Mean Field Models for Self-Gravitating Particles

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We consider in this lecture parabolic-elliptic systems of the form

$$n_t = \nabla \cdot \left(D_*(\nabla p + n\nabla\varphi)\right), \tag{1}$$

$$\Delta\varphi = n\,, \tag{2}$$

which appear in statistical mechanics as hydrodynamical *mean field* models for self-interacting particles, cf. e.g. [4] and other papers by P.-H. Chavanis.

Here $n = n(x,t) \geq 0$ is the density function defined for $(x,t) \in \Omega \times \mathbb{R}^+$, $\Omega \subset \mathbb{R}^d$, $\varphi = \varphi(x,t)$ is the Newtonian potential generated by the particles of density n, and the pressure $p \geq 0$ is determined by the density-pressure relation with a sufficiently regular function $p = p(n,\vartheta)$. The parameter $\vartheta > 0$ plays the role of the temperature, and $D_* > 0$ is a diffusion coefficient which may depend on n, ϑ, x, ...

Such systems can be studied either in the *canonical ensemble* (i.e. the *isothermal* setting), when $\vartheta = \text{const}$ is fixed, or in the *microcanonical ensemble* with a variable temperature: $\vartheta = \vartheta(t)$, and the energy

$$E = \frac{d}{2}\int_\Omega p\,dx + \frac{1}{2}\int_\Omega n\varphi\,dx = \text{const}\,, \tag{3}$$

which, for a given n, defines $\vartheta = \vartheta(t)$ in an implicit way.

In this work we consider examples of density-pressure relations

$$p(n,\vartheta) = \vartheta^{d/2+1} P\left(\frac{n}{\vartheta^{d/2}}\right)$$

more general than Maxwell–Boltzmann, Fermi–Dirac and polytropic.

Interesting questions are:

- existence of entropy functionals and entropy production rates,
- existence of steady states with prescribed mass and temperature, or prescribed mass and energy,
- nonexistence of global in time solutions and their blow up,
- continuation of local in time solutions with polytropic density-pressure relations.

These results have been obtained in collaboration with Tadeusz Nadzieja (Zielona Góra), Philippe Laurençot (Toulouse), and Robert Stańczy (Łódź and Wrocław).

References

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[2] P. Biler, T. Nadzieja, R. Stańczy, Nonisothermal systems of self-interacting Fermi–Dirac particles, *Proceedings of the Conference "Nonlocal Elliptic and Parabolic Problems" 2003*, Banach Center Publications, 1–20, to appear.

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[4] P.-H. Chavanis, J. Sommeria, R. Robert, Statistical mechanics of two-dimensional vortices and collisionless stellar systems, *Astrophys. J. 471*, 385–399 (1996).