International workshop: Dynamics in Coupled Network Systems

Weierstrass Institute for Applied Analysis and Stochastics

Berlin, November 20–22, 2023

www.wias-berlin.de/workshops/dcns23

Organizers:

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Weierstrass Institute for Applied Analysis and Stochastics

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1 General Information

Dear Workshop Participant,

Welcome to the Weierstrass Institute for Applied Analysis and Stochastic in Berlin.
For your convenience please find below more information regarding our workshop:

Entrance to the building will be granted by showing your participant′s badge.

Lectures are given in the Erhard-Schmidt lecture room on the ground floor.

Coffee breaks will be in the lobby in front of the Erhard Schmidt lecture room. We will provide snacks and drinks.

Poster session will be in the lecture rooms 405 & 406 on the 4th floor of the building on **Monday, November 20 from 6:00 pm**. We will provide sandwiches and drinks.

Smoking in the building, also electronic cigarettes, is not allowed. As your coffee breaks take place on the ground floor, please use this opportunity to smoke outside of the building.

To use a **WiFi connection** with your own laptop kindly use the personal card that you received at the registration. Eduroam will of course work too.

Computer facilities are provided for your use in the graphics room on the ground floor to the right-hand side coming from the doorkeeper′s. Any workstation in this room may be used. For login please enter the following:

User name:	dcns231
Password:	dC#rxbs4

For logout, you can use the logout-selection on the K-menu (left down corner). Please be aware that this account is used by all workshops participants. Do not leave any confidential data in its home directory. All remaining files will be deleted after the workshop.

Workshop dinner will be held in the restaurant Maximilians, Friedrichstr. 185-190, 10117 Berlin on
Tuesday November 22 at 7:00 pm.

Assistance will be given by anybody of the WIAS staff participating in the workshop (wearing blue badges).

We wish you a pleasent stay in Berlin.
Yours sincerely,

Organizers.

2 Program

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11:40	**Georg Gottwald (Sydney)** A stochastic approximation for the finite-size Kuramoto-Sakaguchi model
12:20	**Marius Yamakou (Erlangen)** Heterogeneity, Chimera and self-induced stochastic resonance in coupled oscillators
13:00 - 14:30	LUNCH
14:30	**Katharina Krischer (Garching)** Emergence of multifrequency clusters in nonlocally coupled adaptive systems: experiment and model
15:10	**Diego Pazó (Santander)** Volcano transition in populations of phase oscillators with random interactions
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16:20	**Michael Rosenblum (Potsdam)** Feedback control of synchrony in an ensemble with time-varying coupling
17:00	**Michael Zaks (Berlin)** Geometry of continua of steady states in ensembles of globally coupled oscillators
17:40	**Zheng Bian (São Carlos, SP)** Emergence of chaotic cluster synchronization in heterogeneous networks
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10:10	**Bastien Fernandez (Paris)** Proof of feedback delay dependence of the stability of synchronized degrade-and-fire oscillations implying a common activator
10:50	**Giulio Zucal (Leipzig)** Action convergence: from graph to hypergraph limits
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11:40	**Bob Rink (Amsterdam)** A parametrization method for high-order phase reduction in networks of coupled oscillators
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13:00 - 14:30	LUNCH
14:30	**Isabelle Schneider (Berlin)** Symmetry groupies of networks and dynamical systems
15:10	**Eddie Nijholt (London)** Exotic symmetries in network dynamical systems
15:50 - 16:20	COFFEE BREAK
16:20	**Deniz Eroglu (Istanbul)** Network dynamics reconstruction from data: emergent hypergraphs and critical transitions to synchronization
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17:40	**Oleksandr Burylko (Kyiv)** Time-reversible dynamics in a system of two coupled active rotators
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11:50	**Áine Byrne (Dublin)** Macroscopic descriptions for networks of Izhikevich neurons
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14:30	**Dmitry Turaev (London)** Non-ergodicity of high-energy motions of large systems of repelling particles
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3 List of posters

Poster session will be in the lecture rooms 405 & 406 on the 4th floor of the building on **Monday, November 20 from 6:00 pm**. We will provide sandwiches and drinks.

