Time-dependent transport in quantum dot systems: from transient to steady state regime

Valeriu Moldoveanu

National Institute of Materials Physics, Bucharest, Romania

Vidar Gudmundsson

Science Institute, University of Iceland

Andrei Manolescu

Reykjavik University, Iceland

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Transients due to time-dependent couplings.

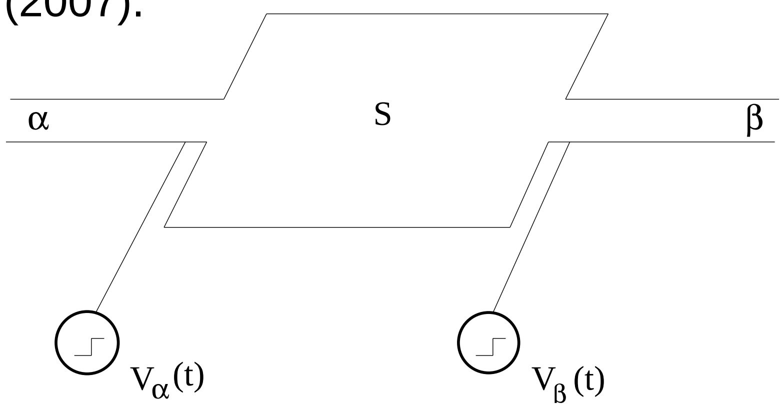
- $H(t) = H_{coupled} + V_{bias}(t)$ vs. $H(t) = H_{bias} + V_{coupling}(t)$

  S. Kurth *et. al* Phys. Rev. B **72** (2005) vs. C. Caroli *et. al* J. Phys. C **4** (1971).

- Step-like potential and turnstile configuration.

- Applications: non-periodic potentials and many-level systems (beyond A.-P. Jauho *et al.*, Phys. Rev. B **50** (1994)).

- Rigorous results: H. D. Cornean *et al.* arXiv:0708.0303 and G. Nenciu , J. Math. Phys. 48, 033302 (2007).



- Time dependent Hamiltonian $H(t) = H_S + H_L + H_T(t)$.

$$\begin{aligned} H_S &= \sum_{m=1}^{N} (\epsilon_m + V_g) d_m^\dagger d_m + \sum_{\langle m,n \rangle} t_{mn} d_m^\dagger d_n, \\ H_T(t) &= \sum_{\alpha=l,r} V_{i_\alpha m_\alpha}(t)(c_{i_\alpha}^\dagger d_{m_\alpha} + h.c), \\ j_\alpha(t) &= \frac{ie}{\hbar}[H(t), N_\alpha] = \frac{ie}{\hbar} V_{i_\alpha m_\alpha}(t)(c_{i_\alpha}^\dagger d_{m_\alpha} - d_{m_\alpha}^\dagger c_{i_\alpha}). \end{aligned}$$

- $\rho(t) = e^{it(H_S+H_L)} \tilde{U}(t,t_0) \rho(t_0) \tilde{U}(t,t_0)^* e^{-it(H_S+H_L)}$
- $\rho(t_0) = \rho(H_L) \oplus \rho(H_S)$.

- $J_\alpha(t) = \mathrm{Tr}\{\rho_0 \tilde{U}(t,t_0)\tilde{j}_\alpha(t))\tilde{U}(t,t_0)\}$
  $= \mathrm{Tr}\{\rho_0 T_C(e^{-i\int_C ds \tilde{H}_T(s)}\tilde{j}_\alpha(t))\}$
- Keldysh contour: $C = (-\infty, t] \cup [t, -\infty)$.
- Main formula (matrix form):

$$J_\alpha(t) = -\frac{2e}{h}\mathrm{Im}\int_{-2t_L}^{2t_L} dE \int_0^t ds e^{-iE(s-t)}\mathrm{Tr}\{\Gamma^\alpha(E;t,s)(G^R(t,s)f_\alpha(E) + G^<(t,s))\}.$$

- Retarded Green function
  $G^R_{m_\alpha i_\alpha}(t,t') = -i\theta(t-t')\langle\{c^\dagger_{i_\alpha}(t'), d_{m_\alpha}(t)\}\rangle$.
- Lesser Green function $G^<_{m_\alpha i_\alpha}(t,t') = i\langle c^\dagger_{i_\alpha}(t')d_{m_\alpha}(t)\rangle$.
- $\Gamma^\alpha_{m_\alpha,n_\alpha}(E;t,s) = \sum_{i_\alpha,j_\alpha}\rho(E)V_{i_\alpha m_\alpha}(t)V_{j_\alpha n_\alpha}(s)$.

