting channels
$n \leq N_2$

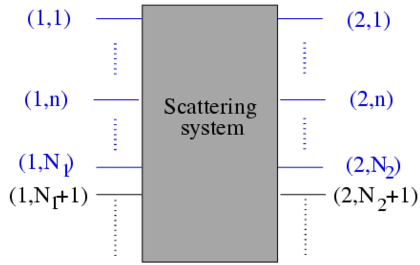
Nonconducting channels
$n > N_1$

Nonconducting channels
$n > N_2$

3. Scattering Problem

Conducting channels
$n \le N_1$

Nonconducting channels
$n > N_1$



Conducting channels
$n \le N_2$

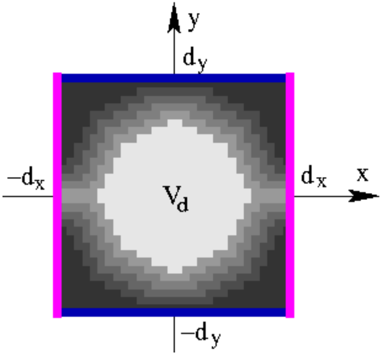
Nonconducting channels
$n > N_2$

Scattering Functions

$$\Psi_n^{(1)}(E;x,y) = \frac{\theta(N_1-n)}{\sqrt{2\pi}} \begin{cases} e^{ik_{1n}(x+d_x)}\phi_n(y) \\ + \sum_{n'=1}^{\infty} \hat{S}^T_{1n,1n'}(E) e^{-ik_{1n'}(x+d_x)}\phi_{n'}(y), & x \le -d_x \\ \\ \sum_{l=1}^{\infty} a_l^{sn}(E)\, \chi_l(x,y), & |\, x \,| \le d_x \\ \\ \sum_{n'=1}^{\infty} \hat{S}^T_{1n,2n'}(E) e^{i\, k_{2n'}\, (x-d_x)}\phi_{n'}(y), & x \ge d_x \end{cases}$$

3. Scattering Problem

Wigner Eisenbud Problem = Eigenvalue problem of the quantum dot artificially closed by Neumann boundary conditions



$$\left[-\frac{\hbar^2}{2m^*}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)+V(x,y)\right]\chi_l(x,y)=E_l\chi_l(x,y)$$

$$\frac{\partial \chi_l}{\partial x}(x=\pm d_x, y)=0$$

$$\chi_l(x, y=\pm d_y)=0$$

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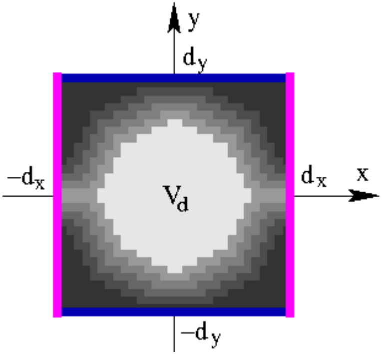

$$\left[-\frac{\hbar^2}{2m^*}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)+V(x,y)\right]\chi_l(x,y)=E_l\chi_l(x,y)$$

$$\frac{\partial \chi_l}{\partial x}(x=\pm d_x, y)=0$$

$$\chi_l(x, y=\pm d_y)=0$$

Scattering Functions inside the Scattering Region

$$\vec{\Psi}(E;x,y)=\frac{i}{\sqrt{2\pi}}\hat{\Theta}\,[\hat{1}-\hat{\mathcal{S}}^T]\,\hat{K}\,\vec{R}(x,y)$$

$$\hat{\Theta}_{sn,s'n'}=\theta(N_s-n)\delta_{ss'}\delta_{nn'}$$

$$\mathbf{K}_{sn,s'n'}=\frac{k_{sn}}{\pi/2d_x}\delta_{ss'}\delta_{nn'}$$

$$\vec{R}(x,y)=\frac{\hbar^2}{2m^*}\frac{\pi}{2d_x}\sum_{l=1}^{\infty}\frac{\vec{\chi}^{(l)}\chi_l(x,y)}{E-E_l}$$

$$\vec{\chi}^{(l)}_{sn}=\int_{-d_y}^{d_y}dy\;\chi_l[(-1)^s d_x,y]\;\Phi_n(y)$$

3. Scattering Problem

Current transmission matrix

$$\hat{\mathbf{S}}(E) = \hat{\Theta}\left[\hat{1} - 2(\hat{1} + i\,\hat{\Omega})^{-1}\right]\hat{\Theta}$$

