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Hierarchical networks of transitions in phase space

Alexander Lohse
Universität Hamburg

Heteroclinic and excitable connections are trajectories in the phase space of a dynamical system that facilitate transitions between different (stationary) states. Several methods exist to systematically construct vector fields that possess a network of such transitions corresponding to a desired connection structure given through a directed graph. We briefly discuss some of these and provide a new way of achieving the same task for a \emph{hierarchical connection structure}, which is given as a finite set of digraphs $G_1,\ldots,G_N$ (the lower level), together with another digraph Γ on the set of vertices $\{V_1,\ldots,V_N\}$ (the top level). The dynamic realizations of $G_1,\ldots,G_N$ are heteroclinic networks and they can be thought of as individual connection patterns on a given set of states. Edges in Γ correspond to transitions between these different connection patterns. In our construction, the connections given through Γ are not heteroclinic, but excitable with zero threshold: such a connection exists between two sets S,S' if in every δ-neighbourhood of S there is at least one initial condition such that its ω-limit is contained in S'. In this sense, we show a theorem that allows the systematic creation of hierarchical networks that are excitable on the top level, and heteroclinic on the lower level. Our results modify and extend the simplex method by Ashwin \& Postlethwaite, which is one of several existing techniques for the construction of vector fields that possess a desired heteroclinic network.