- The integral Dyson equation (matrix form):

$$G^R(t,t') = G_0^R(t,t') + \int_0^t dt_1 G^R(t,t_1) \int_0^{t_1} dt_2 \Sigma^R(t_1,t_2) G_0^R(t_2,t')$$

- The Keldysh equation

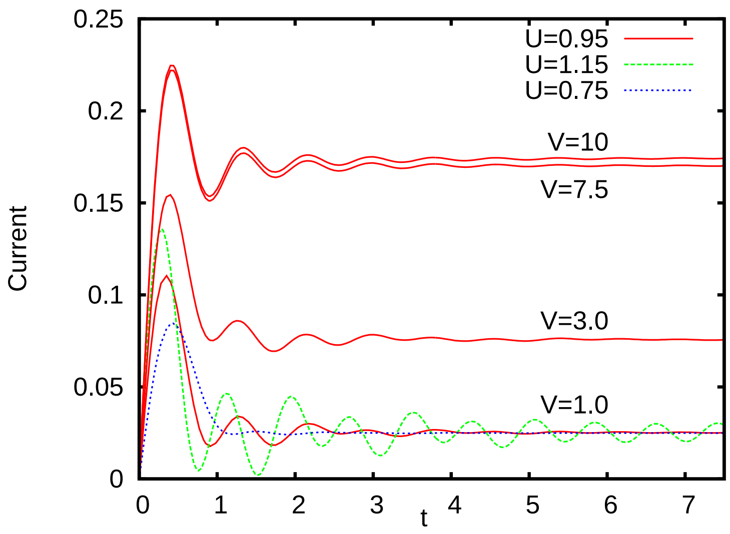
$$G^<(t,t') = \int_0^t dt_1 G^R(t,t_1) \int_0^{t'} dt_2 \Sigma^<(t_1,t_2) G^A(t_2,t')$$

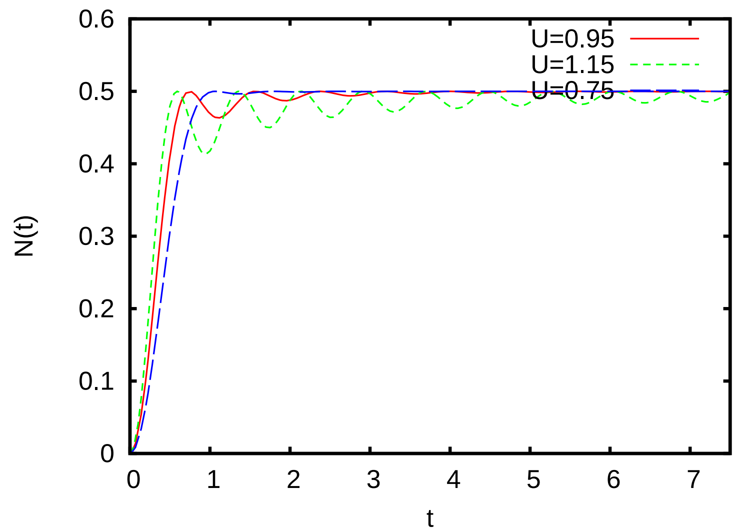
- The retarded self-energy

$$\Sigma^R_{mn}(t,t') = -i\theta(t-t') \sum_\gamma V_\gamma(t) V_\gamma(t') \int_{-2t_L}^{2t_L} dE \rho(E) e^{-iE(t'-t)} \delta_{mm_\gamma} \delta_{nn_\gamma}$$