$\hat{\mathbf{S}} = \hat{\Theta}\hat{K}^{1/2}\hat{\mathcal{S}}\hat{K}^{1/2}$, $\hat{\mathcal{S}}$ = generalized scattering matrix

R matrix

$$\hat{\mathbf{\Omega}} = \sum_{l=1}^{\infty} \frac{\vec{\alpha}_l \vec{\alpha}_l^T}{E - E_l},$$

$\vec{\alpha}_l = \mathbf{K}^{1/2}\,\vec{\chi}^{(l)}$

Transmission probabilities

$$T_{nn'}(E;V_d) = \mid \hat{\mathbf{S}}_{1n,2n'}(E) \mid^2$$

Transmission phases

$$\varphi_{nn'}(E;V_d) = \arg[\hat{\mathbf{S}}_{1n,2n'}(E)]$$

Total transmission through the quantum system

$$T(E;V_d) = \sum_{n=1}^{N_1(E)} \sum_{n'=1}^{N_2(E)} \mid \hat{\mathbf{S}}_{1n,2n'}(E = E_F; V_d) \mid^2$$

4. Resonances

$$\hat{\mathbf{S}}(E) = \hat{\Theta}\left[\hat{1} - 2(\hat{1} + i\,\hat{\Omega})^{-1}\right]\hat{\Theta} \qquad \Rightarrow \qquad \det[1 + i\hat{\Omega}(\bar{E}_{0\lambda})] = 0$$

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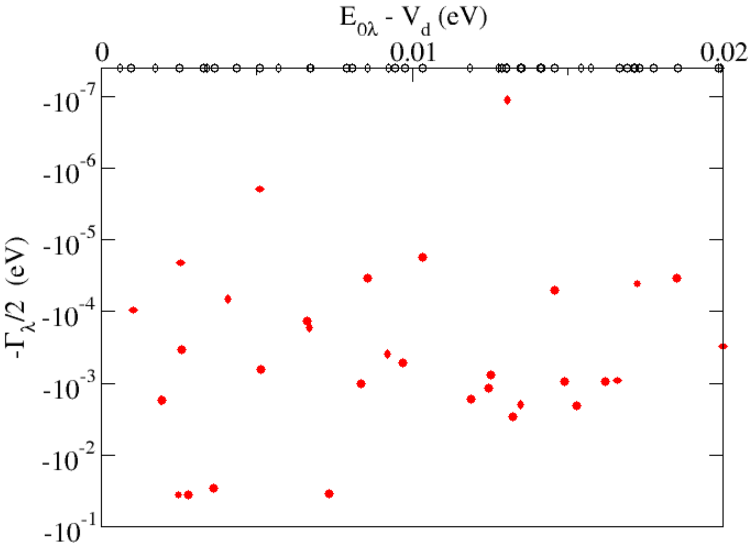


4. Resonances

S-matrix

$$\hat{\mathbf{S}}(E) = \hat{\Theta}\left[\hat{1} - 2(\hat{1} + i\,\hat{\Omega})^{-1}\right]\hat{\Theta}$$

S-matrix around a resonance energy

$$\hat{\mathbf{S}}(E) = \hat{\mathbf{S}}_\lambda(E) + 2i\frac{\vec{\beta}_\lambda \vec{\beta}_\lambda^T}{E - E_\lambda - \bar{\mathcal{E}}_\lambda(E)}$$

$$\hat{\boldsymbol{\Omega}} = \sum_{l=1}^{\infty} \frac{\vec{\alpha}_l \vec{\alpha}_l^T}{E - E_l} = \frac{\vec{\alpha}_\lambda \vec{\alpha}_\lambda^T}{E - E_\lambda} + \hat{\boldsymbol{\Omega}}_\lambda \qquad \vec{\beta}_\lambda = \hat{\Theta}(1 + i\hat{\boldsymbol{\Omega}}_\lambda)^{-1}\vec{\alpha}_\lambda, \qquad \bar{\mathcal{E}}_\lambda(E) = -i\vec{\alpha}_\lambda^T \vec{\beta}_\lambda$$

