Spectral Manipulation in Fabry-Pérot lasers with Non-Periodic Gratings

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Abstract—We consider a Fabry-Pérot laser with a low density of additional discrete features along the cavity, where the real part of the effective refractive index is modified by each feature. While a plain Fabry-Pérot laser in general shows an emission spectrum with many modes, a careful choice of the position of these features can increase the purity of the spectrum dramatically. Here we introduce a perturbative analytical method which allows us to solve the inverse problem relating the index profile along the cavity to the threshold gain modulation in wavenumber space. This method enables the precise tailoring of the Fabry-Pérot laser spectrum.

I. INTRODUCTION

Fabry-Pérot (FP) lasers are the classical type of lasers with an active medium enclosed by two mirrors. In the form of edge-emitting semiconductor lasers they are today among the easiest and cheapest to manufacture. A FP cavity of length L_c supports in principle the oscillation of all modes with wavelength $\lambda_m = 2nL_c/m$, $(m \in \mathbb{N})$, where *n* is the cavity effective index. Depending on the active medium, the resulting spectrum will then feature many frequencies and therefore the use of standard FP lasers in practical applications is often limited.

A well established approach to overcome these difficulties is the phase-shifted distributed feedback (DFB) laser [1], where regions with a periodically varying refractive index provide feedback for exactly one lasing mode. However, such devices require in general complex processing and regrowth steps and are therefore difficult to manufacture.

It has been known for some time, that effective index perturbations within the cavity can improve the spectral purity of a FP laser [1], [2]. Significant progress has been achieved recently using a new method [3] which allows the construction of the grating structure that optimally approximates a desired spectrum when the number N of features is limited. Using this method it was possible to realize a single-mode FP semiconductor laser, with a side mode suppression ratio at twice threshold of more than 40 dB (see Fig. 3). The method proposed in [3] is valid to first order in $N\Delta n/n$.

II. METHOD

We consider lasing in a cavity of length L_c as shown in Fig. 1. The system comprises a FP cavity with complex mirror reflectivities $r_1 = |r_1| \exp(i\varphi_1)$ and $r_2 = |r_2| \exp(i\varphi_2)$ with N additional index steps along the cavity. Each section of the cavity is numbered with an index *i* beginning on the left while the index steps are numbered with an index *j*. We define



Fig. 1. One dimensional model of a FP laser cavity of length L_c and including N index steps. The cavity effective index is n_1 while the additional features providing the index step have effective index n_2 . All cavity sections are numbered $2N + 1 \ge i \ge 1$ beginning on the left. The additional features are also numbered with an index j. The matrix T relates the left and right moving fields at the cavity mirrors. The complex mirror reflectivities are r_1 and r_2 as shown.

 $\theta_i = k_{iz} \cdot L_i$, where $k_{iz} = n_i k_{0z}$ is the wavevector along the z direction in each region. L_i and n_i are the length and the effective refractive index of the *i*th section respectively. The adjusted optical path length across the cavity is then $\sum \theta_i$.

Consider a single index step feature centered at position z_0 and of length L. The step region is of refractive index n_2 while the surrounding regions are of refractive index n_1 . We define F to be the matrix relating the right and left moving electric fields after the right interface and before the left interface. We have

$$\begin{pmatrix} E^{+}(z_{0}-L/2-\epsilon) \\ E^{-}(z_{0}-L/2-\epsilon) \end{pmatrix} = F \begin{pmatrix} E^{+}(z_{0}+L/2+\epsilon) \\ E^{-}(z_{0}+L/2+\epsilon) \end{pmatrix}$$
(1)

We define a general propagation matrix across a region of length L_i given by

$$P(\theta_i) = P_i = \begin{pmatrix} \exp(-i\theta_i) & 0\\ 0 & \exp(i\theta_i) \end{pmatrix}.$$
 (2)

Assuming that the refractive index step is the same for all the additional features, one can show that the matrices F_j can be written with Pauli matrices in the form

$$F_j = P_{2j} - q^{-} \sin \theta_{2j} \,\sigma_y + i[1 - q^+] \sin \theta_{2j} \,\sigma_z, \quad (3)$$

where $q^- = 1/2(q - q^{-1})$ and $q^+ = 1/2(q + q^{-1})$ with $q = n_1/n_2$. We now define the matrix T relating the left and

right moving fields at the cavity mirrors as shown in Fig. 1. i.e.,

$$T = P(\theta_1)F_1P(\theta_3)...F_{N-1}P(\theta_{2N-1})F_NP(\theta_{2N+1}).$$
 (4)