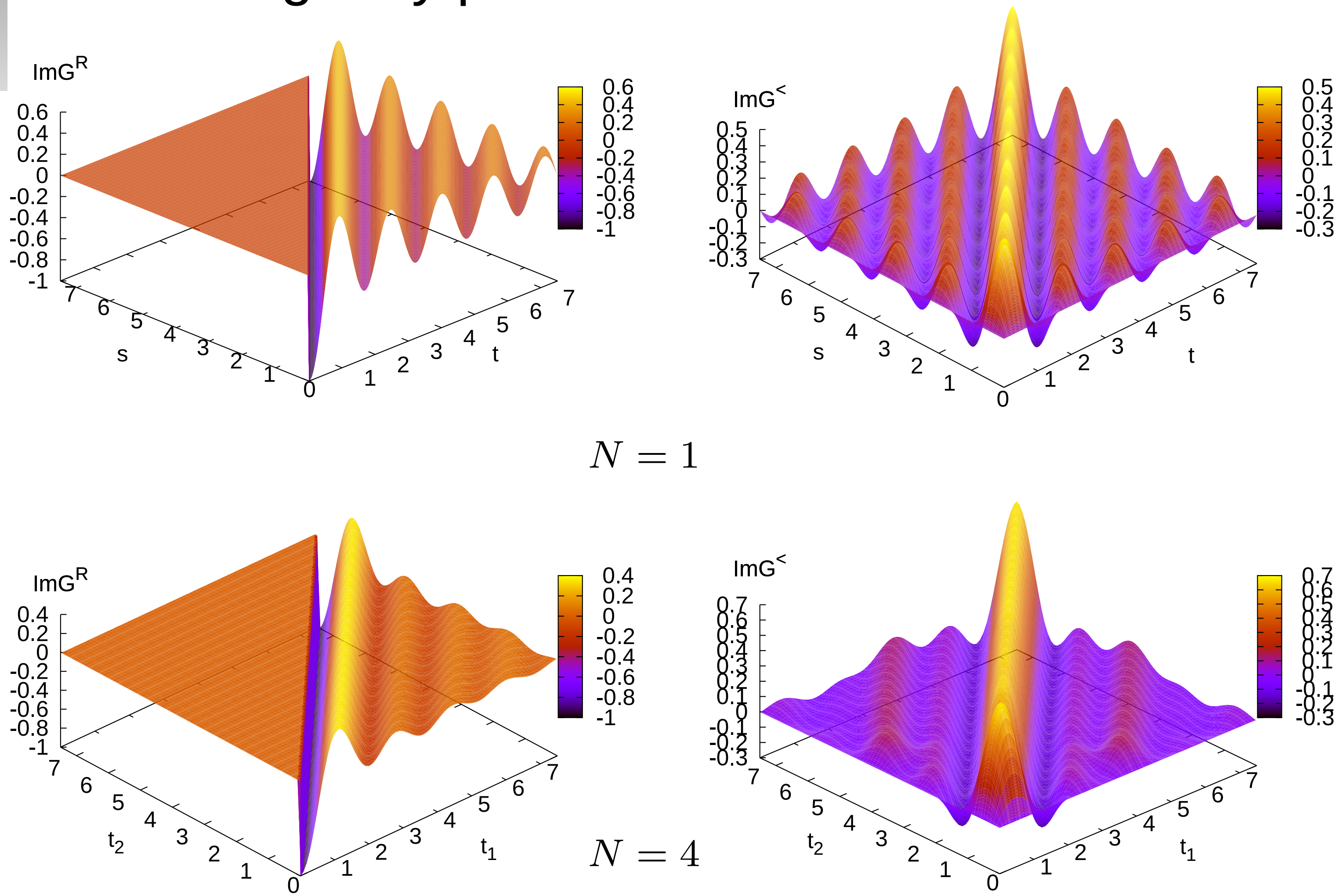
- Wide band limit approximation+ single site: $\Sigma^R(t,t') \sim \delta(t-t')$
- Exact solution of the Dyson equation, no matrix inversion.

