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Model-free structural adaptive smoothing of diffusion weighted images

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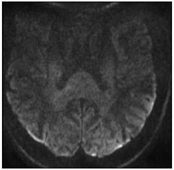
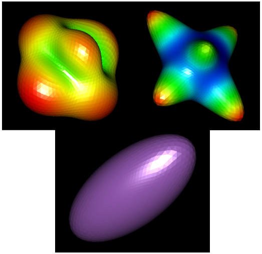
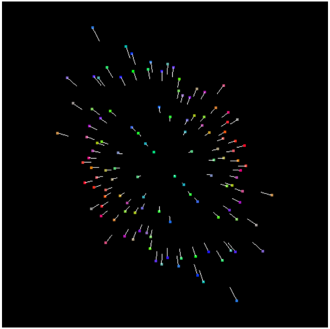
Model-free structural adaptive smoothing of diffusion weighted images

- Diffusion weighted magnetic resonance imaging (dMRI) suffers from significant noise due to motion artifacts, magnetic field inhomogeneity, ghosts,...
- Smoothing enables to reduce number of measured gradients, i.e. scan time
- Nonadaptive smoothing leads to deterioration of structure (blurring)
- Noise creates artifacts for dMRI data resulting in falsified model features.
- There is no generally accepted model (DTI, QBI, mixed tensors,...)





Content



Diffusion weighted magnetic resonance imaging

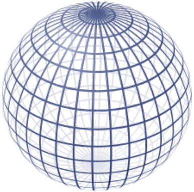
- Focus on brain white matter anatomy, i.e. on neuronal fibers
- Measuring diffusion of water ...
- ... for a direction $\vec{b}$ specified by a magnetic field
- Restricted water diffusion within neuronal fiber bundles

Data acquisition (with fixed b-value):

- Measurements of integral values on a regular grid of voxel (size $\approx 1 - 2mm^3$)
- At least one unweighted image, i.e. $\vec{b} = 0$
- 6-140 diffusion weighted images (3D) with different gradient directions $\vec{b}$ uniformly sampled from the sphere S^2
- Data: $S : \mathbb{R}^3 \rtimes S^2 \rightarrow \mathbb{R}$

Notations

- $S^2 := \{\vec{n} \in \mathbb{R}^3 : \|\vec{n}\| = 1\}$ = 2-sphere
- $\mathrm{SE}(3) := \mathbb{R}^3 \rtimes \mathrm{SO}(3)$ = 3D Euclidean motion group, where $\mathrm{SO}(3) := \{\mathbf{R} \in \mathbb{R}^{3\times 3} : \mathbf{R}^T = \mathbf{R}^{-1}, \det(\mathbf{R}) = 1\}$
- $\mathrm{stab}(\vec{e}_z) := \{\mathbf{R} \in \mathrm{SO}(3) : \mathbf{R} =$ rotation around the z-axis$\}$



http://en.wikipedia.org

Then it holds [Duits, Franken, 2010]

- $S^2 \cong \mathrm{SO}(3)/\mathrm{stab}(\vec{e}_z)$ and $\mathbb{R}^3 \rtimes S^2 \cong \mathrm{SE}(3)/(\mathbf{0} \rtimes \mathrm{stab}(\vec{e}_z))$.
- Any function $F : \mathrm{SE}(3) \to \mathbb{R}$, where $F(\vec{v}, \mathbf{R}_{(\alpha,\beta,\gamma)}) = F(\vec{v}, \mathbf{R}_{(0,\beta,\gamma)})$ for all $\alpha \in [0, 2\pi)$, can be identified one-to-one with a function $f : \mathbb{R}^3 \times S^2 \to \mathbb{R}$.

