Brandenburgische Technische Universität Cottbus

Quantum scaling behavior of nanotransistors

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In cooperation with

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A contribution to the
4^{th} **Workshop for Mathematical Models for Transport in Macroscopic and Mesoscopic Systems**
Berlin, February 2008

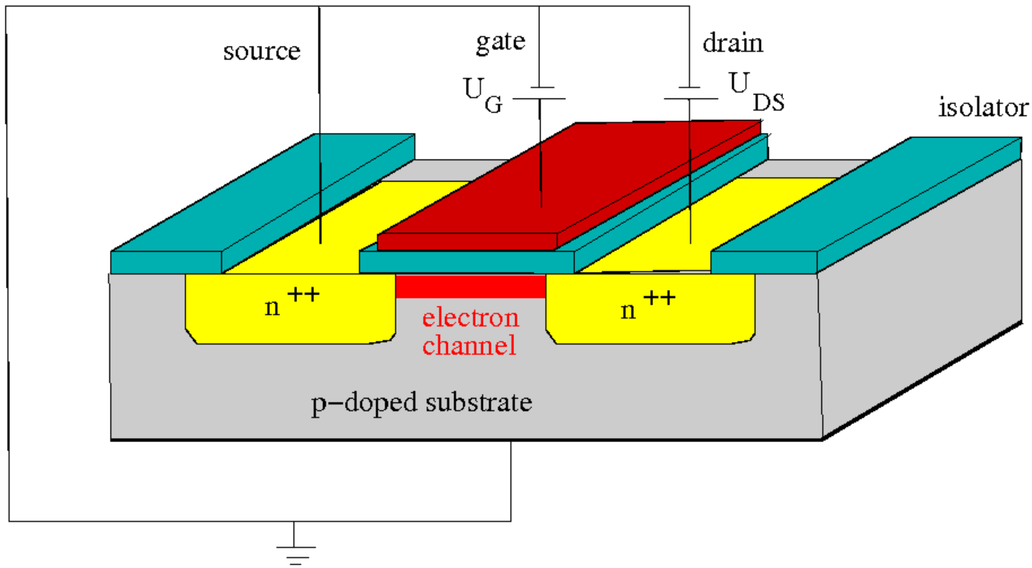
Organization of the talk

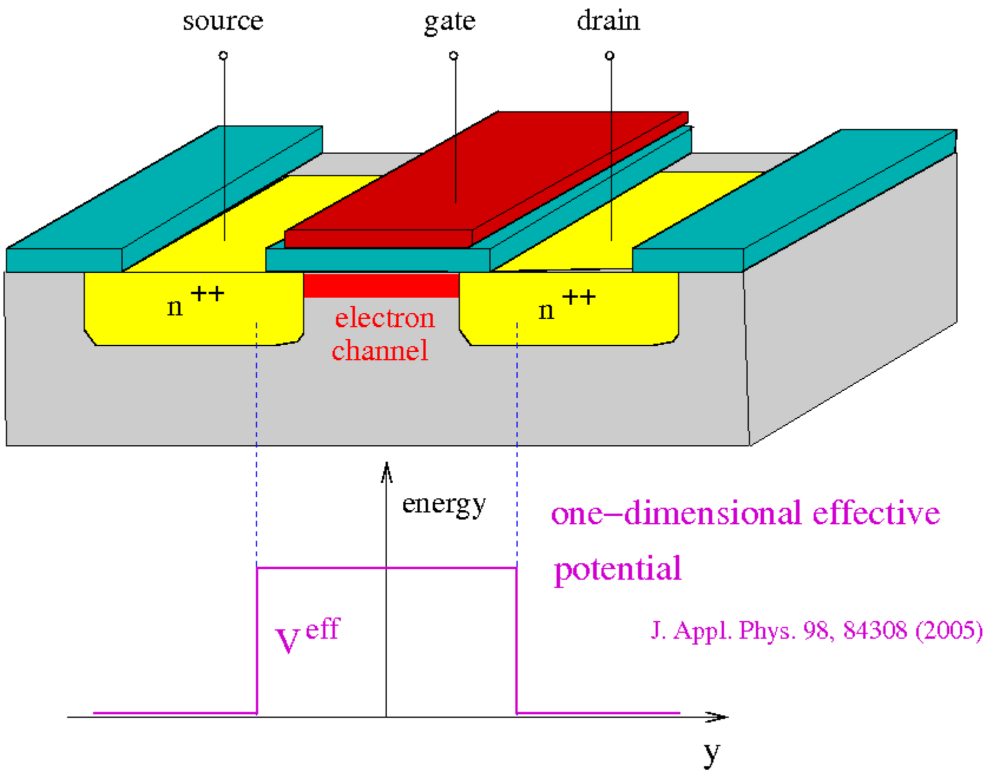