- $J_\alpha(t) = J_\alpha^R(t) + J_\alpha^<(t)$
- Symmetric bias: $\mu_{\alpha,\beta} = \mu_0 \pm eV/2$.
- The coupling controls the transient oscillation
- Occupation number: $N(t) = \mathrm{Im}\sum_{m\in S} G_{mm}^<(t,t)$





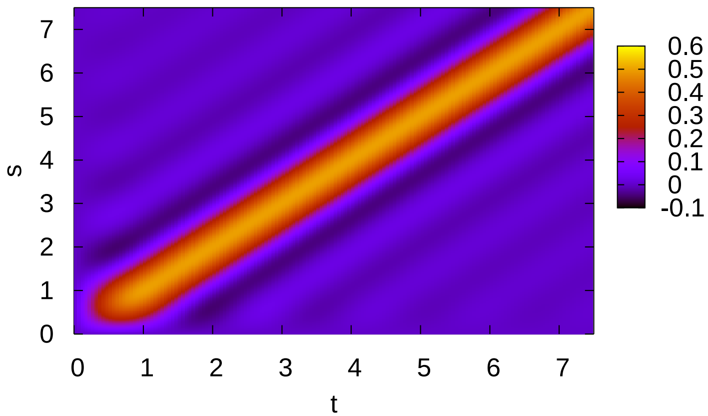
- The imaginary parts are relevant!