Background matrix

$$\hat{\mathbf{S}}_\lambda(E) = \hat{\Theta}\left[\hat{1} - 2(\hat{1} + i\,\hat{\Omega}_\lambda)^{-1}\right]\hat{\Theta}$$

Resonance energies

$$\bar{E}_{0\lambda} - E_\lambda - \bar{\mathcal{E}}_\lambda(\bar{E}_{0\lambda}) = 0$$

5. Fano Approximation

Transmission Probabilities around $E_{0\lambda}$

$$T_{nn'}(E) \simeq T_{1nn'} \mid \frac{1}{e_\lambda + i} + \frac{1}{q_{Fnn'}} \mid^2 - T_{2nn'} + T_{nn'}^{bg}$$

$e_\lambda = (E - E_{0\lambda})/\Gamma_\lambda/2$

Transmission Phases around $E_{0\lambda}$

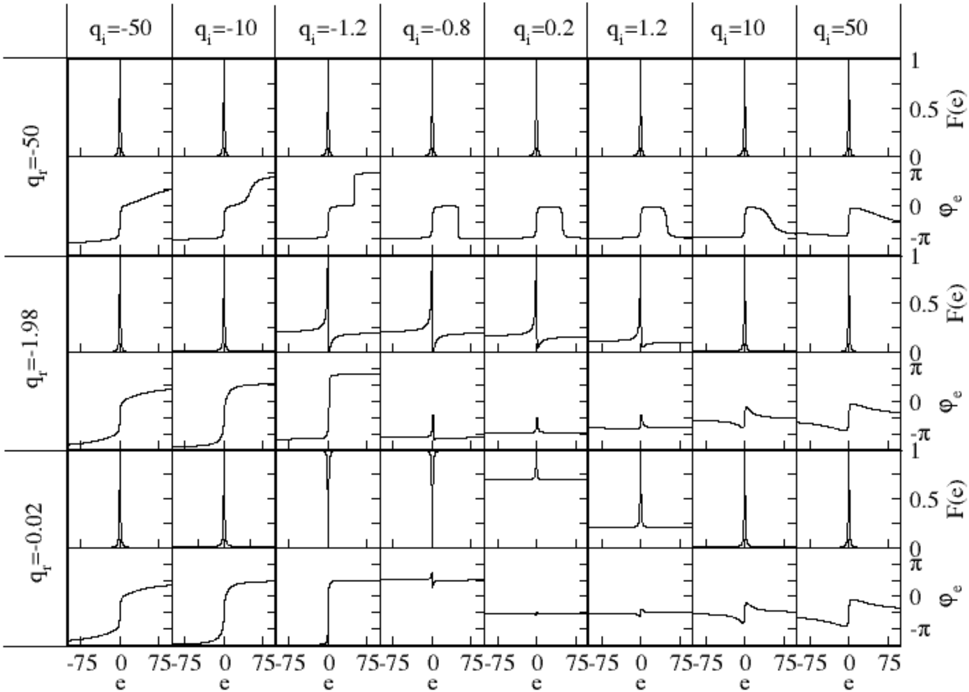
$$\tan(\varphi_{Fnn'}) = -\frac{\mathrm{Im}[q_{Fnn'}](e_\lambda^2 + 1) + \mid q_{Fnn'} \mid^2}{\mathrm{Re}[q_{Fnn'}](e_\lambda^2 + 1) + \mid q_{Fnn'} \mid^2 e_\lambda}$$

Total Transmission around $E_{0\lambda}$

$$T(E) \simeq T_1 \mid \frac{1}{e_\lambda + i} + \frac{1}{q_F} \mid^2 - T_2 + T_{bg}$$

