Some applications of stochastic geometry in Orange Labs

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Introduction

- Goal of the talk : give an overview of some applications of stochastic geometry in Orange

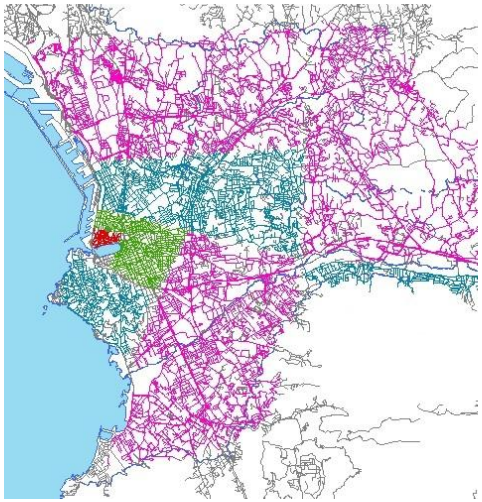
- Sketch of the presentation :
  - Road systems models
  - Fixed networks
  - Mobile networks

Road systems models

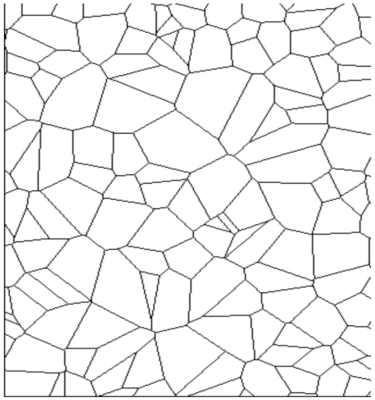
Joint work with C. Gloaguen

Modeling a road system

reality



random model



1 realization of PVT Poisson Voronoï Tessellation (ex for blue part)

macroscopic description

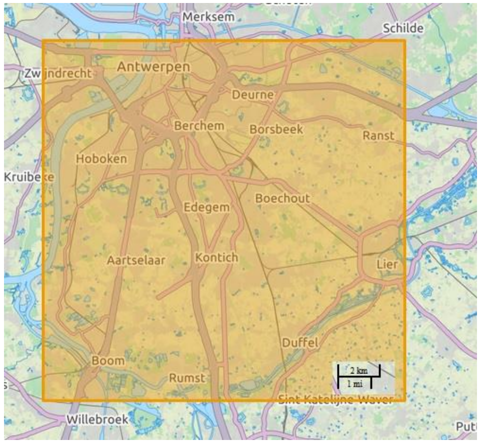
part	area km2	type	parameter km-2
red	0.61	PVT	149.04
green	1.65	PVT	114.63
blue	7.98	PVT	38.56
pink	30.87	PVT	18.68
whole city	65.90	PVT	28.08

- the road system supports the network equipment : its morphology provides structure for the network
- only 3 quantities embed the morphology of a planar tessellation :
  number of crossings, number of street segments and length of streets
- the road system is viewed as a realization of a stationary random planar tessellation
  - a whole city map is replaced with very little information (area, model type & parameters)
  - no need for localizing any street segment

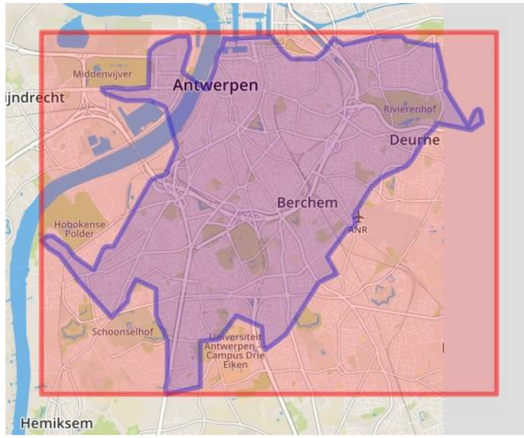
Prototype

map from internet

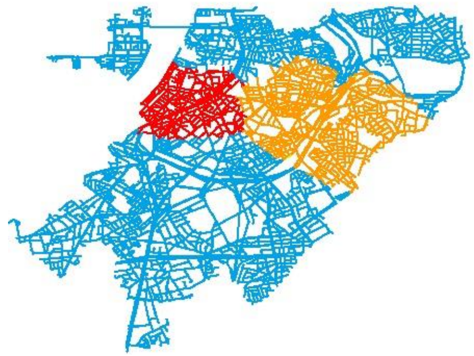
Anvers, Belgique



drawing the city's build up area



street system partition



- Open Street Map is a free database for street maps of (almost) any city in the world
- detailed data : type of streets, administrative contour
- automatization of treatment
  - detection of build up area contour
  - extraction and selection of streets
  - segmentation in homogeneous parts
  - computation of mathematical model -> « voirie file »

part	area km2	type	parameter km-2
1	3.77	PVT	76.32
2	9.32	PVT	60.04
3	40.41	PVT	42.27
whole city	53.50	PVT	47.73

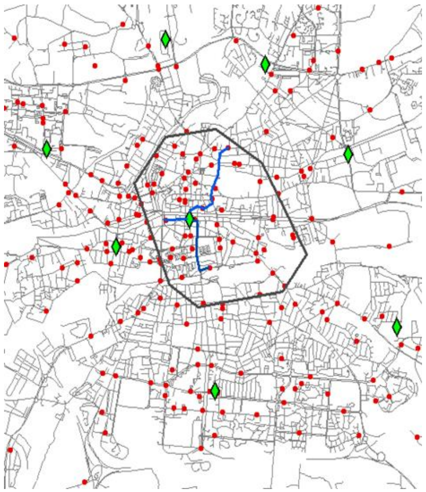
Fixed network

Joint work with C. Gloaguen

Geometry & network : distances

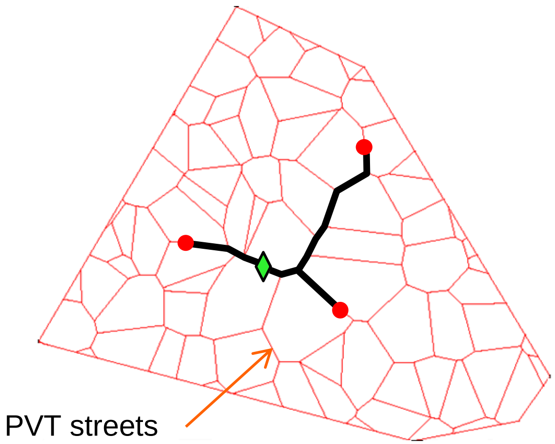
real network

serving zone = Voronoï cell



typical serving zone

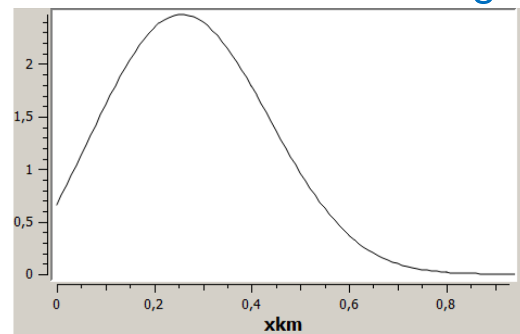
Poisson-Voronoï-Cox-Voronoï cell



macroscopic description

area	type	param.	# node
7.98	PVT	38.56	33

statistics on connexion lengths



- the network is deployed on homogeneous areas (parts or city as a whole)
- the nodes are located on the streets, the links are shortest paths along the streets
- the serving zones of the nodes form a Voronoï tessellation
  - the set of all serving zones is modeled by realisations of a typical serving zone Z
  - the mean area of Z is determined by the number of ◆ nodes
- analytical formulas for the probability distribution of the point to point connexion lengths in Z are available
  - this allowed to check that the model fits reality well