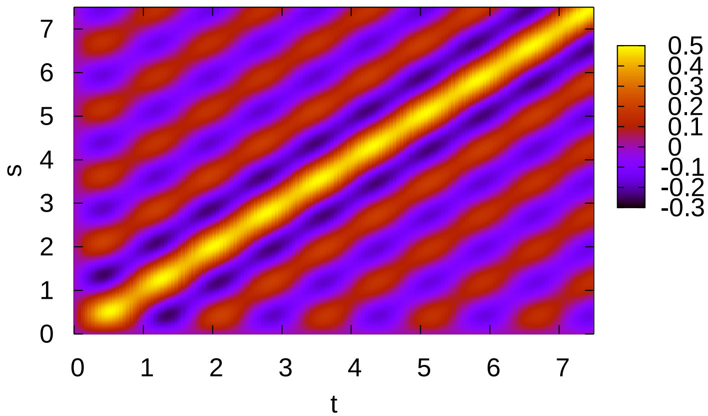


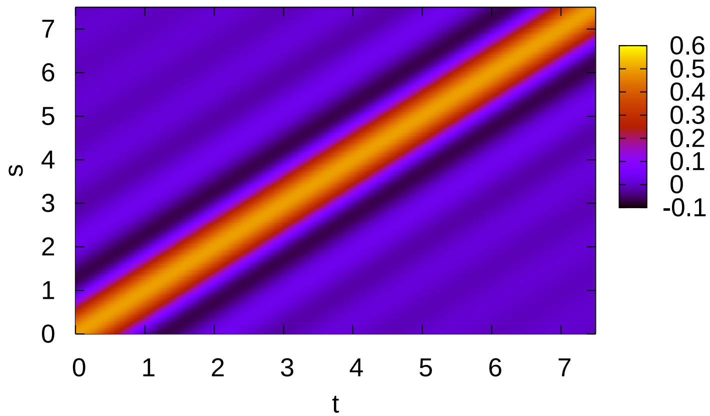
Criteria for the steady state

$$J_\alpha^<(t) = -2U^2 \mathrm{Im} \int_0^t ds G^<(t,s) F_2(s,t))$$

$$G(t,s) = G(t-s,0).$$

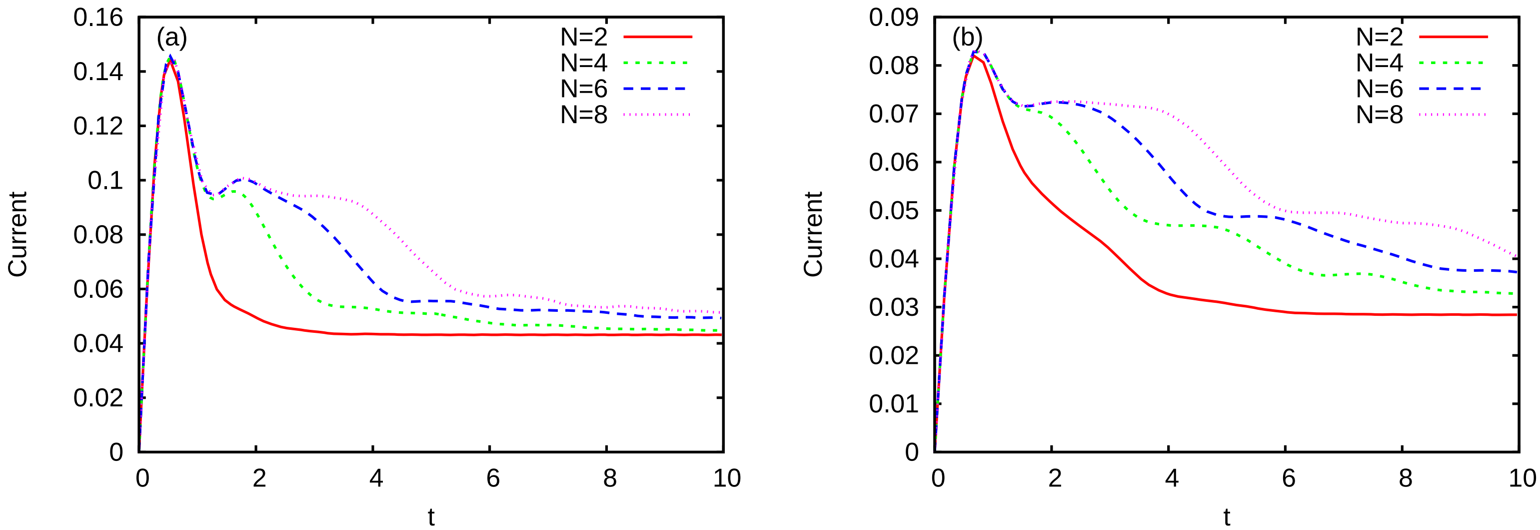






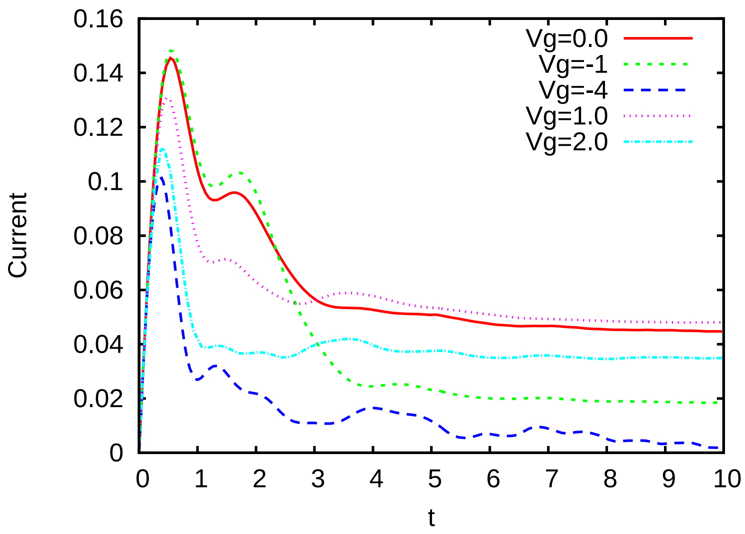
- Non-oscillatory smooth transition to the SS.
- Step-like behavior at moderate coupling.

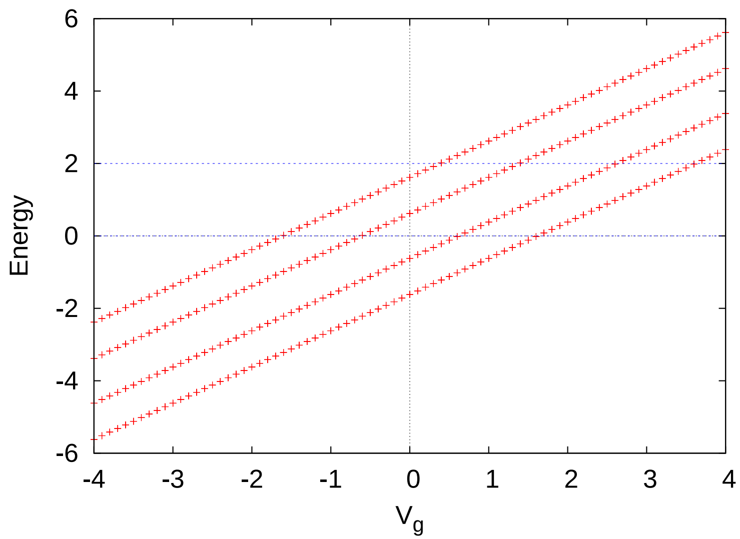
Parameters: (a) $U = 0.75$, (b) $U = 0.5$, $kT = 0.0001$, $V = 1$.