I Introduction

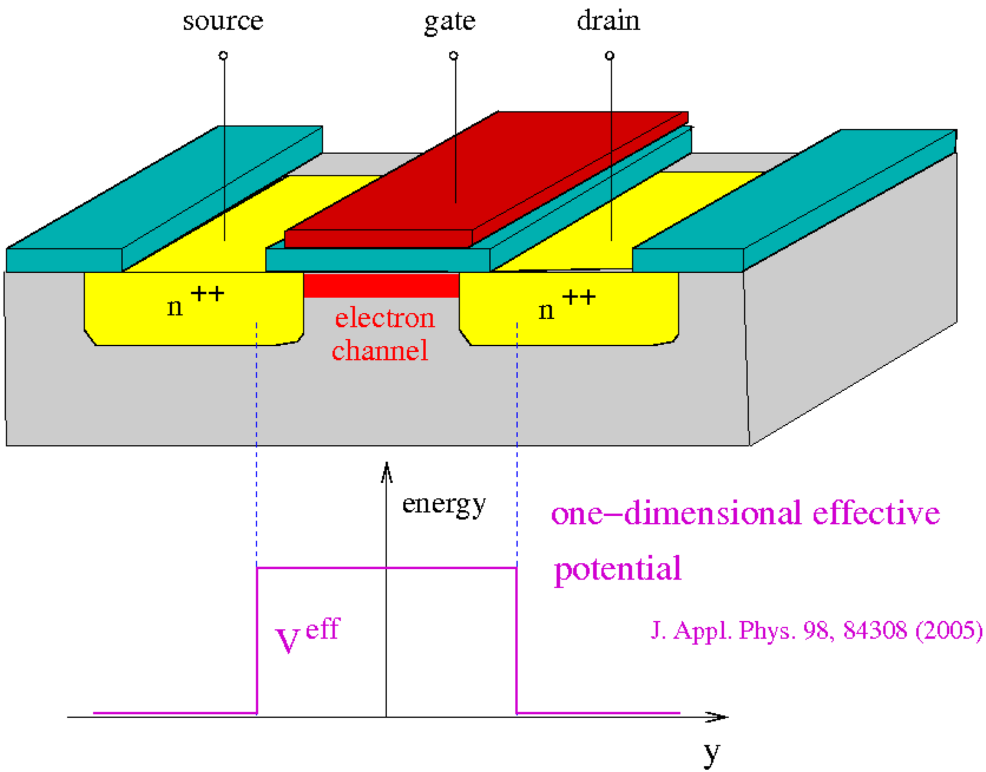
II One-dimesional effective problem

III Scale-invariant description

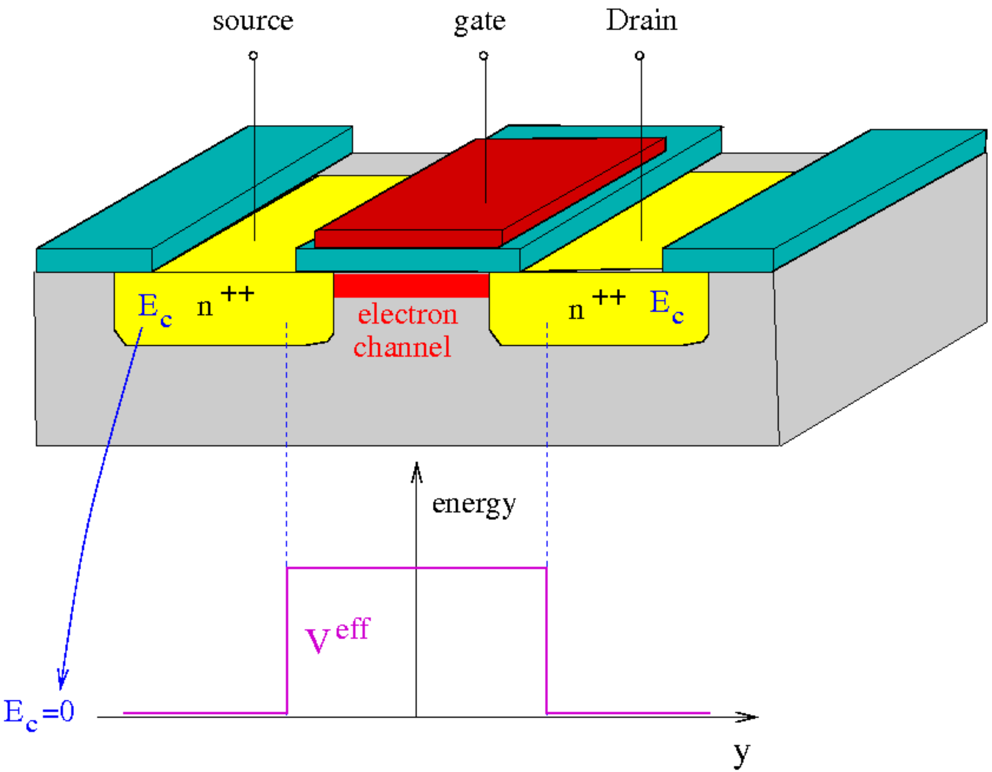
IV I-V-curves in dependence of dimensionless parameters


source
gate
drain
U_G
U_{DS}
isolator
n^{++}
electron
channel
n^{++}
p−doped substrate

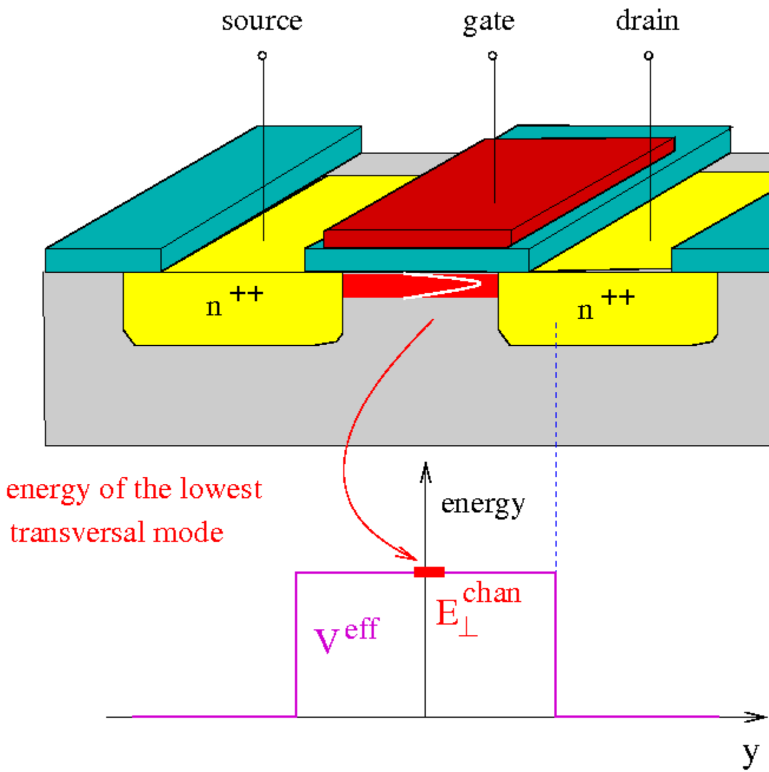

source
gate
drain
n++
n++
electron
channel
energy
one–dimensional effective
potential
J. Appl. Phys. 98, 84308 (2005)
V eff
y

