Adaptive methods and efficient data structures for stabilized finite flement methods

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The numerical approximation of nonstationary convection-diffusion-reaction problems

$$\partial_t u - \nabla \cdot (\varepsilon \nabla u) \boldsymbol{b} \cdot \nabla u \alpha u = f \tag{1}$$

with a small diffusion coefficient $0 < \varepsilon \ll 1$ remains to be a challenging task [3]. Solving those problems numerically naively by standard Galerkin finite element approximations leads to perturbed solutions with spurious unphysical oscillations. To reduce those oscillations close to regions with sharp inner or boundary layers, stabilization concepts are applied to the finite element approximations. Beside those stabilization concepts the application of adaptive mesh refinement has been considered in the past to further eliminate oscillations.

In this contribution we combine stabilized finite element methods with an a posteriori error control mechanism based on a dual weighted residual approach [1]. The dual weighted error estimator assesses the discretization error with respect to a given goal quantity of physical interest. Flexible data structures for the open source `deal.II` library are presented that are indispensable to handle the complex framework for solving the primal and dual problem in the course of the DWR philosophy. In particular, these structures offer huge potential for the application to large scale problems of practical interest. We present the derivation of our adaptive algorithm for SUPG stabilized approximations of Eq. (1). In numerical experiments its applicability is studied and demonstrated for benchmark problems of convection-dominated transport; cf. [2, 4].

Literatur

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