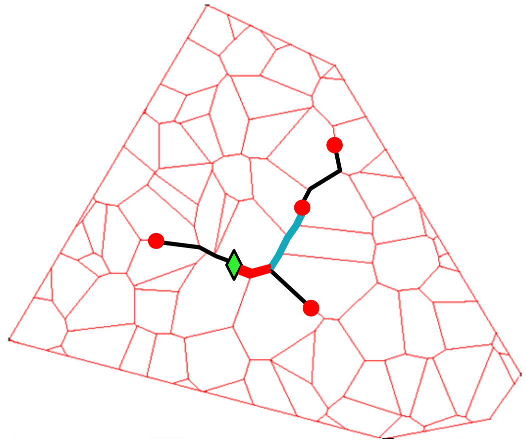
Geometry & network : cables

reality

no comment

typical serving zone

random capacity tree

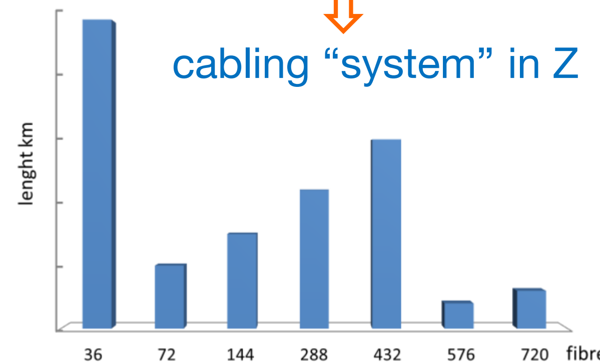


macroscopic description

area	type	param.	# node ◆	# ●	FO / ●
7.98	PVT	38.56	33	321.0	18.5

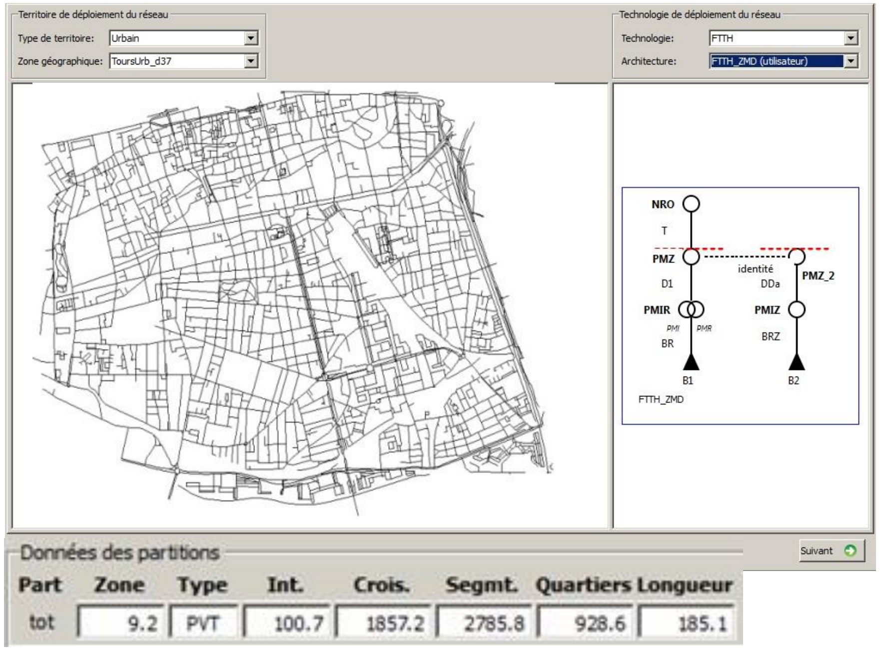
⇩

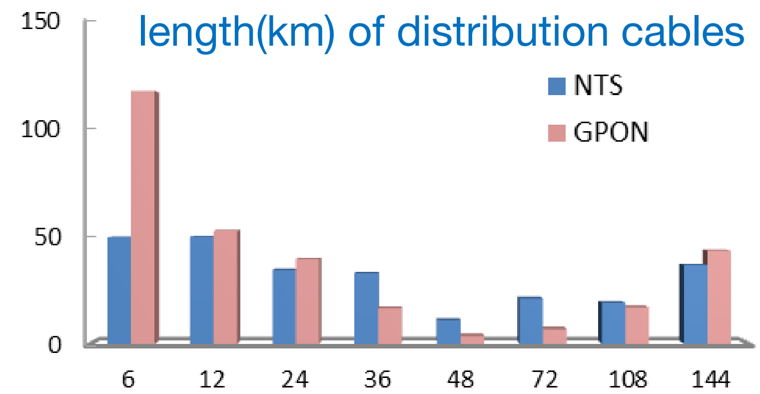
cabling “system” in Z



- the typical serving zone is the key ingredient
- an averaged incoming capacity (# fibres) is associated to each ● node
- a street segment is empty or supports one cable of sufficient size
- no closed parametric formula for the random capacity tree
  - limit results for sparse trees / dense trees
  - fast simulations procedures for the cell and the tree
- statistics on type of cable-length are derived from capacity-length computed on realizations on the typical cell and used in the cost function

Prototype : benchmark on Tours





cost

1-NTS/GPON	cables	Total
Transport	2%	8%
Distribution	8%	7%
Total cost	6%	4%

- GPON optimizer (operational tool in Orange) optimization of global deployment cost
  - on the architecture required by the French regulator Arcep and engineering constraints
  - ☺ optimal location of nodes & cabling scheme, exact path of fibres, detailed cost function
  - ☹ computational time (tens of hours) ⇨ restricted to small areas
- NTStool designed for macroscopic estimation in very large areas
  - available data set Tours (9 km^2) is the limit range
  - ☺ structuring cost units can be addressed
  - ☺ computation in minutes
- detailed comparison : cabling system ok, cost within %

Prototype : benchmark on a real deployment

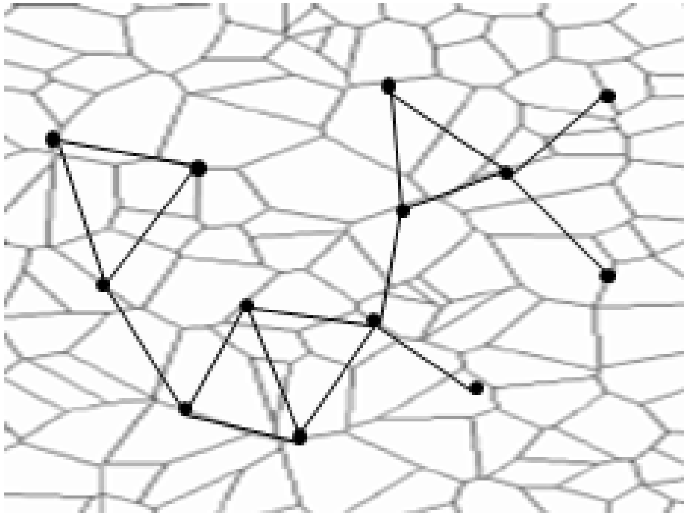
- one city in AMEA
- four zones for comparisons of global costs results

