Discrete dissipative structures and semiconductor device equations

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Dissipative systems can be found everywhere and are of special interest in case the distance to the thermodynamic equilibrium is not very small. While some analytic problems are well understood and often a variational formulation is known [1], [2], the formulation of proper discretizations is not 'main stream'.

Semiconductors are a nice example due to the fact that an AAA battery is sufficient to get the electron-hole gas in devices seriously out of the thermodynamic equilibrium. That is the fundamental bases for the use of semiconductors as fast switches. Additionally and somewhat surprisingly their main functionality can be described by equations of thermodynamic flavor, hence a free energy and its dissipation rate.

The traditional discretization in that field (Scharfetter-Gummel scheme and backward Euler) is dissipative for arbitrary arguments [3], hence fulfills an elementary stability requirement.

In general finite element discretizations destroy the dissipative structure or stringent smallness assumptions have to be introduced. On the other hand, in case of other statistic functions than Boltzmann statistics, generalizations are necessary. Two different approaches will be discussed:
- the 'discrete chain rule' based on preserving basic stability properties of the variational formulation (compare [4]),
- and the integration of a Cauchy problem with the constant current as parameter.
The latter is reproducing the dissipation rate of the related one dimensional boundary value problem and requires the solution of nonlinear equation per edge in the boundary conforming Delaunay mesh. This involves up to now either numerical integration or other approximations. On the other hand mean value theorem based estimates guarantee bounds and provide initial guesses, too.

References

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