Locating QD levels within the bias window

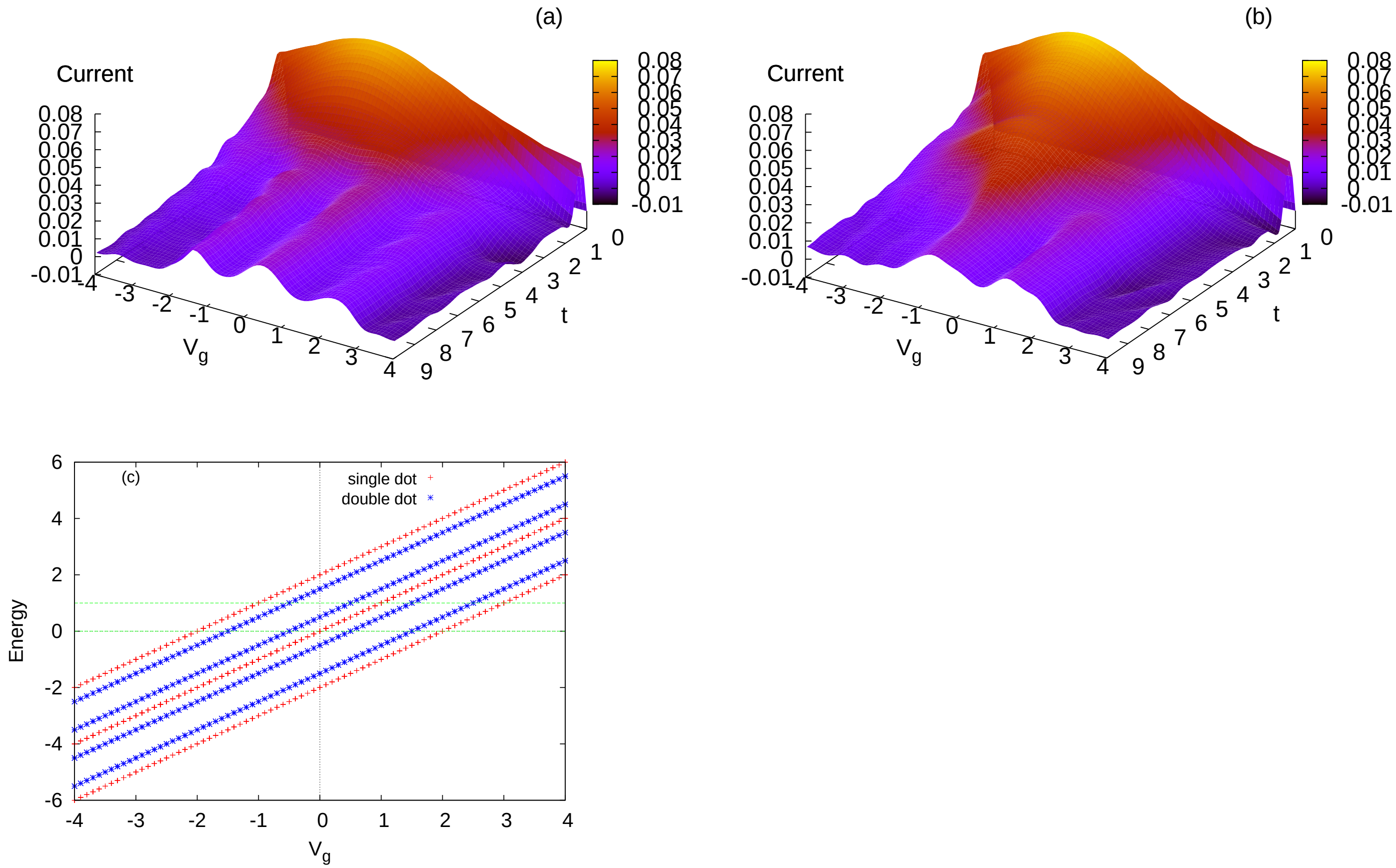
- $E_\lambda \longrightarrow E_\lambda + V_g$ (global shift).
- Tuning the highest energy levels within the BW $\longrightarrow$ steps.
- $J_\alpha(t) \sim$ states within the BW