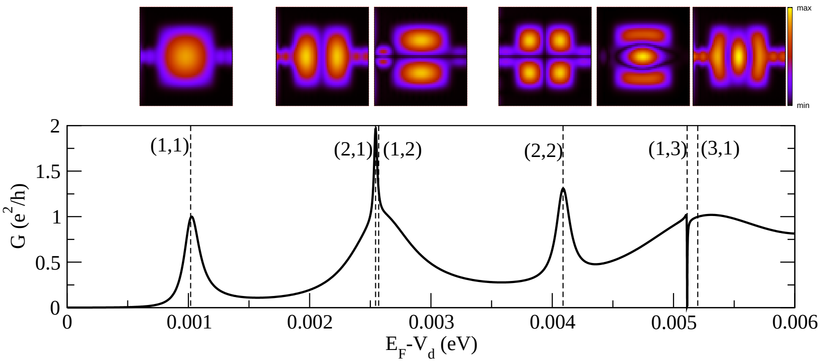
5. Fano Approximation

Fano Function

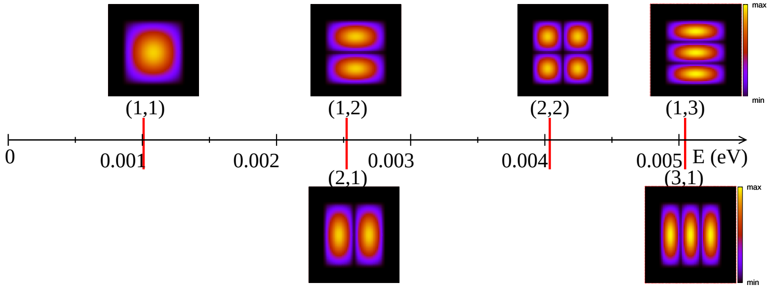


6. Conductance at low temperatures: $G(V_d) = \frac{2e^2}{h} T(E_F; V_d)$

(a)



(b)



E.R. Racec, U.Wulf, P.N. Racec, Phys. Rev. B 82, 085313 (2010)

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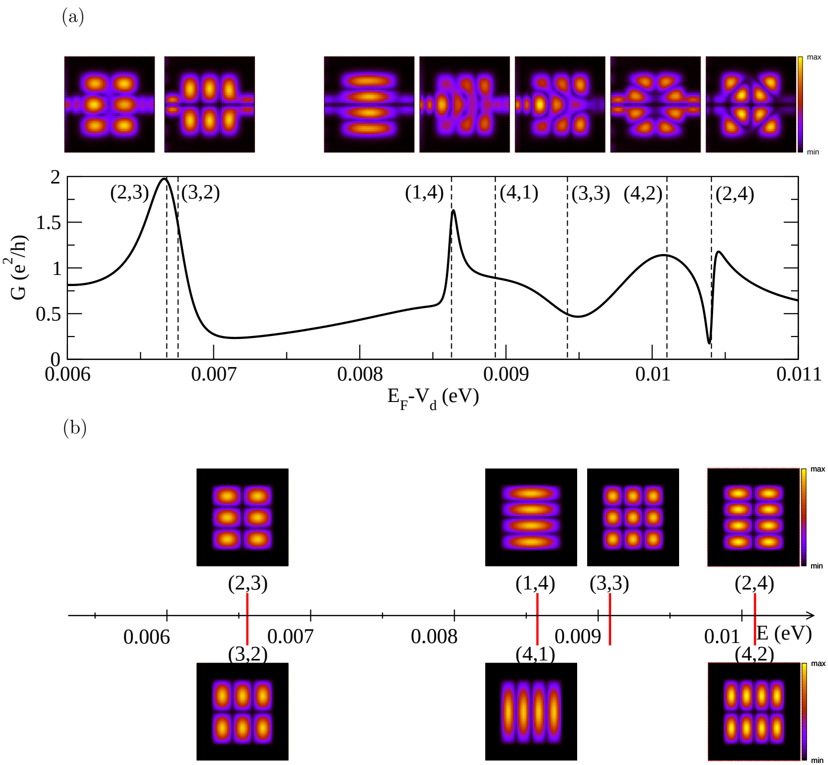
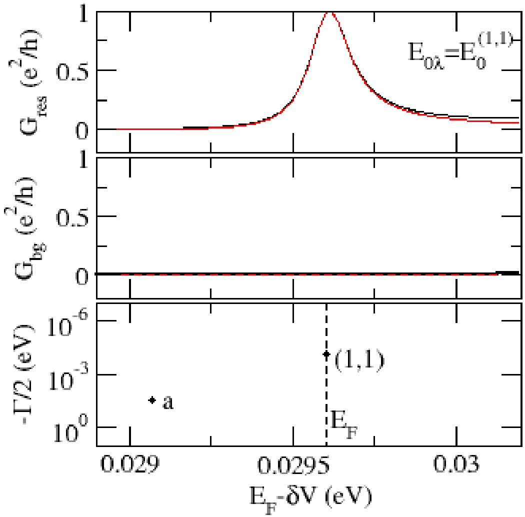