- **Dönmez, Bengi**
  Vrije University Amsterdam, Netherlands
  Synchronization in networks of anticipatory agents

- **Eser, Muhittin Cenk**
  Freie Universität Berlin
  Interlayer synchronization by time-varying network topology

- **Mau, Erik**
  Universität Potsdam, Germany
  Design and analysis of limit cycle systems with phase-isostable coordinates

- **Stöhr, Mina**
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  Square waves and Bykov T-points

- **Thomé, Nicolas**,
  Technische Universität München, Germany
  Exploring the emergence of 3-cluster solutions in Stuart-Landau Oscillators under linear global coupling

4 Abstracts

Multifunctionality in reservoir computing

Amann, Andreas
University College Cork, Ireland

The concept of multifunctionality relates to the ability of a single network to perform multiple tasks, without retraining. This is for example evident in biological neural networks, which are able to seamlessly switch between different previously learned tasks involving the same neurons. Here we try to understand this effect using a reservoir computer, which is a simple trainable dynamical system. We find that multistability and co-existence of attractors lead naturally to multifunctionality. Surprisingly, training of certain attractors often leads to the emergence of other untrained attractors, and the mechanisms which cause this are investigated.

Emergence of chaotic cluster synchronization in heterogeneous networks

Bian, Zheng
University of São Paulo, Brazil

Cluster synchronization plays an important role in the proper functioning of many real-world network systems. The mechanism behind the cluster's emergence and transformation in response to parameter change, especially in heterogeneous networks that lack symmetry, is crucial to the general understanding of collective behavior in realistic large network systems and yet has remained out of reach by existing approaches. Here, we uncover a mechanism for cluster synchronization in heterogeneous networks by developing a heterogeneous mean field approach along with a self-consistent theory to analyze the cluster structure and its stability. The detailed insights gained in our work provide a timely addition to the study of complex systems and reveals the dynamical nature of the cluster synchronization in random networks.

Symmetries and collective dynamics in phase oscillator networks on graphs, graph limits, and beyond

Bick, Christian
Vrije Universiteit Amsterdam, Netherlands

Phase oscillators on graphs provide insights into the emergence of collective oscillatory dynamics. If the number of interacting units is large, graph limits and dynamics thereon are a useful tool to understand the dynamics. We consider dynamical systems on graph limits from the perspective of symmetries and discuss implications for finite-dimensional systems. Moreover, we discuss dynamics on graph limits and their bifurcation, also with respect to higher-order interactions beyond graphs.

Time-reversible dynamics in a system of two coupled active rotators

Burylko, Oleksandr
Institute of Mathematics of NAS of Ukraine, Ukraine

We study two coupled active rotators with Kuramoto-type coupling and focus our attention to specific transitional regimes where the coupling is neither attractive nor repulsive. We show that certain such situations at the edge of synchronization can be characterized by the existence of a time-reversal symmetry of the system. We identify two different cases with such a time-reversal symmetry. The first case is characterized by a non-reciprocal attractive/repulsive coupling. The second case is a reciprocal coupling exactly at the edge between attraction and repulsion. We give a detailed description of possible different types of dynamics and bifurcations for both cases. In particular, we show how the time-reversible coupling can induce both oscillation death and oscillation birth to the active rotators. Moreover, we analyse the coexistence of conservative and dissipative regions in phase space, which is a typical feature of systems with a time-reversal symmetry. We show also, how perturbations breaking the time-reversal symmetry and destroying the conservative regions can lead to complicated types of dissipative dynamics such as the emergence of long-period cycles showing a bursting-like behavior.

