Reversed Doppler Shift in Left-Handed Metamaterials

Yih-Peng Chiou, *Member, IEEE*
Graduate Institute of Electro-Optical Engineering and Department of Electrical Engineering
National Taiwan University, Taipei 106-17, Taiwan
Phone: +886-2-3366-3603 Fax: +886-2-2367-7467
Email: ypchiou@ntu.edu.tw

***Abstract*— The reversed Doppler shift in left-handed metamaterial is simulated by the finite-difference time-domain method. Numerical results shows good agreement between the simulated and the theoretical prediction.**

I. Introduction

Left-handed metamaterials (LHM), first proposed by V. Veselago in 1967 [1], are artificial materials with both electrical permeability and magnetic permittivity been negative. In LHM, the electrical field E, magnetic field H, and wavevector k follow left-handed rule instead of usual right-handed rule, which is the origin of the name. The the directions of group velocity and the phase velocity are opposite in LHM. That is, the direction of phase velocity is opposite to the that of energy velocity. Such behavior results in negative refraction phenomena, both the refracted and the incident waves are on the same side of the normal line. Therefore, a LHM are also called as a negative index metamaterial (NIM). It is also called a doubly negative (DNG) metamaterial since both the electrical permeability and magnetic permittivity are negative. With its extraordinary properties, it is shown that electromagnetic waves can be self-focusing without any lens, perfect lens can be theoretically obtained, subwavelength resolution of images are also plausible [2].

Another interesting property in LHM is the reversed Doppler shift. Normally, electromagnetic waves undergo blue-shift, when they are reflected from a approaching object, or red-shift from a leaving object. However, if an object moves in a LHM, the frequency shift is reversed. In this paper, we investigate such phenomena with finite-difference time-domain (FDTD) simulation of a moving dielectric in a highly dispersive and doubly negative material. To our knowledge, the dynamic phenomena are first numerically demonstrated.

II. Theory and Formulation

There is no LHM found in nature. It is was first realized by electrical dipoles made of strips, and magnetic dipoles made of split-ring resonators [2]. The resonance frequencies of the electrical and magnetic dipoles are tuned to have negative ϵ and μ simultaneously over some frequency range. In our simulation, the Drude model is adopted as

$$\epsilon(\omega) = \epsilon_0 \left(1 - \frac{\omega_e^2}{\omega(\omega - j\Gamma_e)}\right) \tag{1}$$

and

$$\mu(\omega) = \mu_0 \left(1 - \frac{\omega_m^2}{\omega(\omega - j\Gamma_m)}\right). \tag{2}$$

In the FDTD simulation, the auxiliary differential equation method used in [3] is adopted to model the highly dispersive $\epsilon(\omega)$ and $\mu(\omega)$. The conventional perfectly matched layers (PML) do not perform well in LHM. We adopt the modified perfectly matched layers developed by Cummer [4] in the outer boundary to avoid the instability. It is based on a modified stretched coordinate to deal with the doubly negative properties. The stretched coordinate factor is

$$1 + \frac{\sigma}{j\omega\left(1 - \frac{\omega_e^2}{\omega^2}\right)}, \tag{3}$$

instead of $1 + \frac{\sigma}{j\omega}$ in normal PML.

Averaged ϵ and μ are adopted to model the moving object [5]. As shown in Fig. 1, the ϵ is approximated

$$\epsilon_{avg} = \frac{p}{p+q}\epsilon_1 + \frac{q}{p+q}\epsilon_2, \tag{4}$$

and averaged μ can be obtained similarly. Note that $p + q = \Delta z/2$, and p is a function of time and the speed of the moving object. Let $\epsilon_2 = \epsilon_0$, we have

$$\begin{aligned} \epsilon_{avg} &= \frac{p}{p+q}\epsilon_0 \left(1 - \frac{\omega_e^2}{\omega^2}\right) + \frac{q}{p+q}\epsilon_0 \\ &= \epsilon_0 \left(1 - \frac{\left(\sqrt{\frac{p}{p+q}}\omega_e\right)^2}{\omega^2}\right) = \epsilon_0 \left(1 - \frac{\omega_e'^2}{\omega^2}\right). \end{aligned} \tag{5}$$

It works like a new resonance frequency and can be coded as usual.

When a object with speed v moves away from the source in a material of refractive index n, the shifted frequency is

$$f' = f_0 \frac{1 - nv/c}{1 + nv/c}, \tag{6}$$

where f_0 is the original frequency and c the speed of light in free space. The frequency will undergo a red-shift in normal materials, but blue-shift in LHM.

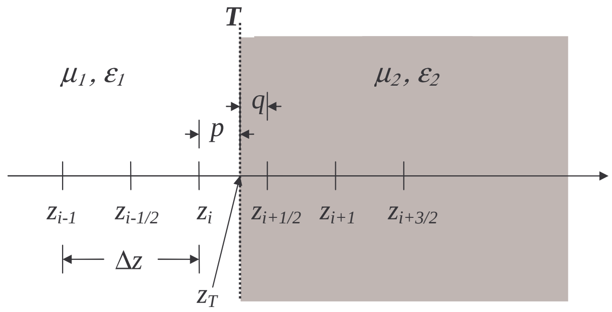

Fig. 1. Interface and sampled fields.