TRIAL #	Housings	(NTS - real)/real
1	9200	1%
2	8500	8%
3	4500	-8%
4	2280	-14%

Mobile networks

Joint work with WIAS (B. Jahnel, A. Hinsen, R. Patterson), C. Hirsch, N. Novita Gafur, T. En Najjary, Y. Wang, B. Blaszczyszyn, Q. Le Gall

Basic model



Hypothesis :
- No interference
- No shadowing
- No mobility

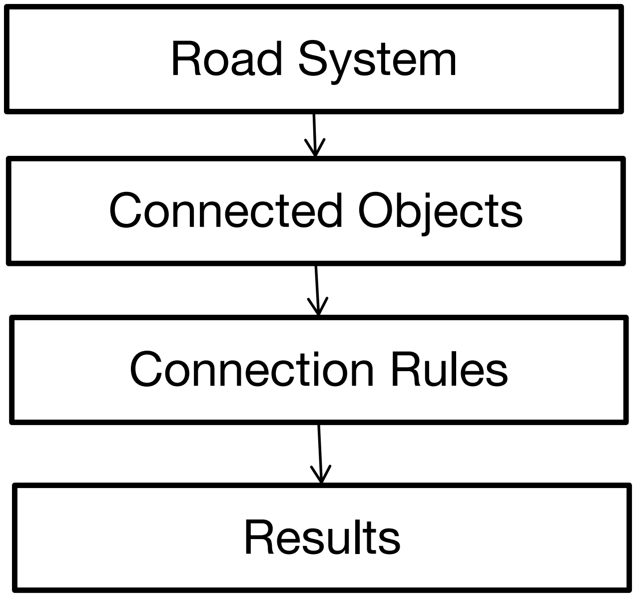
- the road system is modeled by a Poisson-Voronoï tessellation with linear intensity γ (number of street km by km^2)
- the users are placed on the road system according to a Poisson (Cox) process X with linear intensity λ (number of users by street km)
- the range of action of a mobile phone is considered to have a fixed value r
- we consider the graph g(X) with vertex set X and edges connecting two vertices whose distance is smaller or equal to r.

Goals

- long distance communications = continuum percolation of the Gilbert graph :
  - percolation threshold : $\lambda_c(r) = \inf\{\lambda : \mathbf{P}(g_r(X^\lambda) \text{ percolates}) > 0\}$
  - percolation function : $\theta(\lambda, r) = \mathbf{P}(O \leftrightsquigarrow \infty \in g_r(X^\lambda \cup \{O\}))$
  - stretch factor : $\mu(\lambda, r) = \lim_{\substack{|X_i - X_j| \to \infty \\ (X_i, X_j) \in C^\infty}} \frac{N(X_i, X_j)}{|X_i - X_j|}$

- simulations goals :
  - find **the percolation threshold λ_c** : which minimal density of users is needed in order to guarantee communications over large distances
  - trace **the percolation function $\theta(\lambda)$** : what is the probability for a given user to be in the infinite connected component.
  - trace **the stretch factor $\mu(\lambda)$** : what is the ratio of the distance of two users trying to communicate to the number of hops needed to establish the communication.

Generic simulator



Principles:

1. Divide the entire network model into independent components

2. Each component consists of compatible functions

3. Only need the connection graph to evaluate the performance of the network

Results : estimations of λ_c for γ=20

$$\lambda_c \approx \frac{4.51}{\pi \gamma r^2}$$

PBM wrong

PBM>>PDT

PDT-PBM $\Delta > 12\%$

PBM : $\Delta < 3\%$

PBM<PDT

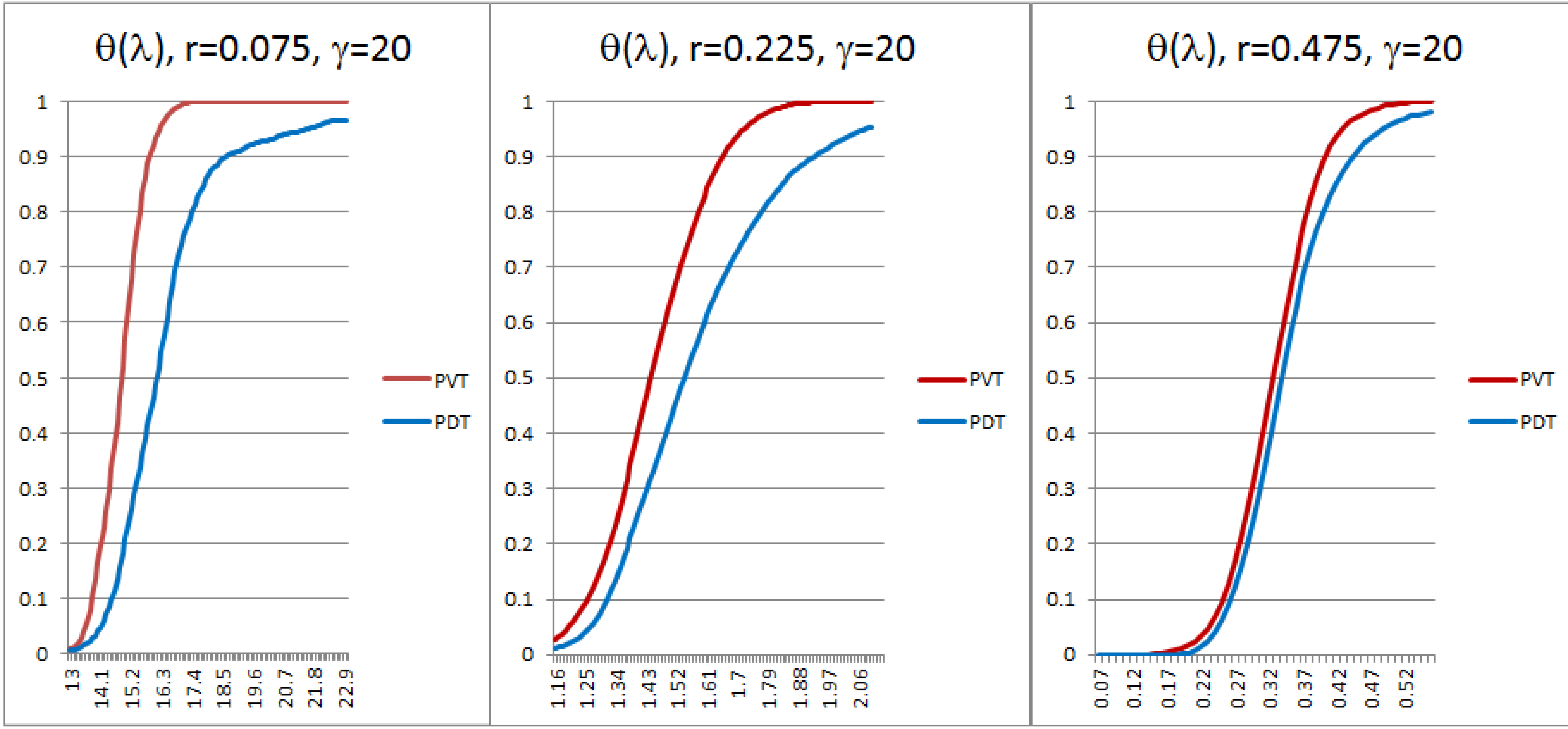
Radius (km)	Percolation Threshold (users/km)		
	PVT	PDT	PBM
0,015	238	248	319
0,025	111,7	116,7	114,8
0,075	14,98	15,77	12,76
0,125	4,81	5,29	4,594
0,175	2,39	2,58	2,343
0,225	1,43	1,51	1,418
0,275	0,95	0,99	0,949
0,325	0,68	0,69	0,680
0,375	0,51	0,52	0,510
0,425	0,40	0,40	0,397
0,475	0,318	0,318	0,318