Parametrizations:

- Parametrization of $\mathrm{SO}(3)$: $\mathbf{R}_{(\alpha,\beta,\gamma)} := \mathbf{R}_\gamma^{\vec{e}_z} \mathbf{R}_\beta^{\vec{e}_y} \mathbf{R}_\alpha^{\vec{e}_z} \in \mathrm{SO}(3)$ for $\beta \notin \{0, \pi\}$
- Parametrization of S^2: $\vec{n}_{(\beta,\gamma)} := (\cos\gamma \sin\beta, \sin\gamma \sin\beta, \cos\beta)^T \in S^2$

Let $g := (\vec{v}, \mathbf{R}_{(\alpha,\beta,\gamma)}) \in \mathrm{SE}(3)$ and $e := (\vec{0}, I) \in \mathrm{SE}(3)$.

$$\|g\|_R := \inf\{\delta > 0 : \exists\, \varphi \in C^1\left([0,1], \mathrm{SE}(3)\right) \text{ with } \varphi(0) = e, \varphi(1) = g,$$

$$\dot{\varphi}(s) = \sum_{i=1}^{6} \varphi_i(s) \mathcal{X}_i|_{\varphi(s)} \text{ and } \left(\int_0^1 \sum_{i=1}^{6} |\varphi_i(s)|^2 ds \right)^{1/2} < \delta\},$$

where $\{\mathcal{X}_i\}_{i=1}^6$ denote the left-invariant vector fields. It holds $\|g\|_R = \left(\sum_{i=1}^6 |k_i|^2\right)^{1/2}$, where the coordinates $\{k_i\}_{i=1}^6$ satisfy (in matrix representation)

$$\prod_{i=1}^{6} \exp(k_i \mathcal{X}_i) = \begin{pmatrix} \mathbf{R}_{(\alpha,\beta,\gamma)} & & & \vec{v} \\ 0 & 0 & 0 & 1 \end{pmatrix} \equiv g \in \mathrm{SE}(3)$$

Discrepancy on $\mathrm{SE}(3)$: $\rho(g,h) := \|h^{-1} \cdot_{\mathrm{SE}(3)} g\|_R, \quad g, h \in \mathrm{SE}(3)$

Discrepancy on $\mathbb{R}^3 \rtimes S^2$: Let $[.]$ denote the equivalence classes of $\mathrm{SE}(3)/(\mathbf{0} \rtimes \mathrm{stab}(\vec{e}_z))$ and κ a parameter balancing between distances on S^2 and in $\mathbb{R}^3$. We consider

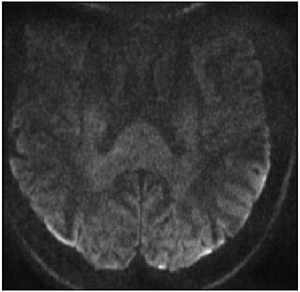
$$\Delta_\kappa(g,h) := \left\| \left[[h]^{-1} \cdot_{\mathrm{SE}(3)} [g] \right] \right\|_{R,\kappa} = \sqrt{\sum_{i=1}^{3} k_i^2 + \kappa^2 \sum_{i=4}^{6} k_i^2}, \quad g, h \in \mathbb{R}^3 \rtimes S^2.$$

Structural assumption

The parameters of the distribution for the data show similarities in the vicinity of any point separated by sharp discontinuities yielding a partition of the space $\mathbb{R}^3 \rtimes S^2$:

$$g_1 \in \mathbb{R}^3 \rtimes S^2 \quad \Rightarrow \quad \mathbb{E}S_{g_2} = \mathbb{E}S_{g_1} \text{ for all } g_2 \in \mathcal{U}(g_1)$$

Generalizations: Local linear or local quadratic model



Propagation-Separation approach [Polzehl, Spokoiny, 2006]

- Iterative estimation of parameters and homogeneity regions
- Propagation: Free smoothing within homogeneous compartments
- Separation: No smoothing among different compartments

Note: The noise distribution of the image S_g is Rician.

Weighted Likelihood estimate: For every $g_1 \in \mathbb{R}^3 \rtimes S^2$ compute

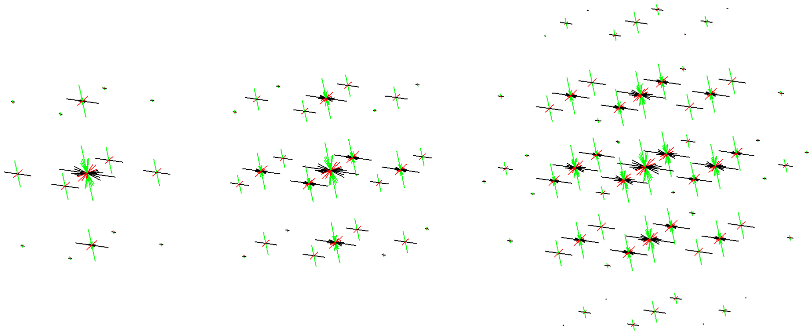
$$\hat{S}_{g_1}^{(k)} := \sum_{g_2} S_{g_2} w_{g_1 g_2}^{(k)} \left(\sum_{g_2} w_{g_1 g_2}^{(k)} \right)^{-1}$$