Macroscopic descriptions for networks of Izhikevich neurons

Byrne, Áine
University College Dublin, Ireland

A number of recent studies have employed the Lorentz ansatz to reduce a network of Izhikevich neurons to a tractable mean-field description. In this talk, I will discuss an equivalent phase model for the Izhikevich model and apply the Ott-Antonsen ansatz to derive the mean-field dynamics in terms of the Kuramoto order parameter. In addition, I will show that by defining an appropriate order parameter in the voltage-firing rate framework, the conformal mapping of Montbrio et al. (2015), which relates the two mean-field descriptions, remains valid. I will present a brief analysis of the model, highlighting the non-trivial relationship between firing rate and within-population synchrony.

Synchronization in networks of anticipatory agents

Dönmez, Bengi
Vrije University Amsterdam, Netherlands

The study focused on a coupled Kuramoto system composed of agents that anticipate the future states of their neighbors based on past data and attempt to align their states accordingly. It is demonstrated that this anticipatory behavior leads to multiple synchronized solutions with varying collective frequencies and distinct stability characteristics. An exact condition for the stability of the synchronized states is derived. It is shown that multistability can be exhibited by the system, with convergence to different synchronized solutions being dependent on the initial conditions.

A quasi-stationary approach to dynamical metastability for weakly interacting particle systems

Engel, Maximilian
Freie Universität Berlin, Germany

We consider systems of N weakly interacting SDEs such as given by the Glauber dynamics associated to the classical mean-field $O(2)$ spin system from statistical mechanics. Well-known results consider the $N \to \infty$-limit of the empirical measures as a weak solution to a nonlocal parabolic PDE of McKean-Vlasov type. However, this approach has so far not been able to fully explain and quantify numerical observations for the small noise regime and bounded interaction potential, indicating fast attraction to a certain concentration within a typical length scale followed by slow leakage on a parabolic time scale. We employ methods involving sub-Markovian semigroups, quasi-stationary distributions and associated spectral methods for the N-dimensional SDE system to resolve these time and length scales, tracking also the dependence on N growing large.
This is joint work with Zachary Adams and Rishabh Gvalani (both MPI MIS Leipzig).

Network dynamics reconstruction from data: emergent hypergraphs and critical transitions to synchronization

Eroglu, Deniz
Kadir Has University, Turkey

Complex systems consist of many interconnected components organized as networks. Moreover, the structure of such networks may evolve by their intrinsic rhythms. Our understanding of dynamical complex systems relies on our ability to infer appropriate models from observations. In this talk, we discuss how to learn equations of motion from observational data that underlie evolution and why the recovered dynamics contain higher-order edges, although they are not initially in the model. The results provide a necessary starting point to predict and control the dynamics.

Interlayer synchronization by time-varying network topology

Eser, Muhittin Cenk
Freie Universität Berlin, Germany

This study explores how a network of oscillators with two layers and changing inter-layer topology syncs. Two factors were analyzed: the number of connections between layers and the timescale of topology changes. A dynamic topology with randomly shifted inter-layer links improves synchronization with fewer connections needed. At a critical switching time, the transition from low to high inter-layer synchronization occurs abruptly. Our first interpretation is that the basin of attraction of the desynchronized state gradually shrinks and disappears, driving synchronization in complex systems. In this study, we delve deeper into the intricate dynamics of this phenomenon, further clarifying the mechanisms driving synchronization in such complex systems.

Proof of feedback delay dependence of the stability of synchronized degrade-and-fire oscillations implying a common activator

Fernandez, Bastien
Centre national de la recherche scientifique, France

Feedback delay plays a crucial role in the quorum sensing synchronization of synthetic gene oscillators. While this role has been evidenced at the theoretical level in a simplified system of degrade-and-fire oscillators coupled via a common activator protein, full mathematical certifications remained to be provided. In this talk, I will prove for the very same model that the synchronized degrade-and-fire oscillations are, in absence of delay, unstable in every arbitrarily large population, and are otherwise asymptotically stable for every positive delay. The proof is part of an extensive study of the population dynamics, which puts emphasis on analyzing the dependence on the number of oscillators.
Joint work with Matteo Tanzi.

