DFG SPP2265 Summer School on

Probability and geometry on configuration spaces

Organisers: Lorenzo Dello Schiavo (ISTA) and

Alexander Zass (WIAS Berlin)

Berlin, July 17–21, 2023

Funding by the Deutsche Forschungsgemeinschaft through the Schwerpunktprogramm 2265 *Random Geometric Systems* and by the Austrian Fonds zur Förderung der wissenschaftlichen Forschung through the ESPRIT programme Grant No. 208.

Contents

Timetable

Registration Monday 8:30–9:15
Welcome Monday 9:15–9:30

	MONDAY	TUESDAY	WEDNES-DAY	THURSDAY	FRIDAY
9:30–10:30	Pulvirenti	Herry	Conache	Nota	Jahnel
10:30–11:00	coffee break				
11:00–12:00	Pulvirenti	Herry	Tamanini	Nota	Jahnel
12:00–13:30	lunch break				
13:30–14:30	Jahnel	Nota	Herry	Pulvirenti	
14:30–15:30	Jahnel	Nota	Herry	Pulvirenti	
15:30–16:00	Müller	Callegaro	Dickson	Lenz	
16:00–16:30	coffee break				
16:30–17:30	Nota	Jahnel	Pulvirenti	Herry	
18:00–21:00			conference dinner		

The venue of the conference is the Harnack-Haus at Ihnestraße 16-20, 14195 Berlin. Most of the event will take place in the Goethe-Saal.

The conference dinner will take place at the Biergarten Alter Krug Dahlem, in Königin-Luise-Straße 52, 14195 Berlin.

Mini courses

Probabilistic and geometric aspects of Poisson point processes

Ronan Herry

Centre Henri Lebesgue, Rennes

Lecture plan

1. Reminders on point processes : Laplace transforms, Palm distribution, thinnings...
   Related literature:
   Kallenberg - Random measures

2. Poisson point processes : various characterizations.
   Related literature:
   Last and Penrose - Lectures on the Poisson Process.

3. Stochastic analysis : Itô integrals, Malliavin calculus, Ornstein-Uhlenbeck semi-group.
   Related literature:
   Last - Stochastic Analysis for Poisson Processes.
   Surgailis - On Mutiple Poisson Stochastic Integrals and Associated Markov Semigroups.

4. Coercive inequalities : Poincaré and modified log-Sobolev inequalities.
   Related literature:
   Bobkov and Ledoux - On Modified Logarithmic Sobolev Inequalities for Bernoulli and Poisson Measures.
   Chafaï - Entropies, convexity, and functional inequalities.
   Wu - A new modified logarithmic Sobolev inequality for Poisson point processes and several applications.

5. Quantitative limit theorems via the Malliavin-Stein approach.
   Related literature:
   Döbler and Peccati - Fourth Moment Theorem on the Poisson space.

Herry - Stable limit theorems on the Poisson space.
Peccati -The Chen-Stein method for Poisson functionals.
Peccati, Solé, Taqqu, Utzet - Stein's Method and Normal Approximation of Poisson Functionals.

6. Entropy-transport and concentration inequalities.
   Related literature:
   Bahcmann and Peccati - Concentration bounds for geometric Poisson functionals: Logarithmic Sobolev inequalities revisited.
   Gozlan, Herry, Peccati - Transport inequalities for random point measures.

7. Transport distance for point processes.
   Related literature:
   Dello Schiavo, Herry, Suzuki - Wasserstein geometry and Ricci curvature bounds for Poisson spaces.

Continuum percolation in random environments

Benedikt Jahnel
TU Braunschweig, WIAS Berlin

Lecture plan

1. Lecture 1: Bernoulli percolation
2. Lecture 2: Percolation in the Poisson–Boolean model
3. Lecture 3: Percolation in more general random graphs
4. Lecture 4: Cox percolation
5. Lecture 5: Applications in telecommunications

Lorentz gas dynamics: Particle systems and scaling limits with some perspectives on the role of long-range interactions

Alessia Nota

University of L'Aquila

Many interesting systems in physics are constituted by a large number of identical components so that they are difficult to analyze from a mathematical point of view. At the same time we are not interested in a detailed description of the system but rather in its collective (statistical) behavior. Therefore, it is necessary to look for all the procedures leading to simplified models which preserve all the interesting physical informations of the original system, cutting away redundant information. The central point is to outline the limiting procedures which lead from the microscopic description based on the fundamental laws of mechanics to a kinetic picture described by integro-differential equations depending on a small number of degrees of freedom. One of the most famous example of kinetic equation is the Boltzmann equation which describes the evolution of a rarefied gas.