with weights $$w_{g_1 g_2}^{(k)} := K_{\mathrm{loc}}\big(\Delta_{\kappa_{(\beta_1,\gamma_1)}^{(k)}}(g_1, g_2)/h_{(\beta_1,\gamma_1)}^{(k)}\big) \cdot K_{\mathrm{st}}\big(s_{g_1 g_2}^{(k)}/\lambda\big),$$

where

- $K_{\mathrm{loc}}, K_{\mathrm{st}} : \mathbb{R}^+ \to [0,1]$ are some decreasing kernels with compact support,
- $\{h_{(\beta,\gamma)}^{(k)}\}_{k=0,\dots,k^*}$ an increasing sequence of bandwidths,
- $\{\kappa_{(\beta,\gamma)}^{(k)}\}_{k=0,\dots,k^*}$ the balancing parameters of discrepancy Δ_κ with $\kappa_{(\beta,\gamma)}^{(k)} := \kappa_0 / h_{(\beta,\gamma)}^{(k)}$,
- the adaptation parameter λ, which can be determined independent of the data,
- and $s_{g_1 g_2}^{(k)} := \mathcal{KL}\left(\hat{S}_{g_1}^{(k-1)}/\hat{\sigma},\ \hat{S}_{g_2}^{(k-1)}/\hat{\sigma}\right) \cdot \left(\sum_{g_2} w_{g_1 g_2}^{(k-1)}\right)$ a statistical penalty depending on the Kullback-Leibler distance $\mathcal{KL}$ between the two standardized Rician distributions with parameters $\hat{S}_{g_i}^{(k-1)}/\hat{\sigma}$, $i = 1, 2$.

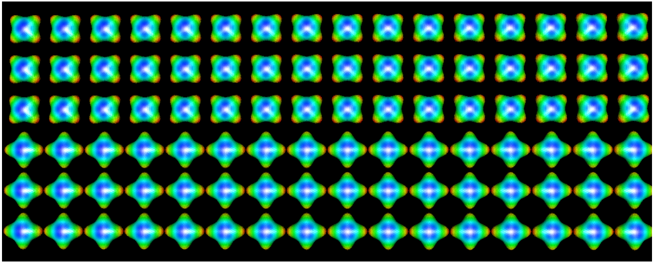
Non-adaptive weights with $\kappa_0 = 0.5$



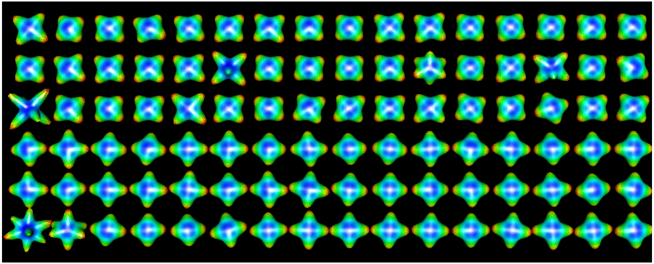
$k^* = 4$ $k^* = 8$ $k^* = 12$

Not mentioned: Smoothing of the non-diffused images S_0, Variance estimation

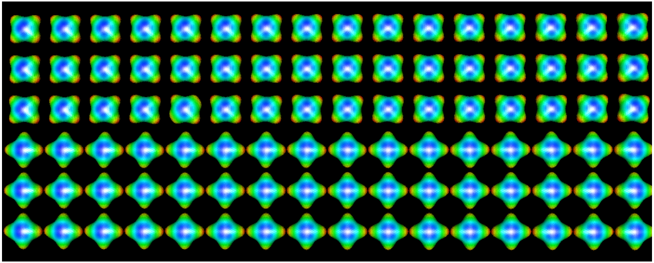
Simulated data: Two homogeneous compartments