7. Conductance in the Fano Approximation

$$G(V_0 + \delta V) \simeq \frac{2e^2}{h} T(E_F - \delta V; V_0)$$

$V_d = V_0$ maximum in conductance for which $E_{0\lambda} \simeq E_F$

Isolated resonances



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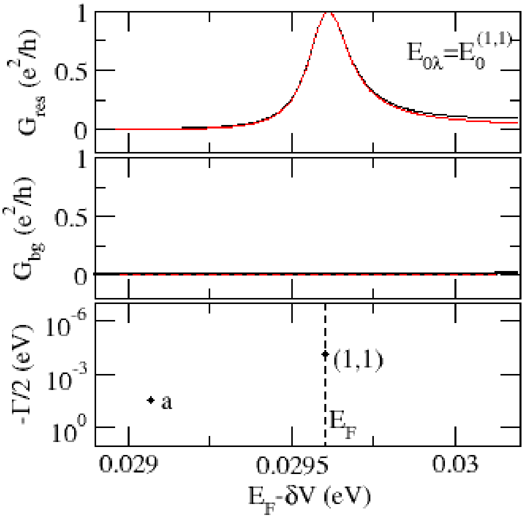
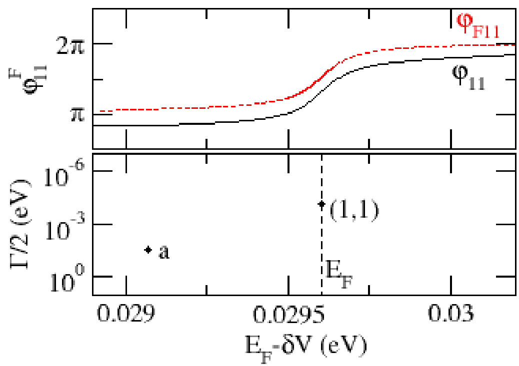
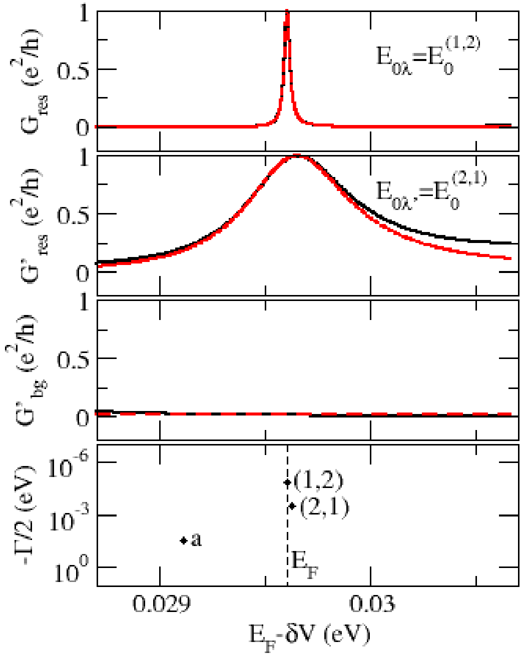