We will begin this mini-course with an introductory lecture in which we present the paradigm of kinetic theory and discuss some fundamental properties of the Boltzmann equation, outlining the limiting procedure which leads from the microscopic description based on the Hamiltonian dynamics of N interacting particles to the Boltzmann equation.

We will then focus on a simple microscopic model, the Lorentz gas (H. A. Lorentz, 1905), which is a gas of non-interacting particles in a random configuration of scatterers. The interaction between the gas particles and the scatterers is specified by a central potential. This model is paradigmatic since it provides a rare source of exact results in kinetic theory. We will see in detail the proof of the rigorous validation of the linear Boltzmann equation from the Lorentz Gas, in the low-density limit. On the other hand, we emphasize that the rigorous derivation of linear kinetic equations from the Lorentz particle systems is so far only available for potentials that are compactly supported or decrease sufficiently fast.

The main open question is how to deal with obstacle configurations yielding long-range interaction potentials. The rigorous validation of kinetic equations for long-range interactions is indeed still open. A first partial result in this direction was obtained in 1999 by Desvillettes and Pulvirenti. More recently, in collaboration with Simonella and Velázquez, we constructed the stochastic force field generated by a Poisson distribution of sources yielding long range potentials and analysed and the precise kinetic equations describing the limit dynamics for the different interactions potentials. The purpose is to report on these results on the problem of the derivation of kinetic equations from particle systems with long-range interactions providing some perspectives in the linear and nonlinear case.

The Widom-Rowlinson model: From phase transition to metastability

Elena Pulvirenti

TU Delft

Lecture plan

1. I will start by stating some basic facts of equilibrium statistical mechanics. I will then briefly explain the theory of phase transitions and metastability and show some examples.

2. I will introduce a model of fluids in the continuum called the Widom-Rowlinson model. The energy of a particle configuration is determined by its halo, given by the union of unit discs centred at the positions of the particles. I will then show that this model has a dual representation and exhibits a *gas-liquid* phase transition.

3. I will introduce a dynamic version of the Widom-Rowlinson model, where particles are randomly created and annihilated inside a torus as if the system were in contact with an infinite reservoir of particles. I will then discuss the metastable behaviour of the WR model, when the dynamics starts from an empty torus. In particular, I will focus on the first time when the torus is fully covered by unit discs, which can be viewed as the crossover time from a *gas phase* to a *liquid phase.*

4. I will show how in order to achieve the transition from empty to full, the system needs to create a sufficiently large droplet, called critical droplet, which triggers the crossover. It turns out that the critical droplet is close to a disc of a certain deterministic radius, with a boundary that is random and consists of a large number of unit discs that stick out by a small distance. I will show how an analysis of the surface fluctuations allows us to derive both a volume term and a surface term in the asymptotics of the average crossover time.

Based on joint works with Frank den Hollander (Leiden), Sabine Jansen (Munich) and Roman Kotecky (Prague).

Invited talks

Survival and complete convergence for a branching annihilating random walk

Alice Callegaro

TU München and JGU Mainz

We study a branching annihilating random walk (BARW) in which particles move on the discrete lattice in discrete generations. Each particle produces a Poissonian number of offspring which independently move to a uniformly chosen site within a fixed distance from their parent's position. Whenever a site is occupied by at least two particles, all the particles at that site are annihilated. This can be thought of as a very strong form of local competition and implies that the system is not monotone. For certain ranges of the parameters of the model we show that the system dies out almost surely or, on the other hand, survives with positive probability. In an even more restricted parameter range we strengthen the survival results to complete convergence with a non-trivial invariant measure. A central tool in the proof is comparison with oriented percolation on a coarse-grained level, using carefully tuned density profiles which expand in time and are reminiscent of discrete travelling wave solutions.

Based on a joint work with Matthias Birkner (JGU Mainz), Jiří Černý (University of Basel), Nina Gantert (TU Munich) and Pascal Oswald (University of Basel/JGU Mainz).