A stochastic approximation for the finite-size Kuramoto-Sakaguchi model

Gottwald, Georg
The University of Sydney, Australia

We perform a stochastic model reduction of the Kuramoto-Sakaguchi model for finitely many coupled phase oscillators with phase-frustration. Whereas coupled oscillators asymptotically exhibit stationary states and a constant order parameter, finite-size networks exhibit non-stationary fluctuations. We derive a stochastic description of the synchronized oscillators. This is achieved by approximating the collective effect of the non-entrained oscillators on the synchronized cluster as a Gaussian process. This allows for an effective closed evolution equations for the synchronized oscillators driven by a two-dimensional Ornstein-Uhlenbeck process. Our reduction reproduces the stochastic fluctuations of the order parameter and leads to a simple stochastic differential equation for the order parameter.

Causality from correlation

Gross, Thilo
Helmholtz Institute for Functional Marine Biodiversity, Germany

Common wisdom has it that correlation does not imply causality. Indeed, naively misinterpreting correlation as evidence of causality is a widespread falacy. However, correlation and causality are not completely disconnected. To explore this connection one first needs to formulate a notion of causality. A common choice is 'Granger causality' which construes causaility as correlation with a time delay. From a dynamical systems perspective this delayed-correlation-as-causality is problematic for several reasons. A more natural representation of causality is offered by the Jacobian matrix of a system, which directly encodes how the system would respond to a change in a variable. Moreover in a dynamical systems correlations between variables are captured (to linear order) by the covatriance matrix. The connection between these two matrices is well known: If we know the Jacobian of a system then it is typically easy to compute the expected covariance. Which shows that we can easily compute correlation from causation, but can we invert this relationship to find causality from correlation? In general this in not possible because some information is lost along the way, but if we have some additional information about the system to make up for this loss then computing causality from correlation is possible. In this talk I argue that in dynamical networks enough information is often available and demonstrate that the proposed method can reconstruct causal relationships in a multiplex ecological network.

The space of power grid dynamics

Hellmann, Frank
Potsdam Institute for Climate Impact Research, Germany

By symmetry arguments, we obtain a normal form of the dynamics of power grids that preserves the full physics of the coupling. This leads to novel matrix and adaptive network formulations of grid dynamics.

On singularities of slowly adaptive networks

Jardón-Kojakhmetov, Hildeberto
University of Groningen, Netherlands

Slow adaptation in networked systems can be regarded as a dynamic bifurcation problem. As such, adaptive systems may exhibit very complicated behavior, particularly in the vicinity of singular equilibria of the (fast) network dynamics. In this talk, I discuss a concrete simple example of where such problems arise. I also present some recent progress in regards to the desingularization of nilpotent singularities of adaptive networks.

Emergence of multifrequency clusters in nonlocally coupled adaptive systems: experiment and model

Krischer, Katharina
Technische Universität München, Germany

In an electrochemical experiment, we observed multifrequency clusters whose emergence we attribute to adaptive, nonlocal coupling. In this talk, I will review the experiments and discuss first simulations employing a prototypical system of coupled oscillators with adaptive and nonlocal coupling. Particular emphasis will be placed on the role of nonlocality in the coupling, and the results will be contrasted with literature results on a fully coupled network with adaptive coupling strength.