PDT > PVT

PBM= Poisson Boolean model
PVT = Poisson Voronoi Tessellation
PDT = Poisson Delaunay Tessellation

Inhomogeneous Bernoulli bond percolation :
Probability for an edge of length ℓ to be open = b^{ℓ}

Results : function θ(λ)



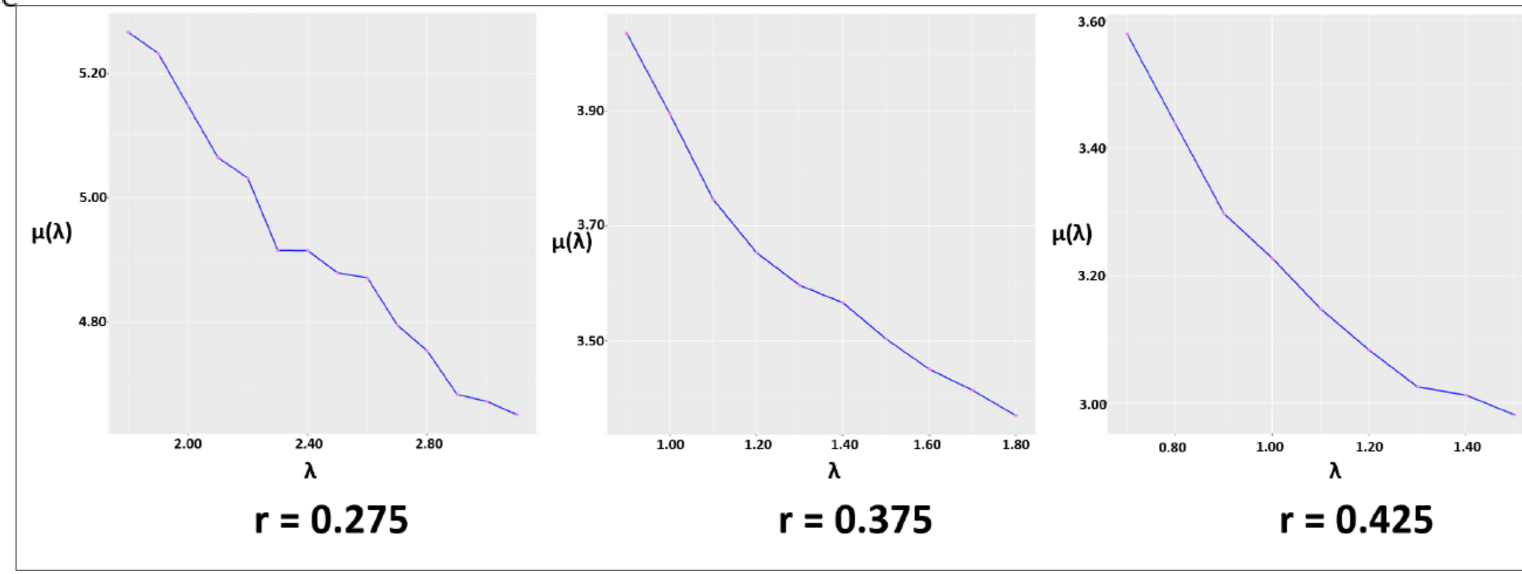
$$\theta_0\left(\frac{\gamma}{a}, r, \lambda\right) = \theta_0\left(\gamma, \frac{r}{a}, \frac{\lambda}{a}\right)$$

orange™

$$\mu(\lambda, r) = \lim_{\substack{|X_i - X_j| \to \infty \\ (X_i, X_j) \in C^\infty}} \frac{N(X_i, X_j)}{|X_i - X_j|}$$

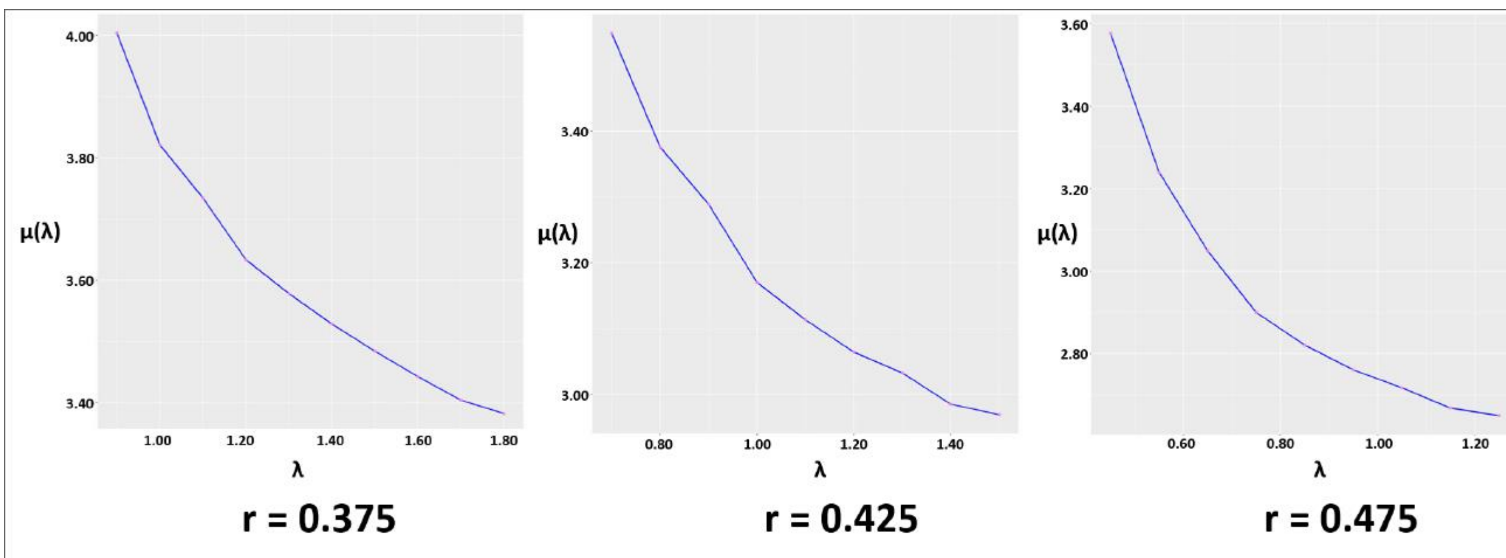
Results : function $\mu(\lambda)$

PDT



γ=20

PVT



Los models : introducing shadowing

Street system
- Random tessellation S
- Typically, Poisson-Voronoi (PVT)