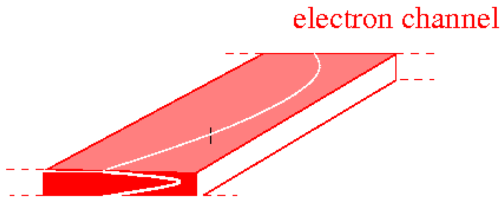


'SMAT'−approximation= single−mode−abrupt−transition

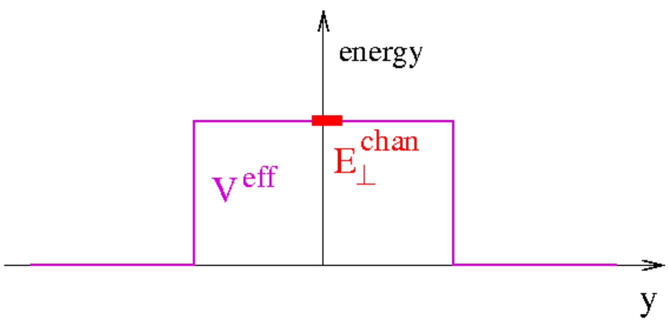

source
gate
Drain
E_c n ++
electron
channel
n ++ E_c
energy
V^{eff}
E_c=0
y

screening


source
gate
drain
n ++
n ++
energy of the lowest
transversal mode
energy
V eff
E⊥ chan
y



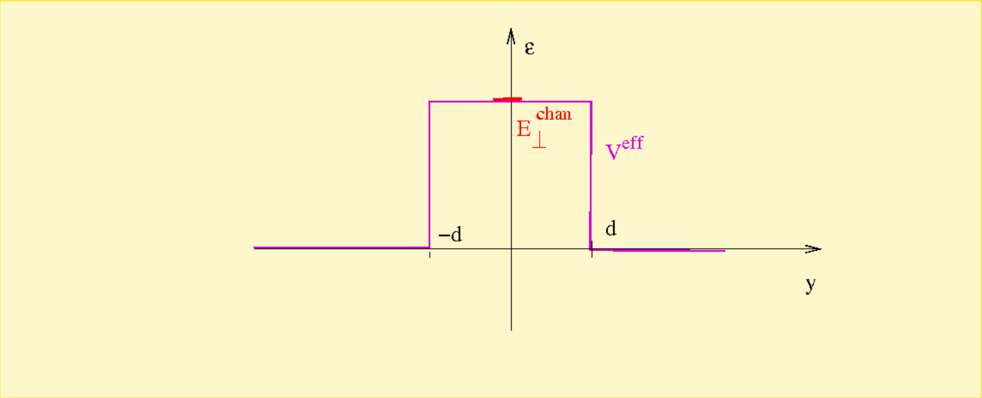
$E_\perp^{chan}$: corresponds to lowest energy with propagating waves in the 'electron wave–guide'



One-dimensional effective problem

$$\left[-\frac{\hbar^2}{2m^*}\frac{d^2}{dy^2} + V^{eff}(y) - \epsilon\right]\phi(y) = 0,$$

$$V^{eff}(y) = \begin{cases} 0, & \text{for } y < -d \\ V_0 + V^F(y), & \text{for } -d \le y \le d \\ -eU_{DS}, & \text{for } y > d. \end{cases}$$



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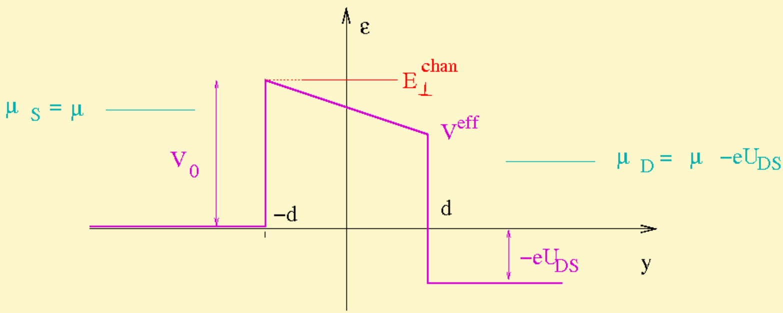


$$I = \frac{2e}{h} \int_0^\infty d\epsilon \left[f\left(\frac{\epsilon - \mu}{kT}\right) - f\left(\frac{\epsilon - \mu + eU_{DS}}{kT}\right)\right] \; T^{eff}(\epsilon).$$

- ϕ are scattering states $\phi^{S/D}$

$$\phi^S(\epsilon, y) = \begin{cases} r^S \exp\left[-ik_S^{eff}(y+d) + \exp\left[ik_S^{eff}(y+d)\right]\right. & \text{for } y < -d \\ \\ t^S \exp\left[ik_D^{eff}(y-d)\right] & \text{for } y > d. \end{cases}$$

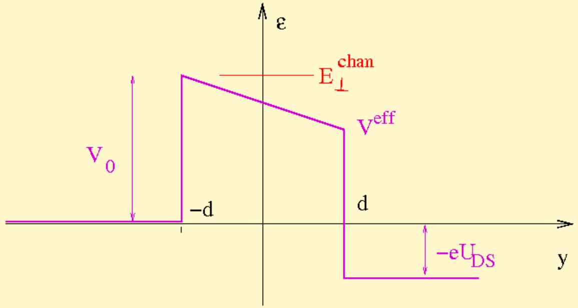
- $k_S^{eff} = \sqrt{2m/\epsilon}/\hbar$ and $k_D^{eff} = \sqrt{2m(\epsilon + eU_{DS})}/\hbar$

- current transmission $T^{eff} = k_S^{eff}(k_D^{eff})^{-1} \mid t^S \mid^2$

Scale-invariant representation of the Schrödinger equation

$$\left(-\frac{1}{\beta}\frac{d^2}{d\hat{y}^2} + \hat{v}^{eff} - \hat{\epsilon}\right)\phi(\hat{y}) = 0$$