Emergence of self-sustained fluctuations in models for the dark marine microbiome

Lücken, Leonhard
Carl von Ossietzky Universität Oldenburg, Germany

The deep-sea microbiome is one of the largest biomes on earth with a fundamental role for regulating global biogeochemical cycles and climate. Yet its temporal variability remains largely unexplored. Being separated from atmospheric influences, the dark biosphere is generally considered to be a static environment. Recent empirical studies of deep-sea microbial community composition, however, revealed remarkable variability among samples, which potentially could be explained by ongoing species turnover. I present simulations for a population dynamics model of marine microbial communities adapted to deep sea environmental conditions. Even in static environments and especially under conditions of energy scarcity, we observe self-sustained chaotic fluctuations. These results may suggest a view of the dark ocean microbiome as a highly dynamic system, where even under stable environmental conditions microbial communities might exhibit a larger intrinsic variability than previously thought.

Design and analysis of limit cycle systems with phase-isostable coordinates

Mau, Erik
Universität Potsdam, Germany

The phase-isostable description, or augmented phase reduction, is a powerful tool for studying limit-cycle dynamics, as it considers amplitude dynamics. Each limit-cycle system has an unambiguous phase-isostable representation valid in the basin of the limit cycle, but finding the appropriate transformation is almost always a numerically demanding task. However, the reverse approach of choosing a transformation from phase-isostable coordinates to some observation coordinates allows for the exact design of a dynamical system with preferred properties of the limit cycle. These include frequency, stability, signal form, and response curves. We demonstrate this method on a two-dimensional system, designing a generalized Stuart-Landau model. Furthermore, we present a new approach to using augmented phase reduction to derive higher terms in the phase reduction of coupled two-dimensional limit-cycle oscillators. We demonstrate the method for the well-known van der Pol model and use it to determine the borders of Arnold's tongue in the second order.

Exotic symmetries in network dynamical systems

Nijholt, Eddie
Imperial College London, United Kingdom

Many coupled cell systems show highly unusual dynamical behavior as a generic phenomenon. Examples include invariant (synchrony) spaces, eigenvalue degeneracy and elaborate synchrony-breaking bifurcation scenarios. These are all observed –and quite well-understood– in systems with finite group symmetry, though many examples exist of non-symmetric networks with the same anomalous behavior. We present a vast generalisation of this classical group set-up, where the symmetry maps need not be invertible and may map between different dynamical systems. These so-called quiver symmetries allow for an equivalent description of many forms of network structure, and capture even more geometric features, such as the presence of invariant synchrony spaces, subnetworks and hidden symmetry. We then show that quiver symmetry may be preserved in various complexity reduction techniques, paving the road to dynamical techniques that are tailor-made for networks.
This is based on joint work with Sören von der Gracht and Bob Rink.

Periodic solutions in next generation neural field models

Omelchenko, Oleh
Universität Potsdam, Germany

We consider a next generation neural field model which describes the dynamics of a network of theta neurons on a ring. For some parameters the network supports stable time-periodic solutions. Using the fact that the dynamics at each spatial location are described by a complex-valued Riccati equation we derive a self-consistency equation that such periodic solutions must satisfy. We determine the stability of these solutions, and present numerical results to illustrate the usefulness of this technique. The generality of this approach is demonstrated through its application to several other systems involving delays, two-population architecture and networks of Winfree oscillators.
This talk is based on joint work with Carlo Laing.

Volcano transition in populations of phase oscillators with random interactions

Pazó, Diego
Instituto de Física de Cantabria, Spain

Populations of heterogeneous phase oscillators with frustrated random interactions exhibit a quasi-glassy state in which the distribution of local fields is volcano-shaped. In a recent work, Ottino-Löffler and Strogatz replicated the volcano transition in a solvable model using a symmetric low-rank, random coupling matrix M. In this contribution, we extend that model including tunable non-reciprocal interactions, i.e. asymmetric M. Our numerical simulations fully confirm the analytical results. To put our work in a wider context, we also investigate numerically the volcano transition in the analogous model with a full-rank random coupling matrix.