Single and double QDs

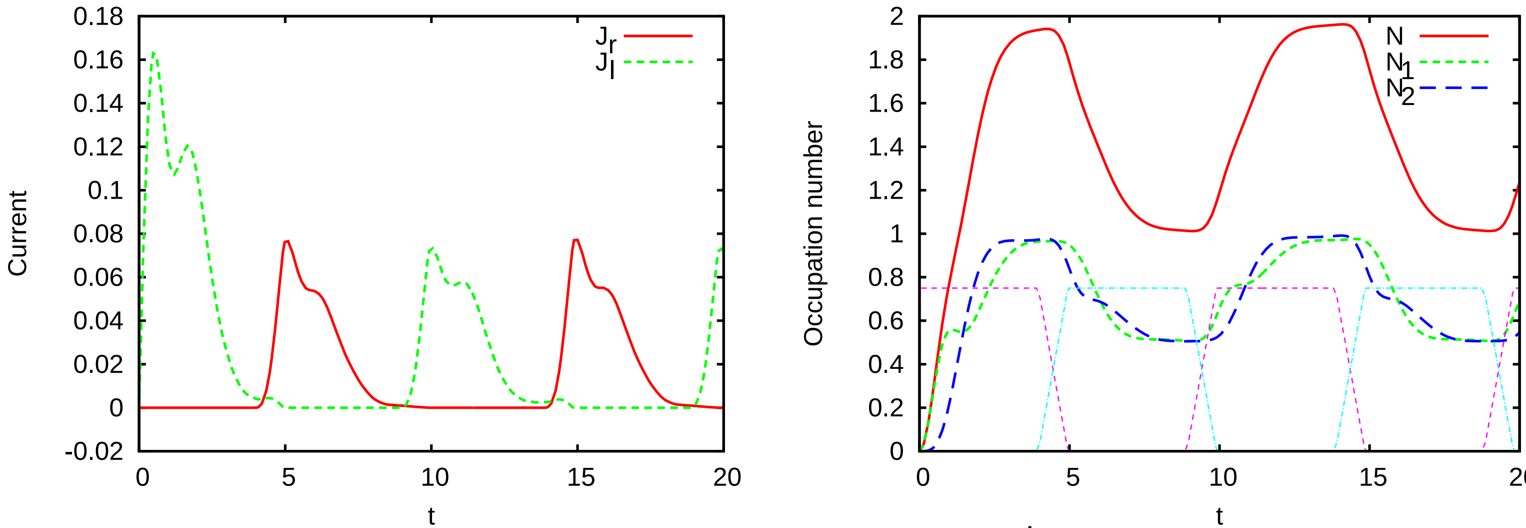
- Single dot $t_{ij} = 1$, $i, j = 1, .., 4 \rightarrow$ 3 resonances.
- Double dot $t_{13} = t_{24} = 0.5$.
- Finite bandwidth $\rightarrow$ negative currents.