- $\hat{y} = y + d/(2d)$ $\quad \hat{\epsilon} = \epsilon/V_0$ $\quad \hat{v}_{ds} = -eU_{DS}/V_0$ $\quad \beta = \frac{2m^*}{\hbar^2}V_0 d^2.$



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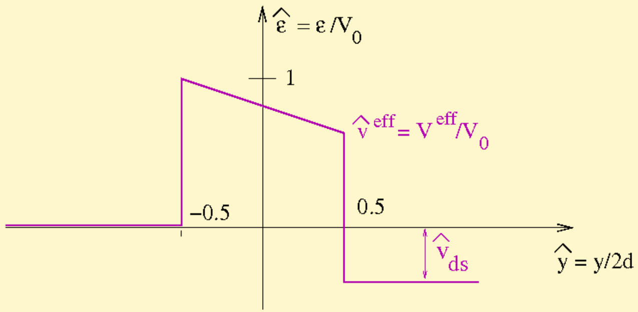
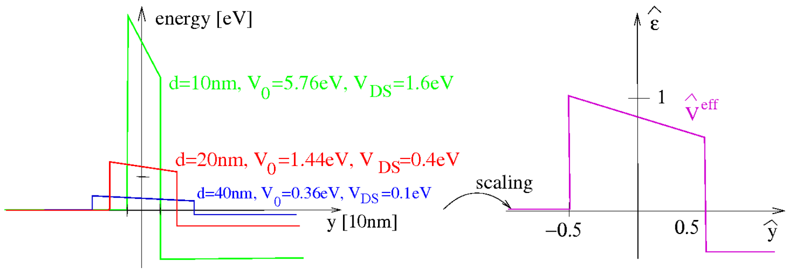


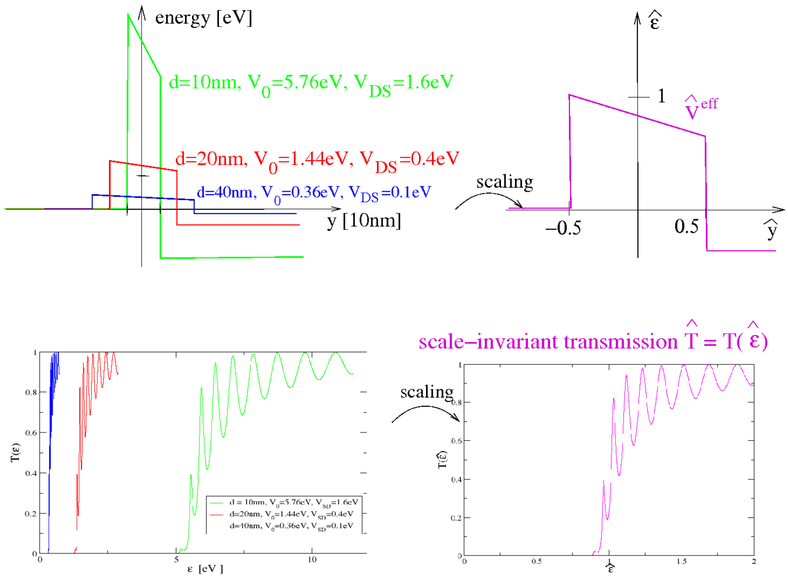
Transmission for fixed $\beta = 1000$ and $\hat{v}_{ds} = 0.5$



three parameters d, V_0, and V_{DS}

two dimensionless parameters β, $\hat{v}_{ds}$

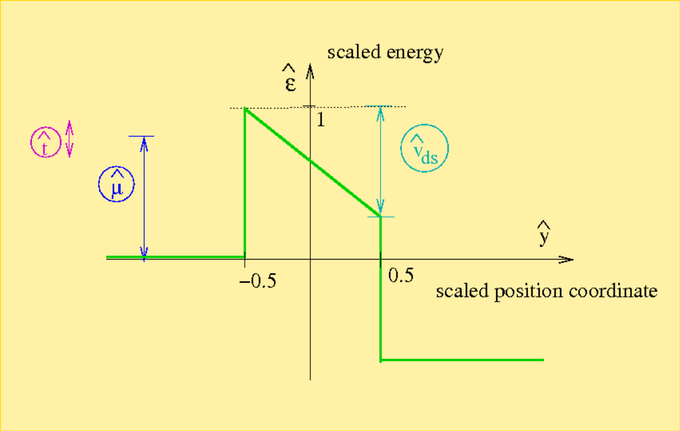
Transmission for fixed $\beta = 1000$ and $\hat{v}_{ds} = 0.5$



$$T = T_{V_0,d,U_{DS}}(E) \rightarrow \hat{T} = \hat{T}_{\beta,\hat{v}_{ds}}(\hat{\epsilon}).$$

The scale-invariant current

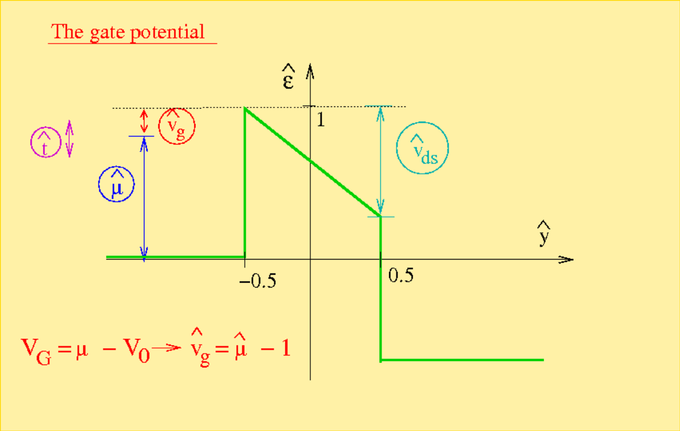
$$I = \frac{2e^2}{h} V_0 \int_0^\infty d\hat{\epsilon} \left[f\left(\frac{\hat{\epsilon}-\hat{\mu}}{\hat{t}}\right) - f\left(\frac{\hat{\epsilon}-\hat{\mu}+\hat{v}_{ds}}{\hat{t}}\right)\right] \hat{T}(\hat{\epsilon}) \quad (1)$$