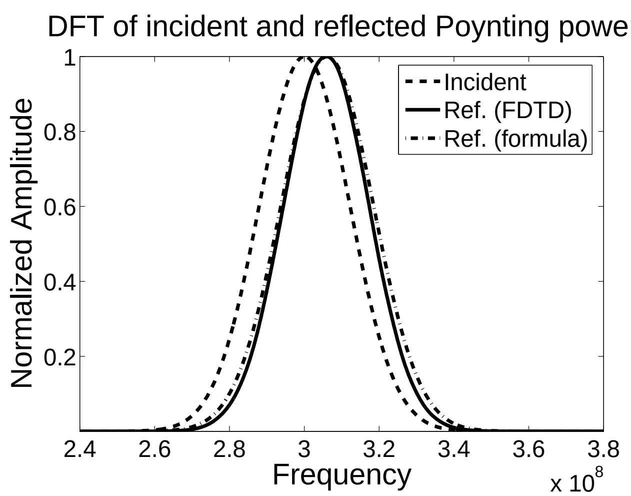

Fig. 2. Reversed Doppler shift in one-dimensional simulation.

III. Numerical Results

First, a one-dimensional FDTD simulation is carried out with a postive refractive index object moving away with speed of $c/100$ from the source in the LHM with $n = -1$. The normalized center frequency is 3×10^7. Fig. 2 shows the normalized Poynting power of the incident wave (dash-doted curve), the reflected wave (solid curve) by FDTD simulation, and reflected wave by equation (6). There is blue Doppler shift instead of red Doppler shift. Our simulation shows the results from FDTD agrees very well with the prediction. The refractive index of the LHM is slightly different from $n = -1$ when the frequency is not the center frequency, which causes slight difference between the simulated results and the predicted results.

A two-dimensional FDTD simulation is also carried out as shown in Fig. 3, which also shows good agreement between the simulated and the prediction. The relative error between the simulated results and the prediction is calculated as

$$\frac{\delta f}{f_0} = \frac{f'_{formula} - f'_{FDTD}}{f_0}. \tag{7}$$

Fig. 4 shows the relative error is less than 0.1 percent around the center frequency.

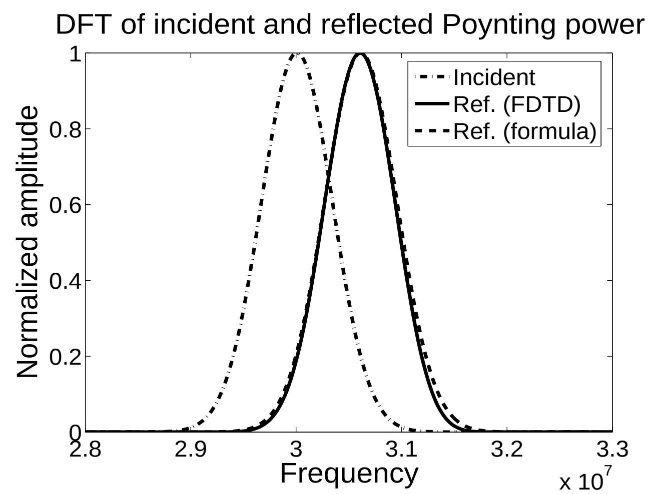

Fig. 3. Reversed Doppler shift in two-dimensional simulation.

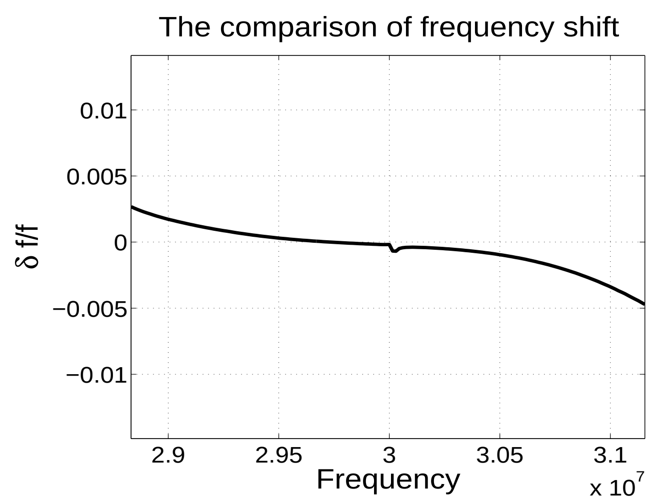

Fig. 4. Comparison between the FDTD simulation and theoretical prediction.

IV. Conclusion

We have demonstrated the FDTD simulation of the reversed Doppler shift in the left-handed metamaterials. The averaged dielectric constant is adopted to model the moving object. The auxiliary differential equation method is applied to the highly dispersive ϵ and μ. The modified PML is used to model the outer boundary. The FDTD results is consistent with the theoretical prediction.

Acknowledgment

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