7. Conductance in the Fano Approximation

Overlapping resonances - weak coupling regime



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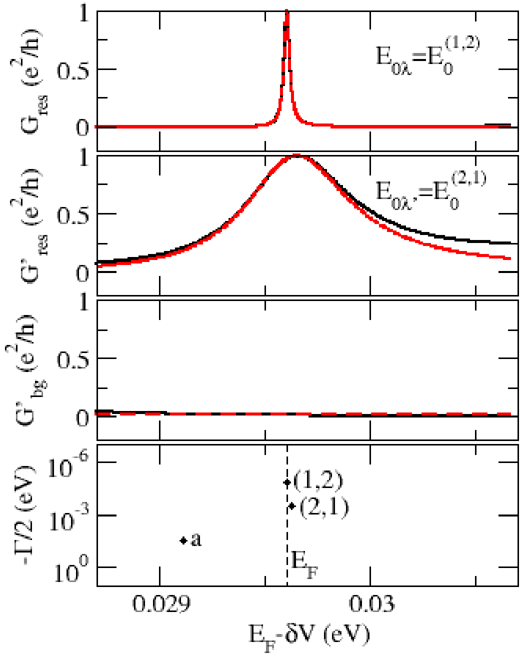

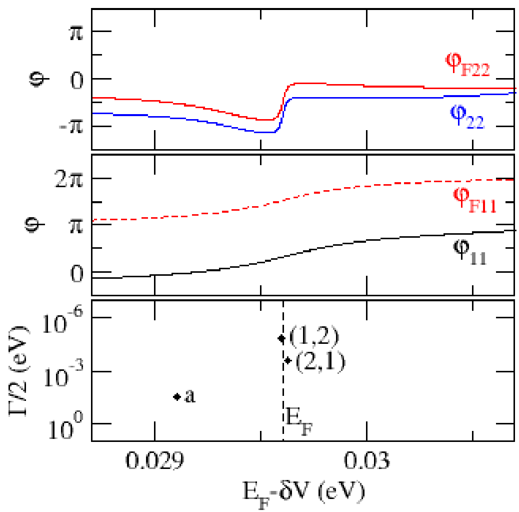

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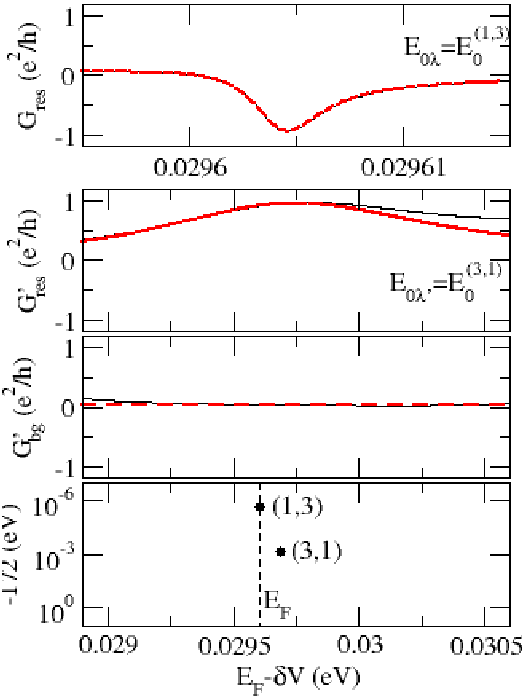

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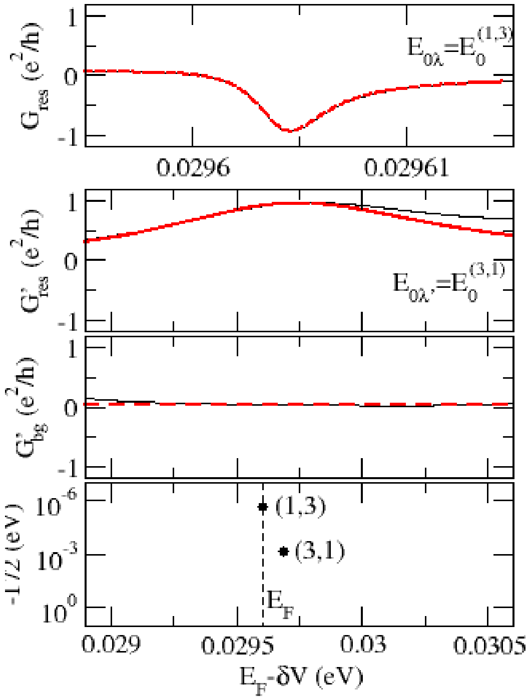