Network users (UE)
- Cox process X^{λ} supported by S
- Given street system, X^{λ} is a Poisson Point Process on streets segments

Relays
- Bernoulli process on crossroads: fraction p of crossroads equipped
- Relays: ensure connectivity between adjacent streets

Connection mechanism
- Two network agents (UEs or relays) are connected iff they are *sufficiently close AND in LOS*
- Only LOS connections allowed → canyon shadowing

Network connectivity
- Network connectivity equivalent to *percolation*
- i.e. existence of *infinite connected component with positive probability*

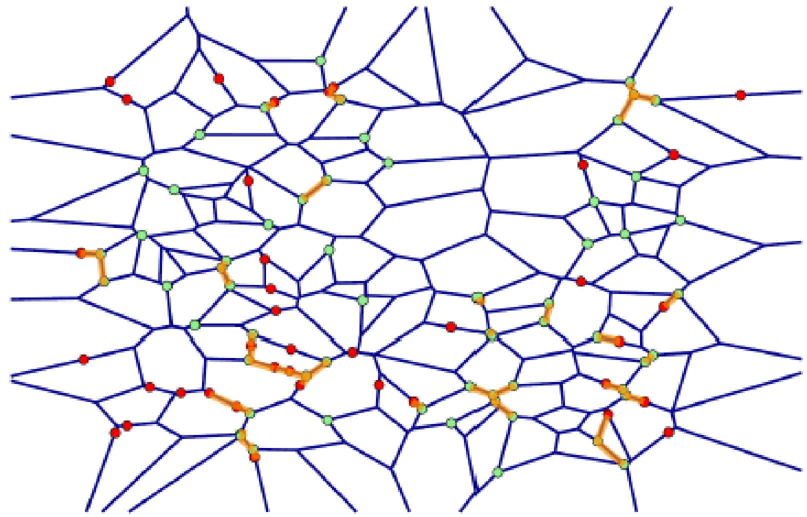
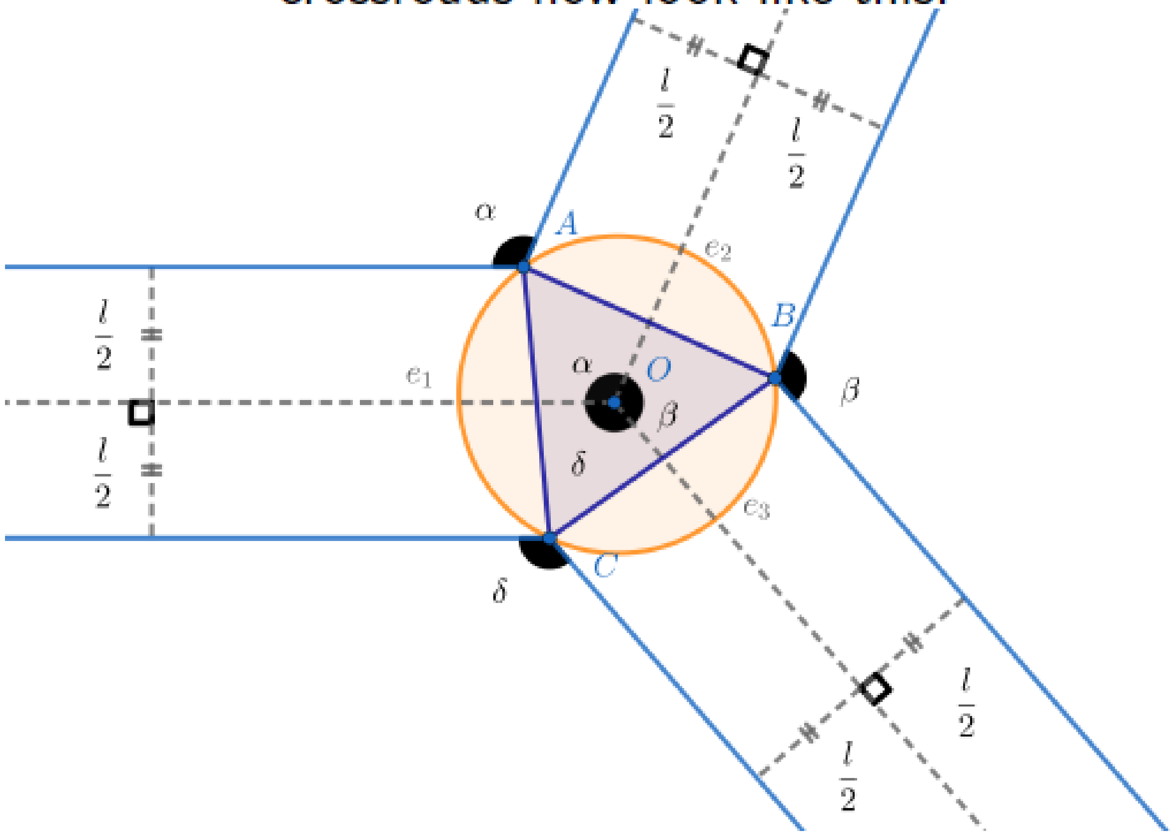


Figure 1: Example of simulated network

Network parameters
Street intensity: γ $(\mathrm{km}/\mathrm{km}^2)$
Users' linear intensity: λ (km^{-1})
Relay proportion: p
D2D radius: r (m)
Area covered by the network: $\mathcal{A}$ (km^2)

Crossroads occupation

After enlargement, each street is given a positive width $l > 0$ and crossroads now look like this:



Two extreme cases: triangle and circle. Denote by $\mathscr{A} := \mathscr{A}(l, \alpha, \beta)$ the corresponding surface.

Los models : results

Typical European urban environment: $\gamma \approx 20\ \mathrm{km/km^2}$; $l = 20\ \mathrm{m}$