$$\hat{\mu} = \frac{\mu}{V_0}, \qquad \hat{t} = \frac{kT}{V_0},$$

The scale-invariant current

$$I \;=\; \frac{2e^2}{h} V_0 \int_0^\infty d\hat{\epsilon} \left[f\left(\frac{\hat{\epsilon}-\hat{\mu}}{\hat{t}}\right) - f\left(\frac{\hat{\epsilon}-\hat{\mu}+\hat{v}_{ds}}{\hat{t}}\right)\right] \hat{T}(\hat{\epsilon}) \qquad (2)$$



$$\hat{\mu} = \frac{\mu}{V_0}, \qquad \hat{t} = \frac{kT}{V_0},$$

The scale-invariant current

$$
\begin{aligned}
I &= \frac{2e^2}{h} V_0 \int_0^\infty d\hat{\epsilon} \left[f\left(\frac{\hat{\epsilon}-\hat{\mu}}{\hat{t}}\right) - f\left(\frac{\hat{\epsilon}-\hat{\mu}+\hat{v}_{ds}}{\hat{t}}\right)\right] \hat{T}(\hat{\epsilon}), \\
&= I_0(1-v_g) \int_0^\infty d\hat{\epsilon} \left[f\left(\frac{\hat{\epsilon}-1-\hat{v}_g}{\hat{t}}\right) - f\left(\frac{\hat{\epsilon}-1-\hat{v}_g+\hat{v}_{ds}}{\hat{t}}\right)\right] \hat{T}(\hat{\epsilon})
\end{aligned}
$$

- Introduce Gate bias V_G for I-V chart $I(V_G, V_{DS})$:

$$V_G = \mu - V_0 \Rightarrow \hat{v}_g = \hat{\mu} - 1$$

- Energy-normalization μ (independent of V_G and V_{DS})

$$v_g = \frac{V_G}{\mu} \Rightarrow \hat{v}_g = \frac{v_g}{1-v_g}$$

- Current normalization $I_0 = \frac{2e^2}{h}\mu$
  $\Rightarrow$ maximum current for given V_G, V_{DS} at $T = 0$, if $T(\epsilon) = 1$.

Dimensionless formulation

$$I/I_0 = (1-v_g)\int_0^\infty d\hat{\epsilon}\left[f\left(\frac{\hat{\epsilon}-1-\hat{v}_g}{\hat{t}}\right)-f\left(\frac{\hat{\epsilon}-1-\hat{v}_g+\hat{v}_{ds}}{\hat{t}}\right)\right]\hat{T}_{\beta\hat{v}_{ds}}(\hat{\epsilon})$$
$$= F(\beta^{th}, t, v_{ds}, v_g)$$

$$t = \frac{kT}{\mu} \Rightarrow \hat{t} = \frac{t}{1-v_g},$$

$$v_g = \frac{V_G}{\mu} \Rightarrow \hat{v}_g = \frac{v_g}{1-v_g}$$

$$v_{ds} = \frac{V_{DS}}{\mu} \Rightarrow \hat{v}_{ds} = \frac{v_{ds}}{1-v_g}.$$

barrier parameter $\beta^{th} = 2m^*\mu d^2/\hbar^2$

$$\beta = \frac{2m^*}{\hbar^2}(\mu - V_G)d^2 = \beta^{th}(1-v_g).$$

Typical values for dimensionless parameters

n^{++}-Si contacts: Ideal non-interacting 3D-Fermi gas, $T = 0$, valley-degeneracy $N_V = 6$, effective mass $m^* = 0.32 m_0$, maximum doping $n = N_D = 10^{21} cm^{-3}$

$$\mu \rightarrow E_F = \frac{\hbar^2}{2m^*} \left(\frac{n}{N_V} \right)^{2/3} \left(3\pi^2\right)^{2/3} = 0.34 eV \left[\frac{N_D}{10^{21} cm^{-3}} \right]^{2/3} .$$

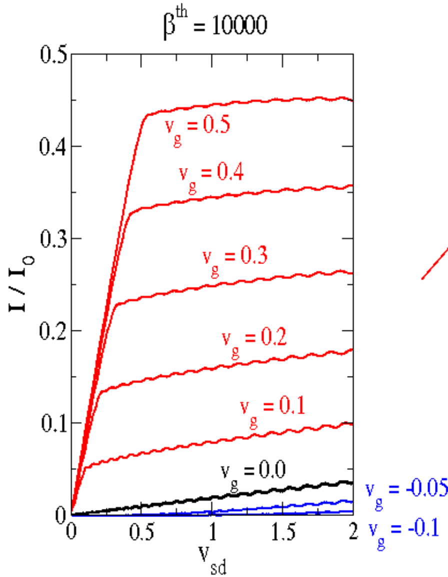
$$\beta^{th} = \frac{2m^*}{\hbar^2} E_F d^2$$

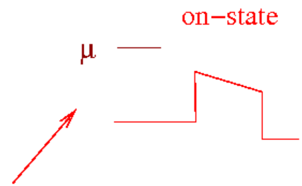
$$d = 10nm \quad N_D = 10^{21} cm^{-3} \quad \Rightarrow \beta^{th} = 135$$
$$d = 30nm \quad N_D = 10^{21} cm^{-3} \quad \Rightarrow \beta^{th} = 1200$$