The effect of the additional features will be largest at the wavelength of a particular mode provided the features are quarter wave resonant; $(s+1/2)\lambda_0/2n_2 = L_{2j}$, where s = 1, 2, ... and L_{2j} is the feature length. Then $F_j = q^+ P_{2j} - q^- \sigma_y$. Using the relation $\sigma_y P_i = [P_i]^{-1} \sigma_y$, we evaluate the matrix T explicitly at this resonant wavelength. The result is

$$T = q^{+N} P_{0(N+1)}$$

$$-q^{-}q^{+N-1} \sum_{j=1}^{N} P_{0j} \left[P_{j(N+1)} \right]^{-1} \sigma_{y}$$

$$+q^{-2}q^{+N-2} \sum_{k>j=1}^{N} P_{0j} \left[P_{jk} \right]^{-1} P_{k(N+1)}$$

$$+\dots$$

$$(-1)^{N} q^{-N} P_{01} \left[P_{12} \right]^{-1} P_{23} \cdots \left[P_{N(N+1)} \right]^{\pm 1} \sigma_{y}^{N}.$$
(5)

In the above expression

$$P_{jk} = P(\sum_{i=2j+1}^{2k-1} \theta_i),$$
(6)

The lasing threshold is defined by the presence of a steady state output through the mirrors with no external input. For a cavity which is open at both ends, this implies that the fraction of left and right moving components at each of the cavity mirrors is given by the mirrors reflectivity, i.e.,

 $\lambda \left(\begin{array}{c} r_1 \\ 1 \end{array} \right) = T \left(\begin{array}{c} 1 \\ r_2 \end{array} \right)$

or

$$T_{11} - T_{22}r_1r_2 = T_{21}r_1 - T_{12}r_2.$$
 (8)

(7)

In the absence of external feedback, the lasing condition is determined simply by the requirement that $T_{11} = 0$. These are the complex poles of the transmission coefficient of the system. We see that in this case, off diagonal terms which are of odd order in σ_y are included in relating the light intensity at points inside the cavity but do not determine the lasing modes or their thresholds. This expansion at the Bragg wavelength thus separates resonant feedback from the nonresonant feedback at the various orders of σ_y .

An expansion of T to first order of $\Delta n/n$ yields a selfconsistent equation for the lasing modes of the cavity. To find the lasing modes of the device we expand about the cavity resonance condition: $\sum \theta'_i = \phi_j^{-'} + \phi_j^{+'} = m\pi + \delta_m - 1/2(\varphi_1 + \varphi_2)$. We find that the threshold gain, $\gamma_{t(m)}$, of the *m*th cavity resonance can be written in terms of its even and odd Fourier components along with a threshold modulation amplitude function which vanishes at the device center where the mirrors are of equal reflectivity. The product of the inverse of this function and the Fourier transform of the desired threshold modulation determines the object feature



Fig. 2. (a) Object feature density function which we approximate. Inset: Laser cavity schematic indicating the locations of the reflective features. (b) Threshold gain of modes for the laser cavity schematically pictured in the inset of (a). The horizontal line is at the value of the mirror losses of the plain cavity.



Fig. 3. Laser spectrum of the index patterned device of Fig. 2 at twice threshold. Inset: Spectrum at twice threshold of an equivalent FP laser.

density function. For a particular HR/AR coated cavity, such a object feature density function is shown in Fig. 2 (a) while a schematic picture of the device, high-reflection (HR) coated as indicated, is shown in the inset. Features are placed on the right of the device center in this case where the amplitude of the threshold modulation they provide is larger. The resultant threshold gain spectrum of the cavity is plotted in Fig. 2 (b). Although we estimate the change in the threshold gain of the selected mode to be less than ten per cent of the plain cavity mirror losses, the side mode suppression ratio at twice threshold exceeds 40 dB, as shown in Fig. 3. For comparison, an equivalent spectrum of a FP laser fabricated on the same bar is shown in the inset.

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