Dynamics of oscillator populations with distributed phase shifts in coupling

Pikovsky, Arkady
Universität Potsdam, Germany

A phase shift in the coupling of an oscillator to an external force occurs naturally due to intertia in the response to the force or due to a delay. We consider a population of oscillators, driven by a global mean field, but with distributed phase shifts of their response. We demonstrate that this population can be described by considering an auxiliary ensemble without phase shifts and by modifying the order parameters. Examples include noise-driven oscillators, Kuramoto-type oscillators with a distribution of natural frequencies, and rotators described by second-order equations for the phases.

A parametrization method for high-order phase reduction in networks of coupled oscillators

Rink, Bob
Vrije Universiteit Amsterdam, Netherlands

I will present a new method for high-order phase reduction in networks of weakly coupled oscillators. This method works by computing an asymptotic expansion for an embedding of a persisting normally hyperbolic torus, as well as for the reduced phase dynamics in local coordinates. The method is not restricted to coupled stable limit cycles or coupled planar periodic orbits. Moreover, it yields the phase dynamics directly in resonant normal form. I will illustrate the method to predict remote synchronization.
This is joint work with Eddie Nijholt and Soeren von der Gracht.

Feedback control of synchrony in an ensemble with time-varying coupling

Rosenblum, Michael
Universität Potsdam, Germany

The study problem is motivated by neuroscience, where in cases of some pathology, e.g., Parkinson's disease, stimulation is necessary to control the level of coherence in a neuronal population. Typically, one exploits a highly interconnected or globally coupled neuronal network to model the pathological brain rhythm. However, these models do not reflect the strong amplitude modulation inherent to experimentally measured data. To mimic these data, we consider an ensemble with time-varying connectivity and use a pulsatile stimulation scheme to control the level of synchrony. Though our model of brain activity is phenomenological and simple, we concentrate on several realistic features: (i) stimulation by charge-balanced pulses, (ii) extraction of the rhythm of interest from its mixture with noise, and (iv) real-time estimation of the phase and amplitude of the signal. We discuss the algorithms for the automated tuning of feedback control parameters.

Symmetry groupies of networks and dynamical systems

Schneider, Isabelle
Freie Universität Berlin, Germany

Symmetries described by group transformations help immensely in our task to qualitatively characterize solutions of dynamical systems providing that they exist. In this talk based on my habilitation thesis, we significantly enlarge the class of dynamical systems which can be studied by symmetry methods, moving our focus from groups to groupoids as the underlying algebraic structure describing symmetry.
Building on the groupoid framework, we fundamentally generalize the notion of equivariance and equivariant bifurcation theory. In conclusion, we present a new unified theory of symmetric spatio-temporal patterns with many applications in networks and dynamical systems.

Square waves and Bykov T-points

Stöhr, Mina
WIAS Berlin, Germany

Complex patterns emerge within a diverse realm of nonlinear systems. Traditionally, these patterns appear as spatial patterns in systems described by partial differential equations. However, intriguingly, similar phenomena can also manifest in various time-delay systems. For instance, in models of optical and optoelectronic systems featuring feedback loops, such phenomena arise in a purely temporal context, devoid of spatial variables. We study a delay-algebraic equation that models a vertical external-cavity Kerr–Gires–Tournois interferometer subjected to anti-resonant injection. Within this system, we observe the intriguing formation of complex coexisting square-wave patterns. These patterns are arranged on snaking branches around a Maxwell point. We present a systematic methodology for the study of these patterns, akin to approaches employed in the analysis of localized patterns. Our approach relies on homoclinic bifurcation theory, enabling a comprehensive exploration of the dynamics within these systems. Specifically, we elucidate the inner workings of the collapsed snaking scenario of square waves within this model through a Bykov T-point.