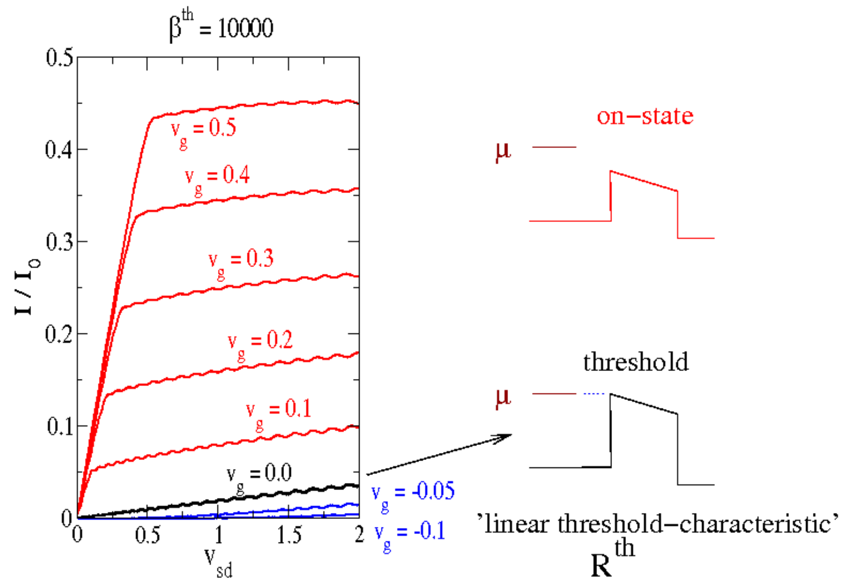
Furthermore

$$I_0 = \frac{2e^2}{h} \mu = 78 \mu A \times \mu[eV]$$

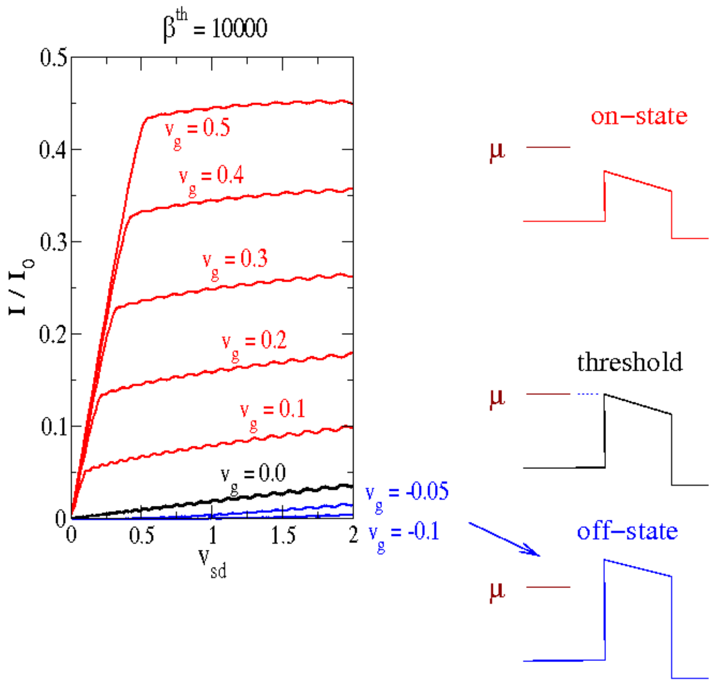
Drain characteristics, strong Barriere, $T \to 0$



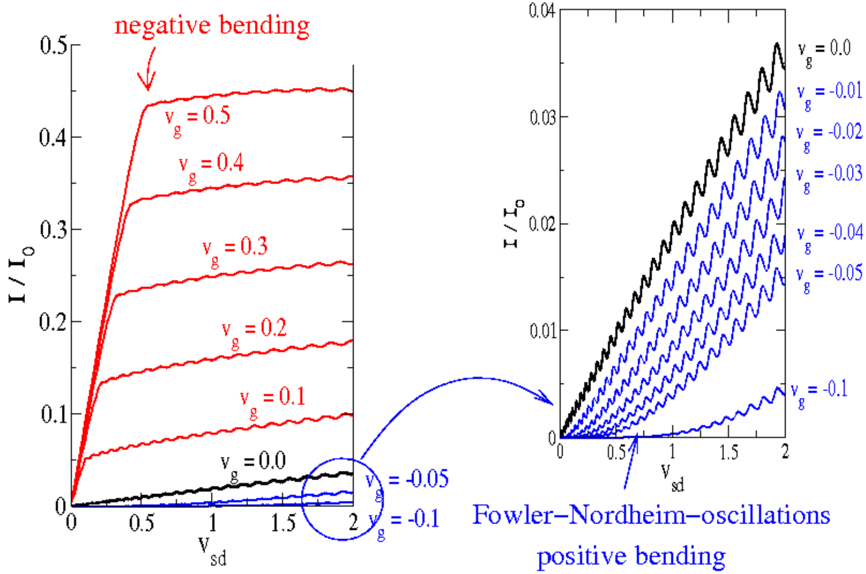




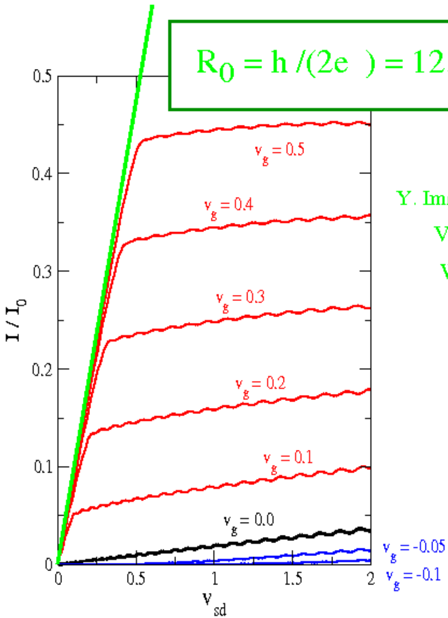
β^th = 10000
I / I_0
v_sd
v_g = 0.5
v_g = 0.4
v_g = 0.3
v_g = 0.2
v_g = 0.1
v_g = 0.0
v_g = -0.05
v_g = -0.1
on−state
μ
threshold
μ
'linear threshold−characteristic'
R^th