A two-level turnstile pump

- $V_l(\omega t) = v_l f(\omega t)$ and $V_r(\omega t) = v_r(1 - f(\omega t))$

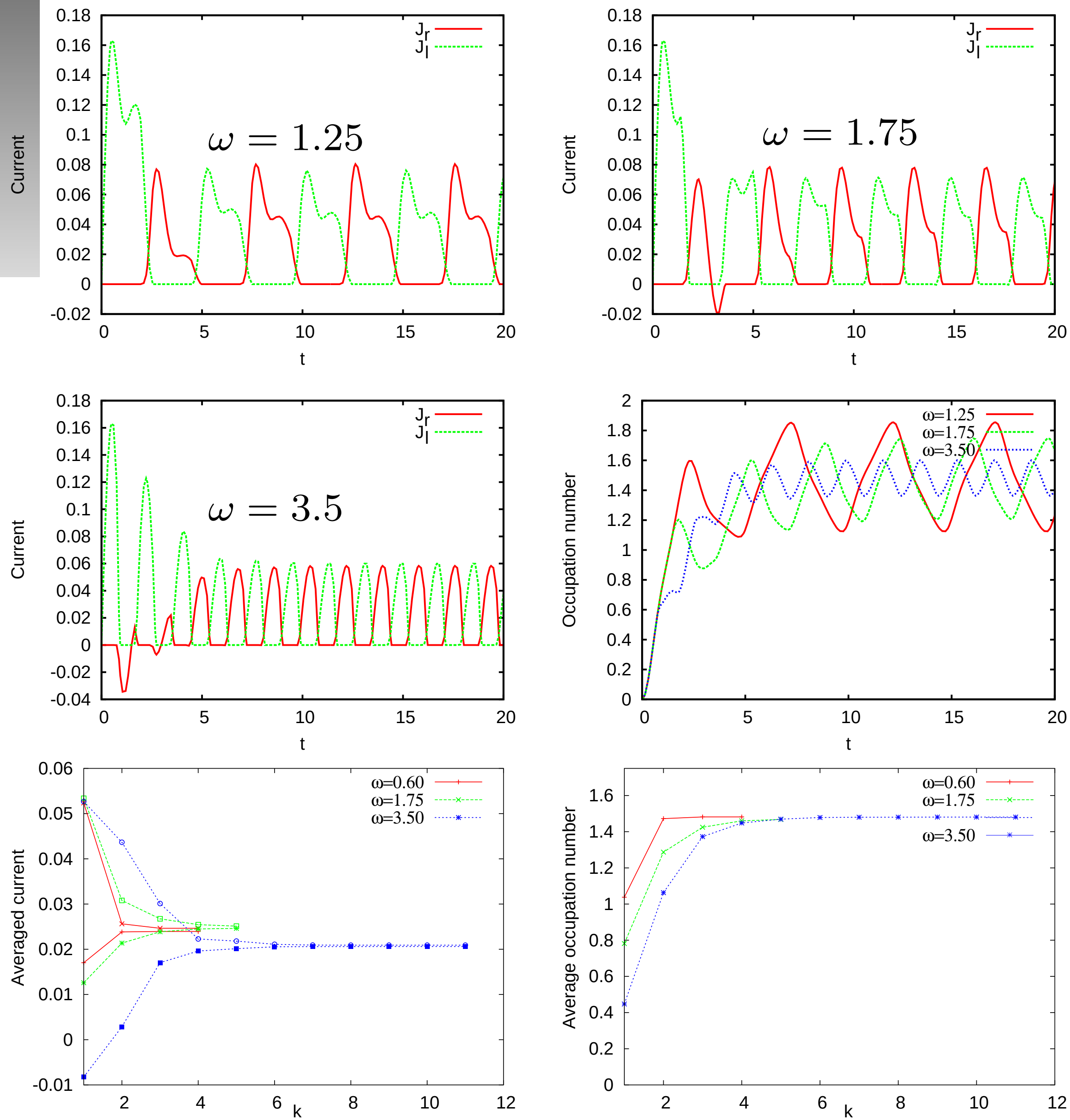
  L. Kouwenhoven *et al.* Phys. Rev. Lett. **67** (1991)



Period-averaged current $\overline{J}_{\alpha,k} = \frac{1}{T}\int_{t_{k-1}}^{t_k} dt J_\alpha(t), \qquad \alpha = l, r.$

Loading/unloading sequence $\rightarrow Q_{\rm pumped} \sim 1.$

Transient effects and nonadiabatic regime



$$\overline{J}_l = \overline{J}_r$$

Conclusions and further (open) problems

- Many-level QDs have more interesting transients.
- Informations on the location of the QD levels.
- Turnstile pump.
- Magnetic field effects.
- Quantum Interference.