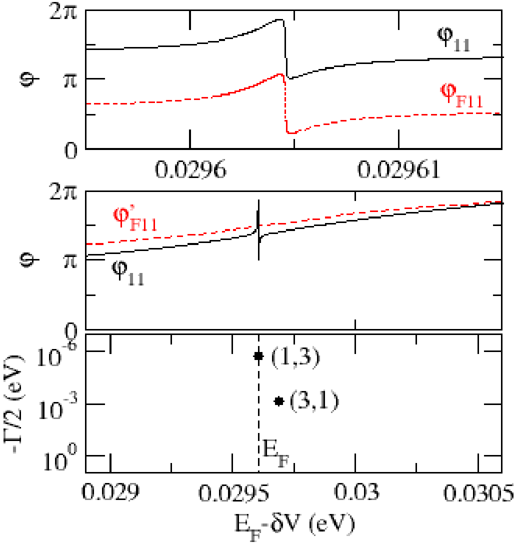

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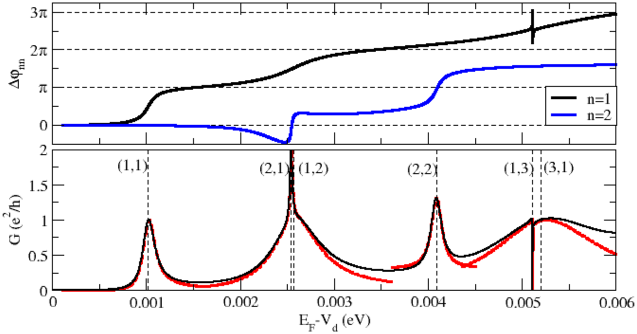

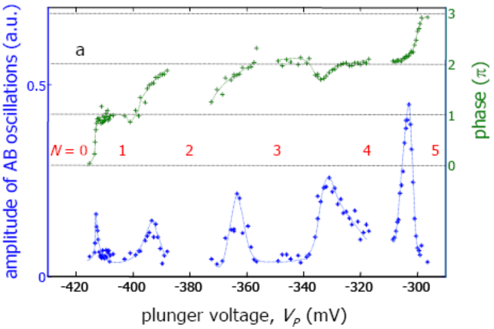

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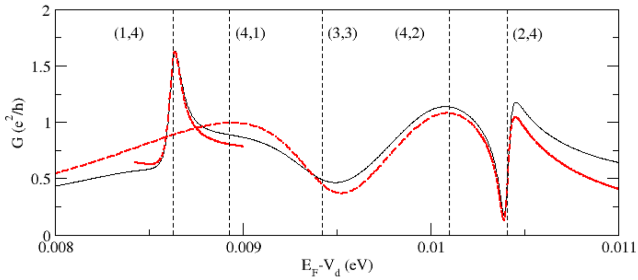

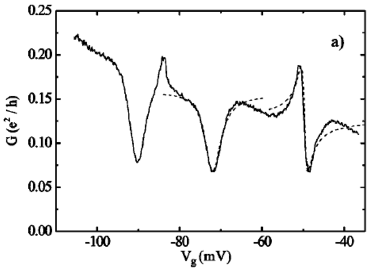

8. Conclusions

- The strong coupling regime of an quantum system to the contacts is characterized by the phenomenon of *resonance trapping* responsible for the existence of the quasi-bound states in continuum.

- In open quantum systems the neighbor resonances do not only overlap, but they also interact with each other.

- The open quantum systems support qualitatively new quantum modes, the hybrid resonant modes.

- The Fano approximation describes accurately the transmission probabilities and phases through an open quantum system.

- The transmission phase is a more sensitive quantity for describing Fano effect in open quantum systems.