β^th = 10000
0.5
0.4
0.3
0.2
0.1
0
I / I0
0
0.5
1
1.5
2
Vsd
vg = 0.5
vg = 0.4
vg = 0.3
vg = 0.2
vg = 0.1
vg = 0.0
vg = -0.05
vg = -0.1
on−state
μ
threshold
μ
off−state
μ


negative bending
0.5
0.4
0.3
0.2
0.1
0
I / I0
0
0.5
1
1.5
2
Vsd
vg = 0.5
vg = 0.4
vg = 0.3
vg = 0.2
vg = 0.1
vg = 0.0
vg = -0.05
vg = -0.1
0.04
0.03
0.02
0.01
0
I / I0
0
0.5
1
1.5
2
Vsd
vg = 0.0
vg = -0.01
vg = -0.02
vg = -0.03
vg = -0.04
vg = -0.05
vg = -0.1
Fowler–Nordheim–oscillations
positive bending

$R_0 = h/(2e\ \ ) = 12.8\ k\Omega$



contact resistance:

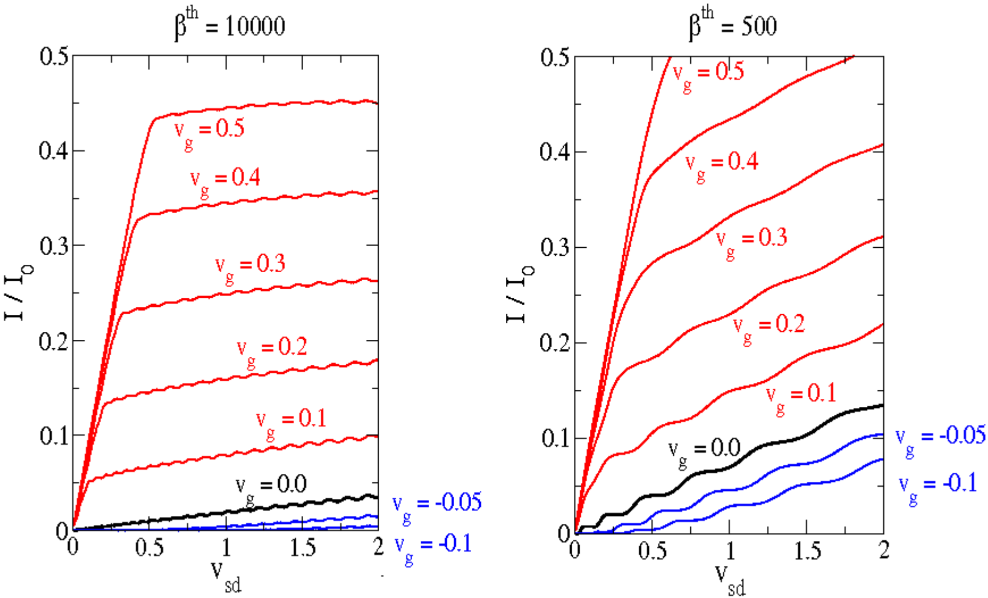
Y. Imry in Directions of Condensed Matter Physics
Vol. 1, Ed. G. Grinstein and G. Mazenko,
World Scientific, Singapore, 102 (1986)

quantization of conductance
in point contacts

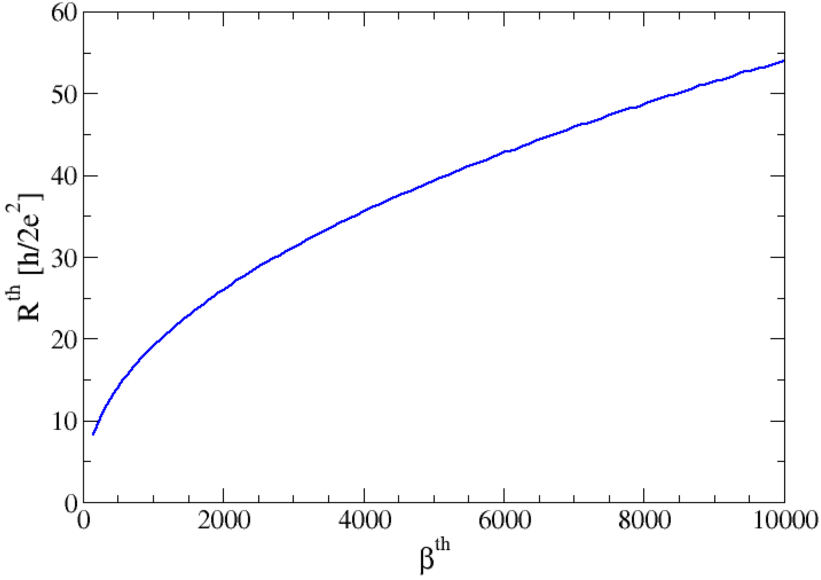
in $\frac{e^2}{h}$ (v. Klitzing constant)

B. J. van Wees et al. Phys.Rev. Lett. 60, 848 (1988)

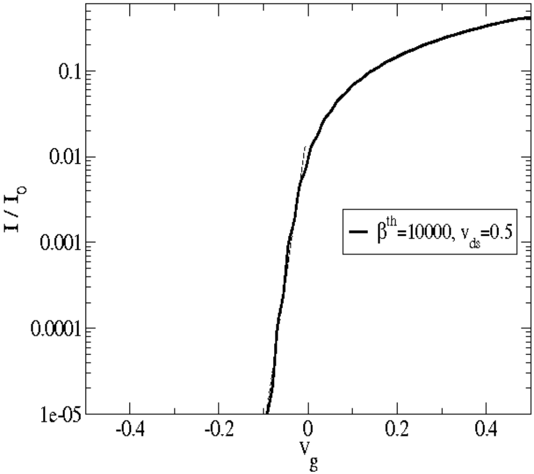
Comparison between stronger- and weaker barrier