Uniformly expanding coupled maps: Self-consistent transfer operators and propagation of chaos

Tanzi, Matteo
King's College London, United Kingdom

Recently, much progress has been made in the mathematical study of self-consistent transfer operators which describe the mean-field limit of globally coupled maps. Conditions for the existence of equilibrium measures (fixed points for the self-consistent transfer operator) have been given, and their stability under perturbations and linear response have been investigated. In this talk, I am going to describe some novel developments on dynamical systems made of N uniformly expanding coupled maps when N is finite but large. I will introduce self-consistent transfer operators that approximate the evolution of measures under the dynamics, and quantify this approximation explicitly with respect to N. Using this result, I will show that uniformly expanding coupled maps satisfy propagation of chaos when N tends to infinity, and I will characterize the absolutely continuous invariant measures for the finite dimensional system.

Exploring the emergence of 3-cluster solutions in Stuart-Landau Oscillators under linear global coupling.

Thomé, Nicolas
Technische Universität München, Germany

Our current research delves into the interaction between Stuart-Landau oscillators under linear global coupling, with a particular focus on understanding the formation of various cluster configurations, including the less common 3-cluster solution. Building upon earlier work by Kemeth et al. [1], which introduced the cluster singularity as an organizing center for the formation of 2-cluster solutions, our work focuses on the emergence of 3-cluster solutions. To investigate the emergence of 3-cluster solutions, we use a reduced model [2], introducing two new parameters to fix the mass ratios between the clusters and thus constraining the solution space. Using the bifurcation continuation software AUTO-07p we uncovered the emergence of 3-cluster solutions. We found that stable solutions appear through saddle-node bifurcations of periodic orbits (SNP). In parameter space, the SNP points for different cluster distributions merge with the transversal instability of the 2-cluster solution into a pair of distinct codimension-2 points. We refer to these new bifurcation points as secondary type-II cluster singularity points. Here, the term 'secondary' signifies that there was one cluster singularity preceding it, and 'type-II' indicates the presence of two distinct frequencies. Furthermore, we show that these two codimension-2 points depend on just one of the earlier introduced parameters. The presented work clarifies the emergence of 3-cluster solutions and we assume that a similar approach is applicable to higher-order cluster solutions. In the future, we want to analyze bifurcations transverse to the 3-cluster solutions, which we have not done yet.

[1] Felix P Kemeth et al 2021 J. Phys. Complex. 2 025005
[2] Ku WL, Girvan M, Ott E.. Chaos. 2015 Dec;25(12):123122.

Non-ergodicity of high-energy motions of large systems of repelling particles

Turaev, Dmitry
Imperial College London, United Kingdom

We prove that the motion of any number of particles interacting by a smooth repelling potential and confined to a bounded region is, under mild conditions, non-ergodic for all sufficiently large energies. Specifically, we show that the "choreographic"motions where all particles follow approximately the same path close to an elliptic periodic orbit of the single-particle system are described by a generalized Fermi-Pasta-Ulam chain of oscillators and are KAM-stable in the high-energy limit. The same holds true for more general choreographic motions where the particles follow a grid formed by parabolic periodic orbits of the single-particle system.

Coherence resonance in influencer networks

Tönjes, Ralf
Universität Potsdam, Germany

Complex networks are abundant in nature and many share an important structural property: they contain a few nodes that are abnormally highly connected (hubs). Some of these hubs are called “influencers"because they couple strongly to the network and play fundamental dynamical and structural roles. Strikingly, despite the abundance of networks with influencers, little is known about their response to stochastic forcing. Here, we show that subjecting influencers to an optimal intensity of noise results in enhanced network synchronization. This new network dynamical effect, which we call “coherence resonance in influencer networks", emerges from a synergy between network structure and stochasticity and is highly nonlinear, vanishing when the noise is too weak or too strong. Our results reveal that the influencer backbone can sharply increase the dynamical response of complex systems.