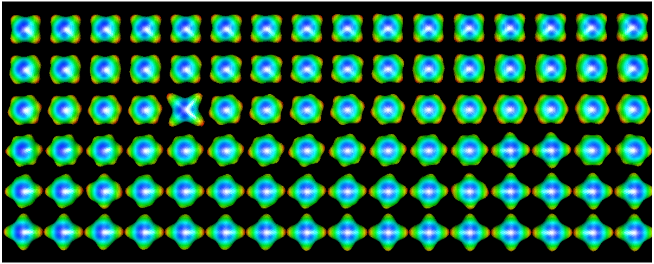
Exact data



Noisy data



Adaptive smoothing on $\mathbb{R}^3 \rtimes S^2$



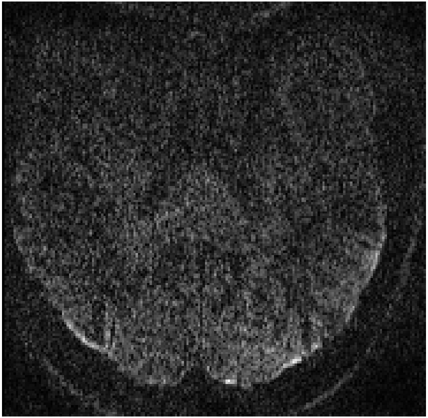
Adaptive smoothing of DTI

Scans were performed on five healthy young volunteers (age 25 ± 3 years) with

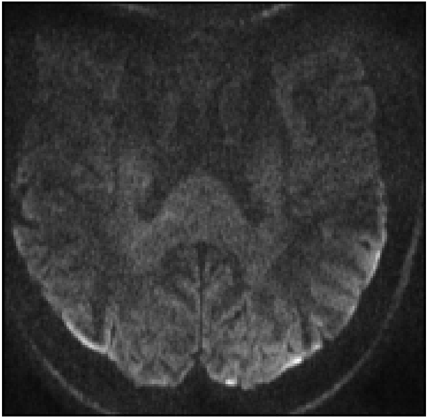
- 7T whole body MR scanner
- Single channel transmit, 24-channel receive phased array head coil
- Optimized monopolar Stejskal-Tanner sequence (Morelli 2010) in conjunction with the ZOOPPA approach (Heidemann 2008)
- 91 slices with 10% overlap and **0.8 mm isotropic resolution**
- TR $14.1\,\mathrm{s}$, TE $65\,\mathrm{ms}$
- ZOOPPA acceleration factor $= 4.6$
- Diffusion weighted scans were performed with **60 directions** with a b-value of $1000\,\mathrm{s}/\mathrm{mm}^2$ and 7 interspersed S_0 images.

For averaging the **scans were repeated 4 times** resulting in a total acquisition time of $65\,\mathrm{min}$.

Real data: Slices of diffusion weighted images



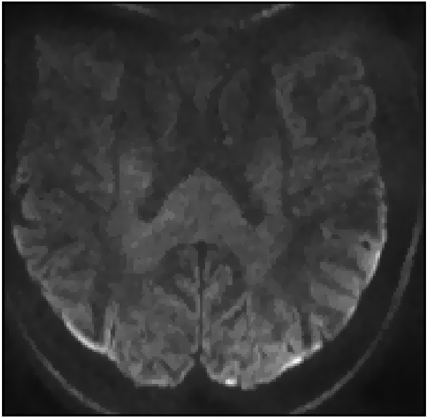
Original data



Averaged data

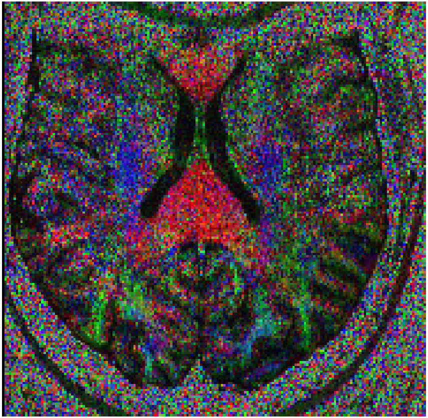
Real data: Slices of diffusion weighted images - Results