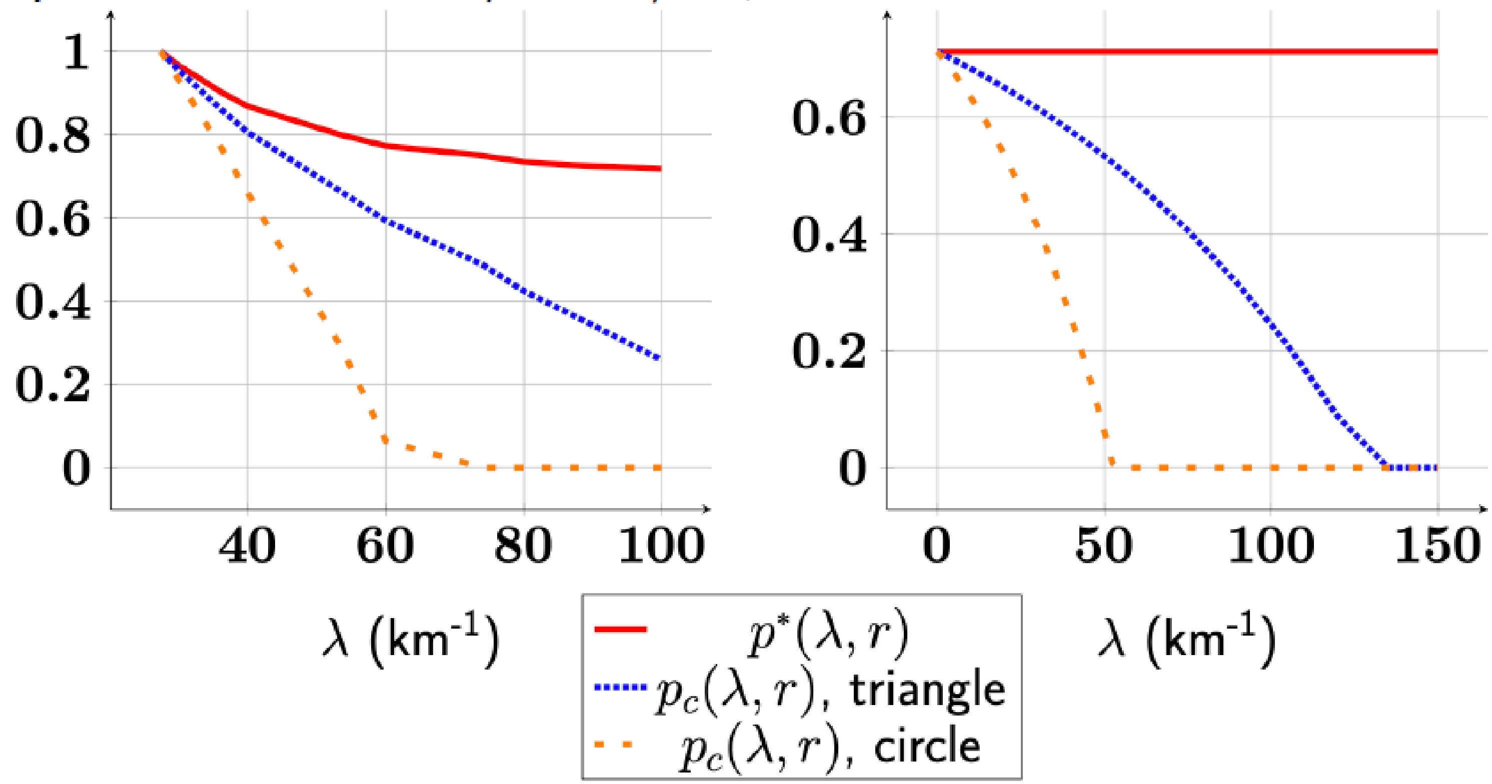
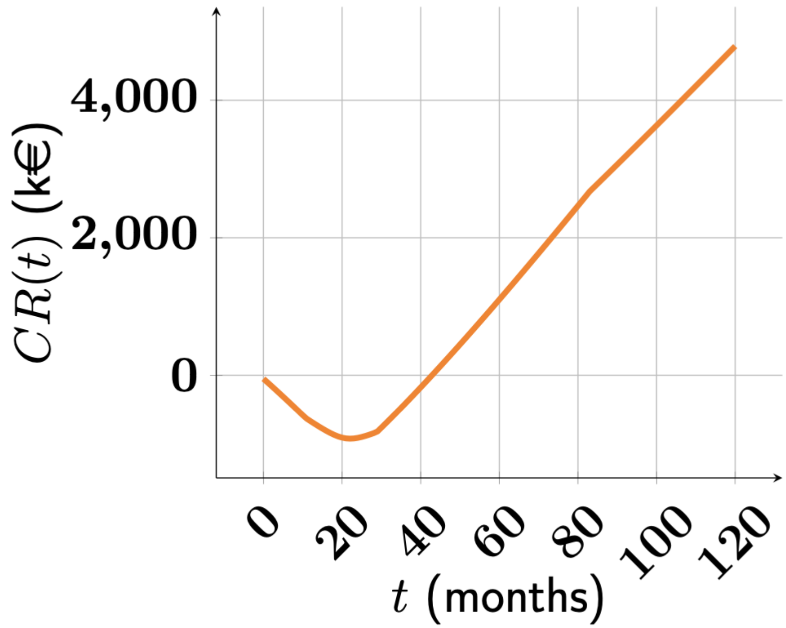
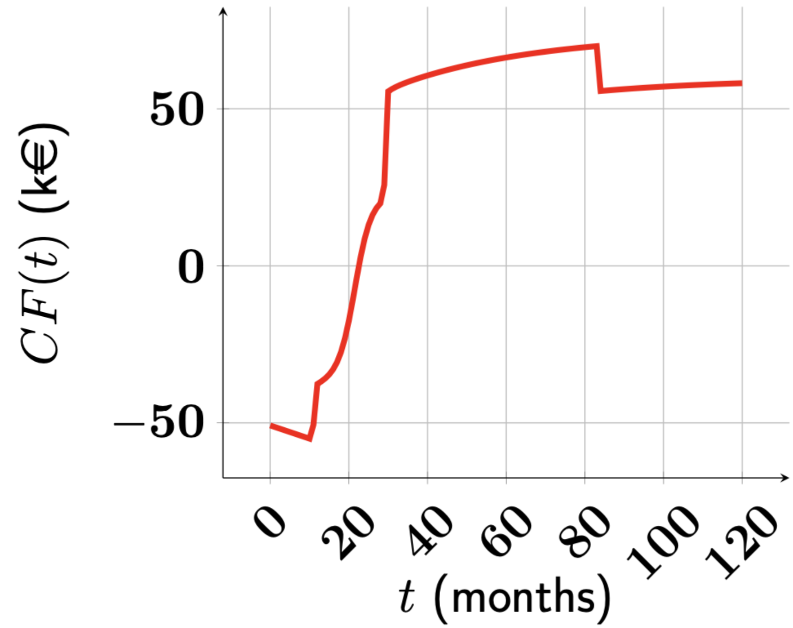


Figure 4: **Proportion of occupied crossroads: plain red. Proportion of relay-equipped crossroads: dotted blue (triangle case) and dashed orange (circle case). Left:** $r = 50\ \mathrm{m}$ **Right:** $r = 200\ \mathrm{m}$

Los models : results



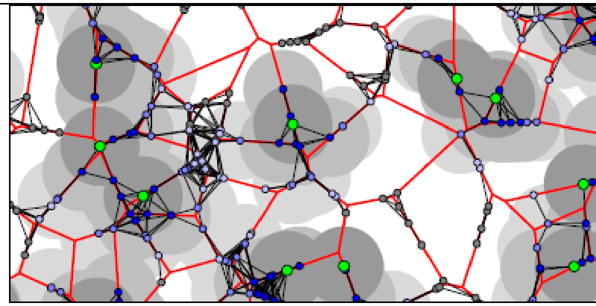
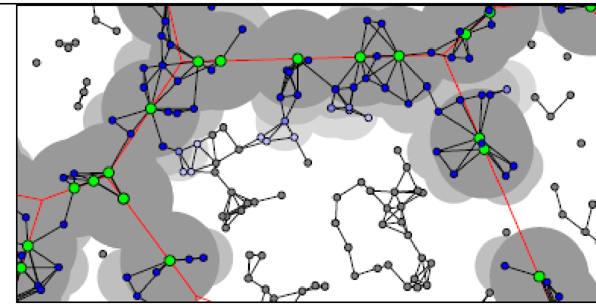
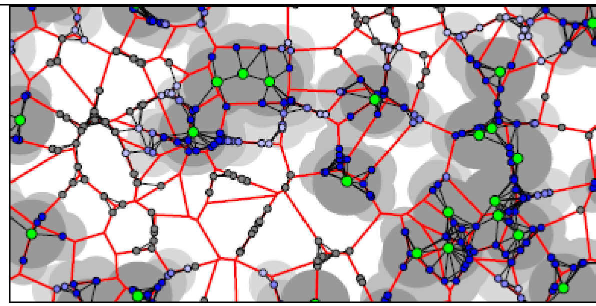
Cumulated cash flow CR as a function of time.
ROI reached at $t = 43$ months.



