

# Universal relaxation equation for disordered systems

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Slower-than-exponential relaxations often occur in disordered systems and, lacking a microscopic theory, are commonly fit to the empirical Kohlrausch-Williams-Watts (KWW) stretched exponential or the Curie-von Schweidler (CvS) power-law algebraic decay. In this work an anomalous-diffusion limited, mixed second-order reaction equation is used to unify the above relaxation laws as different limits of the same overall behavior. Here, relaxation is modeled as a mixed second-order reaction between a concentration of reactants that undergo anomalous diffusion, and a concentration of stationary reactants. The resulting general expression is able to unify transients for a broad class of physical systems. The fit equation uses four parameters: the minority-to-majority reactant ratio  $0 \leq m \leq 1$ , the anomalous power-law exponent  $0 \leq \beta \leq 1$ , the characteristic relaxation time  $\tau$ , and the relaxation amplitude  $f_\delta$ . With the power-law  $\beta < 1$ , the  $m = 0$  and  $m = 1$  limits, respectively, of the minority reactant ratio are shown to correspond to the KWW and CvS expressions, respectively, and the intermediate  $m$  values represent a new class of previously unrecognized relaxation functions. A fitting algorithm is introduced that identifies confidence intervals for each of the four experimental parameters. Three parameters are observed to have quadratic variance around the best fit values, allowing a Wronskian formulation to yield confidence intervals. The mixing parameter  $m$ , on the other hand, has a variance which is non-quadratic, making the confidence interval highly asymmetric. This analysis unifies two empirical laws that were previously considered distinct and provides new physical insight to prominent previously published experimental transients. Prominent examples of disordered systems from biomechanics, energy storage, and dielectric relaxation show excellent fits to the proposed relaxation expression.

**Acknowledgments:** This work was supported by NSF DMR-1729016.