Adaptive smoothing of original data

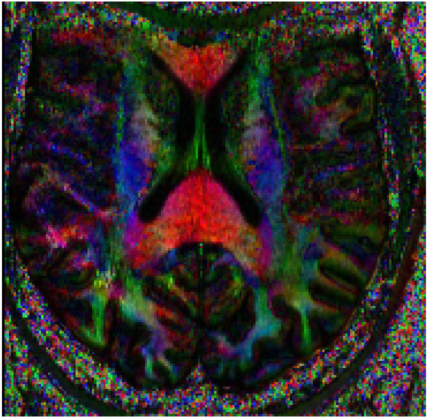


Adaptive smoothing of average

Real data: Color coded FA maps

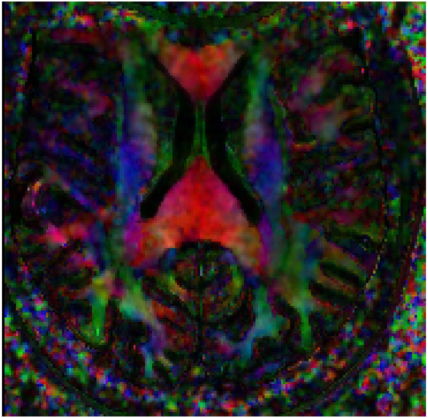


Original data

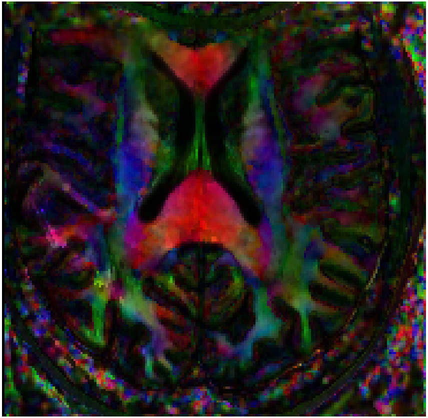


Averaged data

Real data: Color coded FA maps - Results

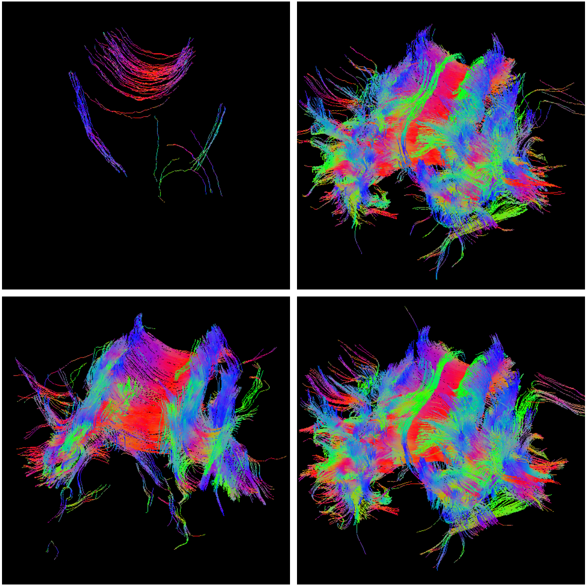


Adaptive smoothing of original data



Adaptive smoothing of average

Fiber tracks showing all fibers with minimum length of 25 segments



Original data

Averaged data

Adaptive smoothing of original data

Adaptive smoothing of average

Conclusion

Diffusion weighted imaging data have the form $S : \mathbb{R}^3 \rtimes S^2 \to \mathbb{R}$

Advantages of the introduced method

- Direct smoothing of the data before application of a specific model
- All information (spatial **and** angular) of the data is used for smoothing
- The quality of the best intermediate result holds up to a constant
- Intrinsic stopping criterion
- Propagation and separation property

Drawbacks of introduced method

- Smoothing on the sphere can introduce a bias (choice of κ_0)
- Running time
- Estimation of variance needed

Joint Work with:

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- Henning U. Voss (Weill Medical College, Cornell University) and
- Alfred Anwander and Robin Heidemann (Max Planck Insitute for Human Cognitive and Brain Sciences)

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- Citigroup Biomedical Imaging Center, Weill Medical College, Cornell University
- Max-Plank Institute for Human Cognitive and Brain Sciences, Leipzig

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Bibliography

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