Dislocation lines in three-dimensional solids

Diana Conache

TU München

This talk is related to one of the most difficult mathematical problems of solid-state physics, namely that of crystallization. One wants to prove that at positive low temperature and high density, there exist corresponding infinite volume Gibbs measures that are non-trivially periodic with the symmetry of a crystal lattice. This problem is so far out of reach. We will introduce and analyze some models for three-dimensional solids with a statistical mechanical description of dislocation lines in thermal equilibrium, which exhibit a form of symmetry breaking at low-temperature. In particular, we will focus on a mesoscopic model, motivated by the one introduced by Kosterlitz and Thouless to describe crystallization and the melting transition in two dimensions, and on a simplified microscopic model, introduced by Ariza and Ortiz.

Asymptotic Expansion of the critical intensity for the random connection model

Matthew Dickson

University of Augsburg

The Random Connection Model (RCM) is a class of spatial random graph model where the vertex set is given by a Poisson point process on $\mathbb{R}^d$ with intensity $\lambda > 0$, and an edge exists between two vertices independently with a probability that depends only on the positions of the two vertices in question. Under very minor conditions these models exhibit a percolation phase transition: there exists a critical λ_c such that for $\lambda < \lambda_c$ clusters are almost surely finite and for $\lambda > \lambda_c$ there is a positive probability that a given cluster is infinite. Evaluating λ_c is not currently possible, but as the dimension $d \to \infty$ the leading order asymptotic behaviour is known (and is universal). Here we derive further terms in the asymptotic expansion and show that after a point a qualitative difference emerges between different models. This is based on joint work with Markus Heydenreich.

Metastability for the dilute Curie Weiss Potts model with Glauber dynamics

Vicente Lenz

TU Delft

I will first introduce the general Potts interacting spins model and the Curie-Weiss Potts model model as a mean field version of it. Then I will introduce the Dilute Curie Weiss model as a random perturbation of the Curie Weiss Model, and present random graph interpretations of. I will introduce metastability through an intuitive approach to then formalize it. I then present quantitative estimates for quantities in the large volume limit under Glauber dynamics, i.e. where spins flip with Metropolis rates at inverse temperature β. The main result ensures that with high probability the DCWP model behaves like the CWO model with respect to the same metastable sets. Our proofs use the potential-theoretic approach to metastability in combination with concentration inequalities.
Based on my PhD research project with Johan Dubbeldam, Elena Pulvirenti and Martin Slowik.

Optimal transport of stationary point processes

Bastian Müller

University of Münster

We develop a theory of optimal transport for stationary random measures with a particular focus on stationary point processes. This provides us with a notion of geodesic distance between distributions of stationary random measures. In the setting of stationary point processes we leverage this transport distance to characterise the evolution of infinitely many Brownian motions as the gradient flow of the specific relative entropy w.r.t. the Poisson process. Further, we establish displacement convexity of the specific relative entropy along optimal interpolations of point processes.

The Schrödinger problem from Brownian to interacting particles: An introduction

Luca Tamanini

Università Cattolica

The Schrödinger problem dates back to 1931 but it is only in the last decade that it has gained a great success, motivated first and foremost for its interpretation as "noised" optimal transport and the related numerical applications. However, the question originally addressed by Schrödinger was not at all a transport problem, but rather a problem of fluctuations, rare events: a matter of large deviations.
Aim of the talk is to give an introductory explanation of the particles system interpretation of the Schrödinger problem (SP) as well as its analytical/PDE formulation. Building on top of this, we will have a closer look at the behaviour of the optimal value as a function of the temperature parameter. In particular, we will be interested in the long-time behaviour (where the ergodic/probabilistic character of the underlying particle system will appear) and in the convergence rate. We will then tackle the problem of interacting particles, starting with the so-called "Mean-Field" Schrödinger problem (introduced by Backhoff-Conforti-Gentil-Léonard), and show that behaviour and convergence rate are the same as in the classical SP.
Finally, we take the first (formal) steps in the study of SP for lattice gases, thus aiming to study more complex particle systems. In this direction, a dynamical characterization of optimizers through a coupled system of PDEs will be highlighted.
Based on joint works with A. Chiarini and G. Conforti.

Author Index