Cash-flow CF as a function of time.

- ROI of 43 months, i.e. 31 months after commercial launch of the service → Not unrealistic!
- Generic model → Take any other parameters you want

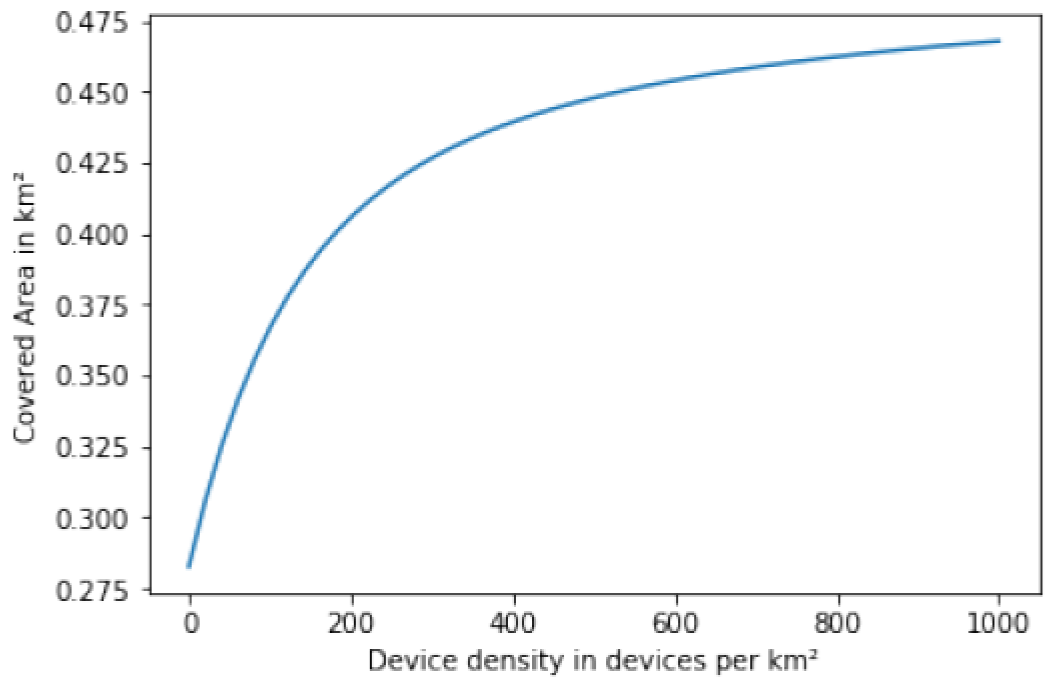
Coverage extension

Infra \ devices	road	not road
road		
not road		

- The road system is modeled by a Poisson-Voronoï tessellation with linear intensity γ (number of street km by km^2)
- The infrastructures and the devices are placed either in the plane or on the road system according to a Poisson (Cox) process X with linear intensity λ_I and λ_D (number of infrastructures/devices by street km)
- The range of action of an infrastructure (resp. device) is considered to have a fixed value r_1 (resp. r)
- Connectivity is modeled by a random graph : the number of authorized hops is k

Coverage extension : toy example

A single antenna an the center of an African village => what is the coverage extension due to D2D ?

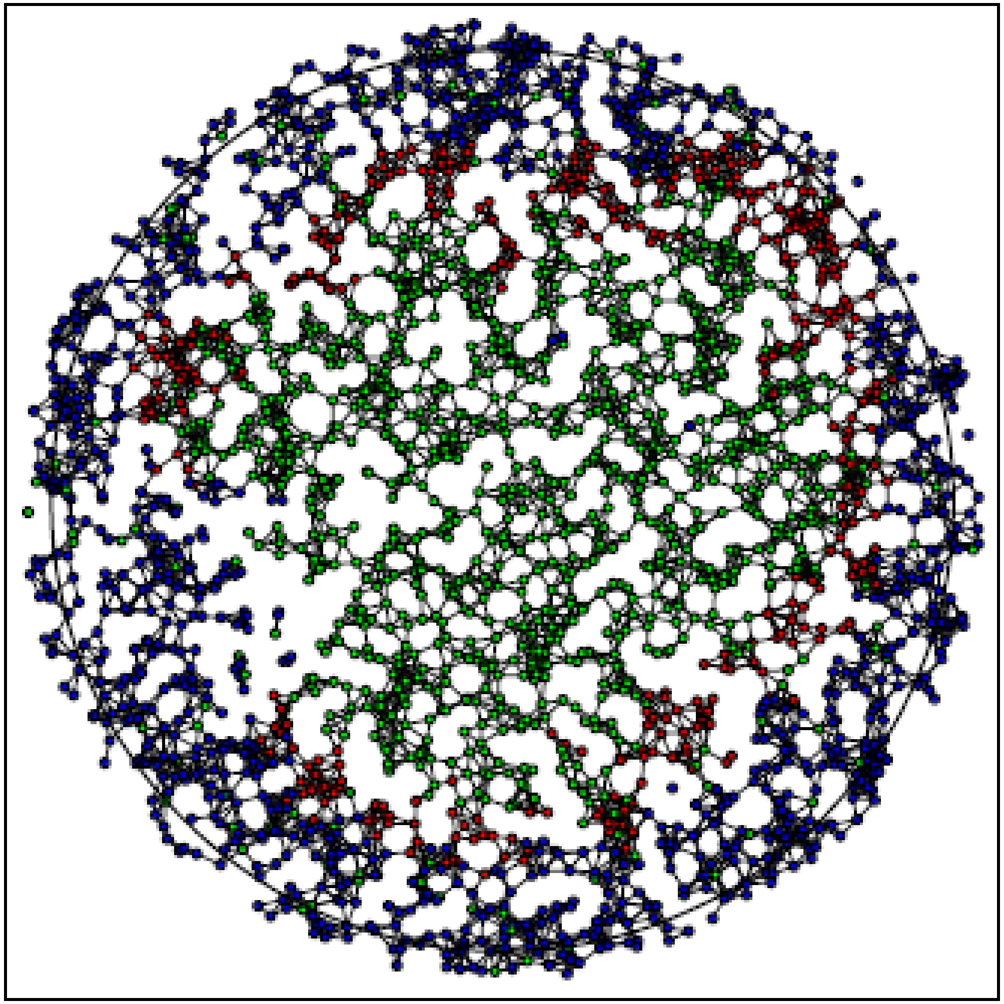


- Parameters :
  - One infrastructure at the origine
  - r_1 = 300m
  - r = 100m
  - k = 1