The threshold resistance



Subtheshold charakteristic



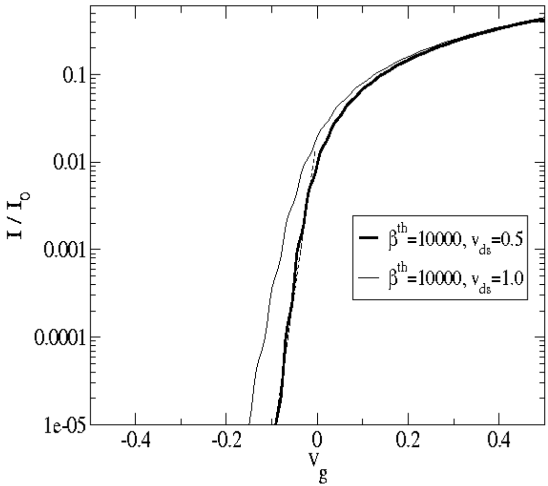
○exponentieller subthreshold current

classical model (drift−diffusion)

electron density weak inversion

quantum model:

source−drain tunneling



○ Drain current dependence

classical model (drift−diffusion)

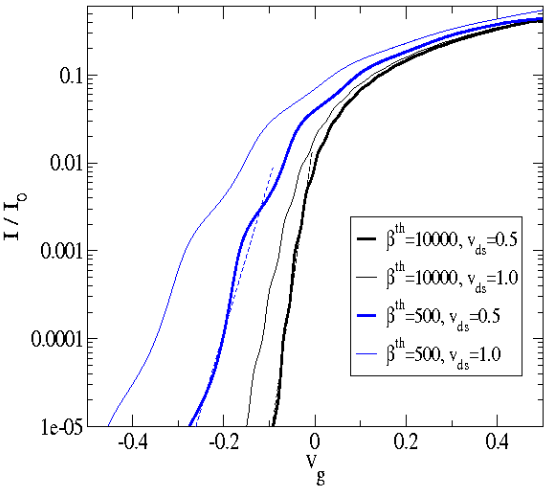
−'short channel effects'

−'Punsh through' between source− and drain depletion zone

quantum model:

drain bias dependence of tunneling

quantum short channel efect



By increasing β^{th}

quantum short−channel effects

can be reduced

Discussion

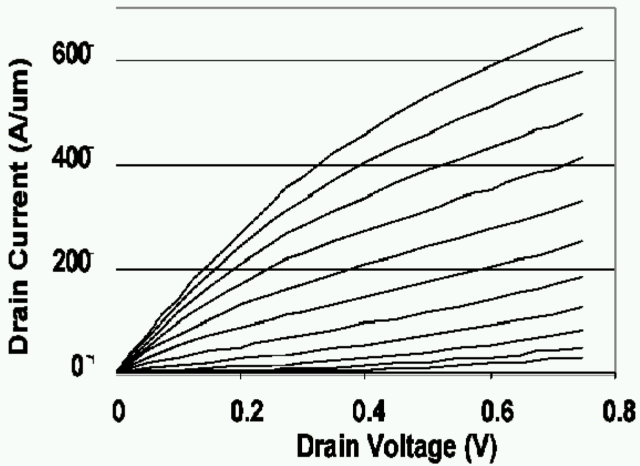
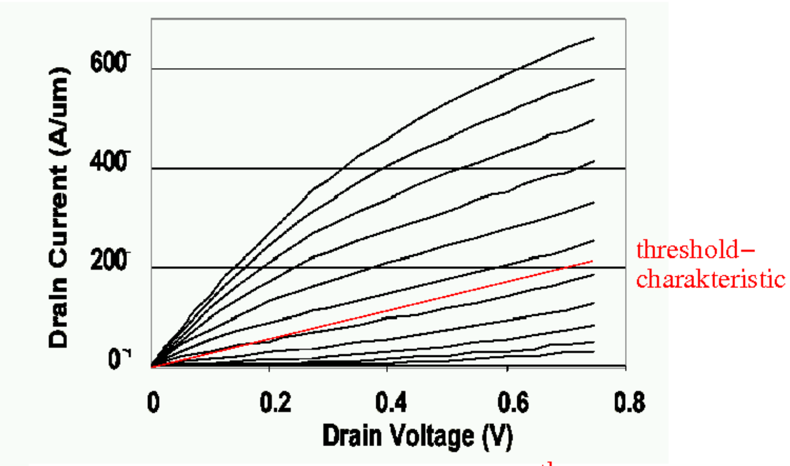
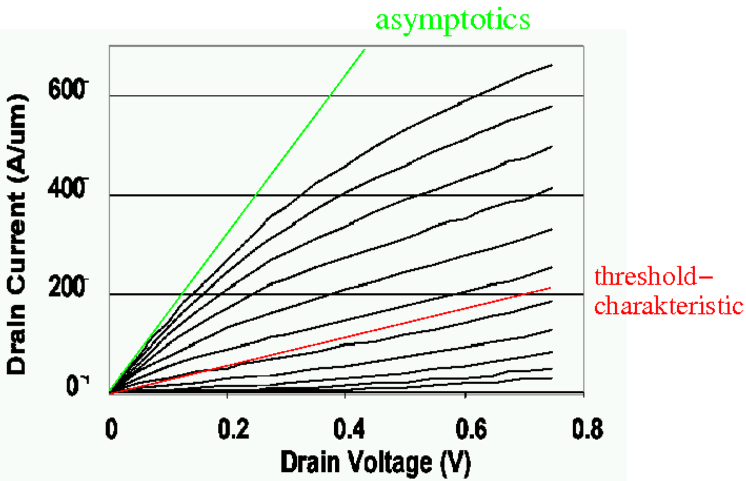


Figure 13: Id-Vd curves of 10nm transistor. V_g to 0.75V, steps of 0.1V

B. Doyle et. al., Intel Technology Journal 6, 42 (2002)



$$R^{th} = 28\ h/(2e) \longrightarrow \beta^{th} = 2000$$



$R^{th} = 28\ h/(2e) \longrightarrow \beta^{th} = 2000$

$R^{0} = 5\ h/(2e)$

Conclusion

I Effectively one-dimensional problem

- SMAT = single mode, abrupt transition

II Scale-invariant description of transport

- Dimensionless barrier strength parameter

$$\beta^{th} = 2m^*\mu d^2/\hbar^2$$

III I-V curves in dependence of β^{th}

- In agreement with INTEL-transistor: linear threshold charak-teristik R^{th}
- 'Quantum short-channel effects' reduced with increasing β^{th}