Heterogeneity, Chimera and self-induced stochastic resonance in coupled oscillators

Yamakou, Marius
Friedrich-Alexander-Universität Erlangen-Nürnberg, Germany

In the first part of the talk, we uncover the mechanism of self-induced stochastic resonance (SISR) in an excitable oscillator. Then, we show that, in contrast to the prevailing results in the previous literature, showing that network heterogeneity can always be optimized to enhance collective behaviors such as synchronization or resonance in coupled oscillators, the effect of heterogeneity on SISR can only be antagonistic. In the second part of the talk, we present yet another form of chimera, which we refer to as self-induced-stochastic-resonance breathing chimera (SISR-BC). SISR-BC combines the mechanism and effects of SISR, the symmetry breaking in the rotational coupling between the slow and fast subsystems of the coupled oscillators, and the property of breathing chimera — a form of chimera state characterized by non-stationary periodic dynamics of coherent-incoherent patterns with a periodically oscillating global order parameter. Unlike other types of chimeras, SISR-BC demonstrates remarkable resilience to a relatively wide range of stochastic perturbations and persists even when the purely excitable system is significantly distant from the Hopf bifurcation threshold — thanks to the mechanism of SISR— and globally attracts random distributions of initial conditions.
This is joint work with Els Heinsalu, Marco Patriarca, Stefano Scialla, and Jinjie Zhu.

Synchronization-desynchronization transitions in neural networks

Zakharova, Anna
Bernstein Center for Computational Neuroscience Berlin, Germany

Synchronization of neurons is believed to play a crucial role in the brain under normal conditions, for instance, in the context of cognition and learning, and under pathological conditions such as Parkinson?s disease or epileptic seizures. In the latter case, when synchronization represents an undesired state, understanding the mechanisms of desynchronization is of particular importance. In other words, the possible transitions from synchronized to desynchronized regimes and vice versa should be investigated. It is known that such dynamical transitions involve the formation of partial synchronization patterns, where only one part of the network is synchronized. The most prominent example is given by chimera states [1]. In the present talk, we discuss an alternative scenario. We show how the so-called solitary states in networks of coupled FitzHugh-Nagumo neurons can lead to the emergence of chimera states. By performing bifurcation analysis of a suitable reduced system in the thermodynamic limit we demonstrate how solitary states, after emerging from the synchronous state, become chaotic in a classical period-doubling cascade [2].

[1] A. Zakharova, Chimera Patterns in Networks: *Interplay between Dynamics, Structure, Noise, and Delay, Understanding Complex Systems* (Springer, Cham, 2020) doi: 10.1007/978-3-030-21714-3
[2] L. Schülen, A. Gerdes, M. Wolfrum, A. Zakharova, *Solitary routes to chimera states*, Phys. Rev. E Letter 106, L042203 (2022) doi: 10.1103/physreve.106.l042203.

Geometry of continua of steady states in ensembles of globally coupled oscillators

Zaks, Michael
Humboldt-Universität zu Berlin, Germany

Equations, governing dynamics of globally coupled oscillators (including the Kuramoto oscillators and active rotators) have been recently shown to possess in their phase spaces, under appropriate conditions, high-dimensional continua of steady states. Depending on the coupling coefficients, these continua may be attracting or repelling as a whole, as well as consist of adjacent attracting and repelling segments. We investigate scenarios under which such continua evolve from generic isolated states of equilibrium. Close to their birth, the newborn continua are localized in the phase space; further away they may become unbounded. We characterize the global changes in the structure of invariant manifolds that accompany the transitions.

Action convergence: from graph to hypergraph limits

Zucal, Giulio
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The theory of graph limits considers the convergence of sequences of graphs with a diverging number of vertices. From an applied perspective, it aims to represent large networks conveniently. Recently, dynamics on graph limits attracted interest as an approximation of coupled dynamical systems on large networks. In this talk, I will give a brief introduction to action convergence, a recent unified approach to graph limits based on functional analysis and measure theory. Moreover, I will present an extension of action convergence to multi-linear operators, investigate its properties and explain how this notion can be applied to sequences of hypergraphs and tensors which may be useful in modelling dynamics on higher-order networks.
This talk is based on the preprint *"Action convergence of general hypergraphs and tensors"* (arXiv:2308.00226).

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