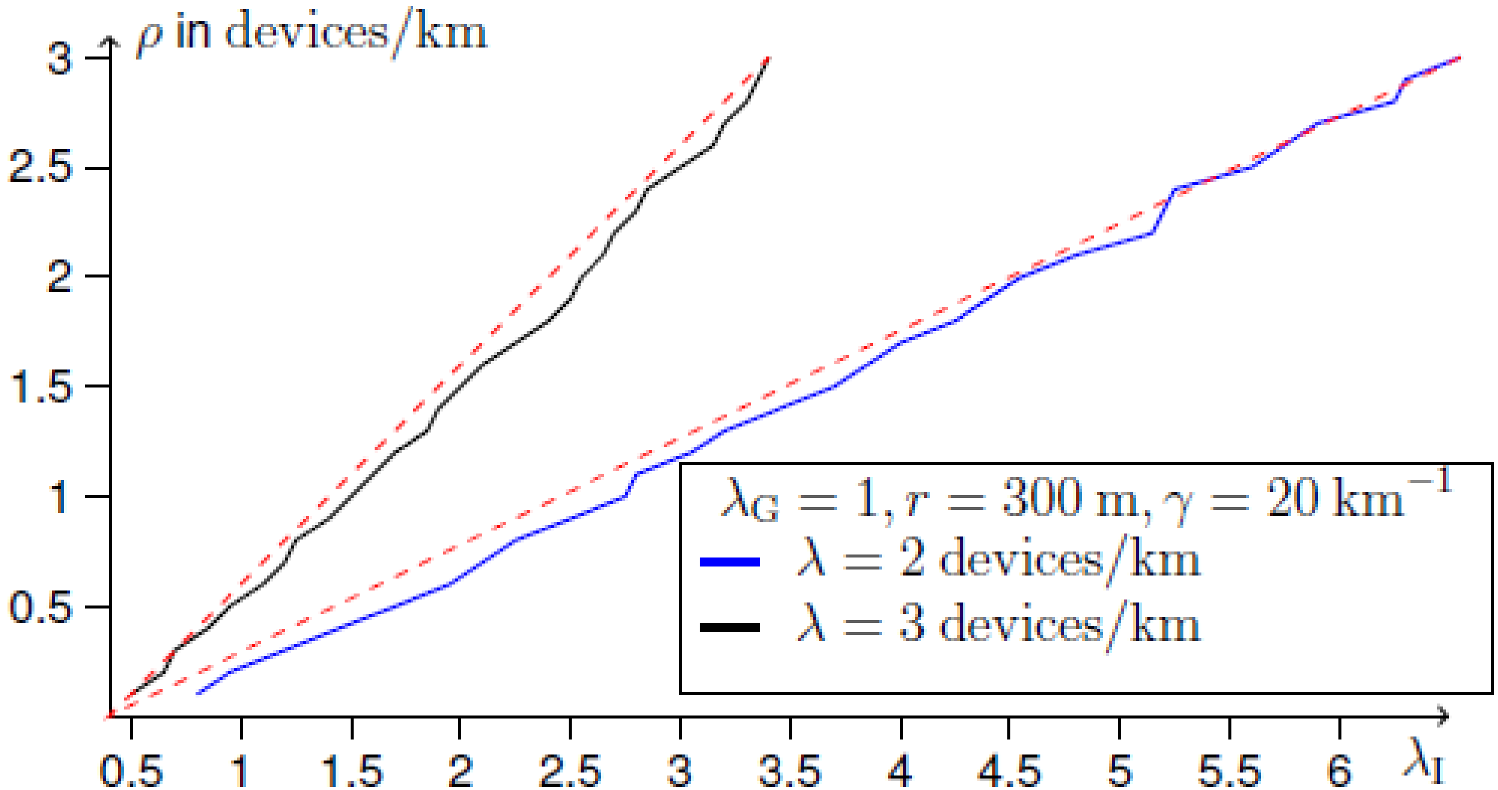
- Resulting curve :
  - Covered surface = f(devices intensity)

Malware propagation

- We use the basic model for streets, users and communication rules (no mobility or radio effects)
- At time t=0, some of the users (intensity ρ) are White Knights
- We introduce a malware in O at time t=0. The malware propagates through communication channels with an infection rate λ_I
- Once attacked, a White Knights heals the attacker with patch rate λ_G and transforms it into a White Knight
- At time t, a user has three possible states : Susceptible, Infected or White Knight.



Malware propagation



Estimated phase-separating curve for the survival of the malware in the SIG-model, in the Markovian setting for two values of the linear device intensity. The dashed red lines indicate possible linear relationships. Below the line the malware is in the survival regime. Above the line, the malware dies.

Conclusion

- Many fields of application for stochastic geometry, for fixed as well as mobile telecommunications
- Many other subjects are ongoing (mobility and buffering, multi-agent simulations, antipodal simulation, …)
- Other teams in Orange apply stochastic geometry for various applications (mainly for mobile communications)
- Many points still need to be investigated (interference, capacity, …).

references

[1] Cali E., Gafur N.N. and al. Percolation for D2D Networks on Street Systems, Proceedings of SpaSWiN '18 (Workshop on Spatial Stochastic Models for Wireless Networks), held in conjunction with WiOpt '18, Shanghai, China, May 2018.

[2] Courtat T. (2012). Promenade dans les cartes de villes. Phénoménologie mathématique et physique de la ville : une approche mathématique – PhD Thesis, Paris Diderot.

[3] Gloaguen C. and Cali E., “Cost estimation of a fixed network deployment over an urban territory,” Annals of Telecommunications, 2017. [Online]. Available: https://doi.org/10.1007/s12243-017-0614-3

[4] Gloaguen C. , Fleischer F., Schmidt H., Schmidt V. (2006). Fitting of stochastic telecommunication network models via distance measures and Monte-Carlo tests. Telecommun Syst 31(4):353-377.

[5] Gloaguen C., Voss F., Schmidt V. (2011). Parametric Distributions of Connection Lengths for the Efficient Analysis of Fixed Access Networks. Ann Telecommun 66:103-118.

[6] A. Hinsen, B. Jahnel, E. Cali, JP. Wary. Phase transitions for chase-escape models on Poisson–Gilbert graphs. Electron. Commun. Probab. 25 (2020).

[7] C. Hirsch, B. Jahnel and E. Cali. Continuum percolation for Cox point processes. Stochastic Processes and their Applications129, 3941–3966, 2019.

[8] Q. Le Gall, B. Błaszczyszyn, E. Cali, T. En-Najjary, The Influence of Canyon Shadowing on Device-to-Device Connectivity in Urban Scenario, Proceedings of IEEE WCNC (Wireless and Communications Networking Conference), 2019

[9] Q. Le Gall, B. Błaszczyszyn, E. Cali, T. En-Najjary, Relay-assisted Device-to-Device Connectivity: Connectivity and Uberization Opportunities, Proceedings of IEEE WCNC (Wireless and Communications Networking Conference), 2020

[10] OpenStreetMap. http://openstreetmap.org. Accessed October 28th, 2020.

Merci