ERC Workshop

MoMatFlu 2016

Modeling Materials and Fluids using Variational Approaches

Weierstrass Institute for Applied Analysis and Stochastics
February 22 – 26, 2016

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ERC Workshop on

Modeling Materials and Fluids using Variational Approaches

Weierstrass Institute for Applied Analysis and Stochastics, Berlin, Germany
February 22–26, 2016

Organized by

Elisabetta Rocca
Alexander Mielke

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ERC projects: Analysis of multiscale systems driven by functionals
Entropy formulation of evolutionary phase transitions

Weierstrass Institute for Applied Analysis and Stochastics

Monday, February 22, 13:00 - 17:35

13:00 - 13:50	REGISTRATION AND SOUP
13:50 - 14:00	OPENING
14:00 - 14:50	A new Visco-Energetic incremental minimization scheme for rate-independent evolution problems **Giuseppe Savaré (Pavia)**
14:50 - 15:40	Finite plasticity in $P^T P$ **Ulisse Stefanelli (Vienna)**
15:40 - 16:10	COFFEE BREAK
16:10 - 17:00	Rate-dependent elastoplasticity at finite strain: existence and approximation results **Riccarda Rossi (Brescia)**
17:00 - 17:35	Diffusive interface dynamics in lattices **Michael Herrmann (Münster)**

Tuesday, October 23, 09:00 - 17:15

09:00 - 09:50	Cahn-Hilliard-Navier-Stokes systems with moving contact lines **Maurizio Grasselli (Milano)**
09:50 - 10:40	Recent results on some diffuse-interface models for incompressible binary fluids with nonlocal interaction **Sergio Frigeri (Berlin)**
10:40 - 11:10	COFFEE BREAK
11:10 - 12:00	On a nonstandard viscous nonlocal Cahn-Hilliard system **Jürgen Sprekels (Berlin)**
12:00 - 12:50	Modeling error estimates in fluid mechanics: An a posteriori approach **Julian Fischer (Leipzig)**
12:50 - 14:30	LUNCH
14:30 - 15:20	Damage problems in bulk domains and interfaces **Elena Bonetti (Pavia)**
15:20 - 15:55	Global-in-time well-posedness for a phase field system describing rate-dependent damage phenomena **Christian Heinemann (Berlin)**
15:55 - 16:25	COFFEE BREAK
16:25 - 17:15	Discussion of different time-discretization schemes for rate-independent damage models **Dorothee Knees (Kassel)**

Wednesday, February 24, 09:00 - 17:15

09:00 - 09:50	Nucleation and microstructure in martensitic phase transformations **John Ball (Oxford)**
09:50 - 10:25	A topological obstruction related to nematic shells: Morse's index formula for VMO vector fields **Giacomo Canevari (Oxford)**
10:25 - 10:55	COFFEE BREAK
10:55 - 11:45	Liquid crystal inertia in the Q-tensor framework **Arghir Zarnescu (Brighton)**
11:45 - 12:20	Wulff shape emergence and sharp $n^{3/4}$ law for crystals **Paolo Piovano (Vienna)**
12:20 - 14:30	LUNCH **Workshop Photo**
14:30 - 15:20	tba **Felix Otto (Leipzig)**
15:20 - 15:55	Nematic elastomer ribbons **Virginia Agostiniani (Trieste)**
15:55 - 16:25	COFFEE BREAK
16:25 - 17:15	The peridynamic model in nonlocal elastodynamics **Etienne Emmrich (Berlin)**
19:00 - 22:00	DINNER **Restaurant Deponie No 3**

Thursday, February 25, 09:00 - 17:35

09:00 - 09:50	Analyses and control for a class of Cahn–Hilliard type phase field systems modelling tumor growth **Pierluigi Colli (Pavia)**
09:50 - 10:25	A Gamma convergence approach to a sharp-interface limit of a phase transition problem, with application to a tumor growth model **Riccardo Scala (Berlin)**
10:25 - 11:00	A diffuse interface tumour model with chemotaxis and active transport **Kei Fong Lam (Regensburg)**
11:00 - 11:30	COFFEE BREAK
11:30 - 12:05	A-free Rigidity and Applications to the Compressible Euler System **Elisabetta Chiodaroli (Lausanne)**
12:05 - 12:55	Relative energies and problems of stability in fluid dynamics **Eduard Feireisl (Prague)**
12:55 - 14:30	LUNCH
14:30 - 15:20	E-convergence to the quasi-steady-state approximation in systems of chemical reactions **Karoline Disser (Berlin)**
15:20 - 15:55	Energy-reaction-diffusion systems **Sabine Hittmeir (Linz)**
15:55 - 16:25	COFFEE BREAK
16:25 - 17:00	On Entropy-Transport problems and distances between positive measures **Matthias Liero (Berlin)**
17:00 - 17:35	Non existence and instantaneous extinction for very fast fractional diffusion equations **Antonio Segatti (Pavia)**

Friday, February 26, 09:00 - 12:45

09:00 - 09:50	Variational methods for steady-state Darcy/Fick flow in swelling-exhibitting or poro-elastic solids **Tomáš Roubíček (Prague)**
09:50 - 10:40	From adhesive contact to brittle delamination in visco-elastodynamics **Marita Thomas (Berlin)**
10:40 -11:10	COFFEE BREAK
11:10 - 11:45	A free boundary problem for the flow of viscous liquid bilayers **Dirk Peschka (Berlin)**
11:45 - 12:35	Flow in a porous visco-elasto-plastic solid **Krejčí (Prague)**
12:35 - 12:45	CLOSING AND SNACKS

Nematic elastomer ribbons

Virginia Agostiniani, and Antonio DeSimone

SISSA, Italy

In this talk, we present a plate model describing the bending behavior of nematic elastomer thin films where the orientation of the nematic director along the thickness has a *twisted* geometry and the typical appearance of the minimal energy configurations is that of wide ribbons. The reduced energy functional is derived from a three-dimensional description of the system using rigorous dimension-reduction techniques. As a result, the (new) two-dimensional model is a nonlinear plate theory in which deviations from a characteristic target curvature tensor cost elastic energy. Moreover, the stored energy functional cannot be minimised to zero, thus revealing the presence of residual stresses, as observed experimentally and in numerical simulations.

Nucleation and microstructure in martensitic phase transformations

John Ball

University of Oxford, U.K.

When a new phase is nucleated in a martensitic phase transformation, it has to fit geometrically onto the parent phase. Likewise, microstructures in individual grains of a polycrystal have to fit together across grain boundaries. The talk will describe some mathematical issues involved in understanding such questions of compatibility and their influence on metastability, drawing on collaborations with C. Carstensen, P. Cesana, B. Hambly, R. D. James, K. Koumatos and H. Seiner.

Damage problems in bulk domains and interfaces

Elena Bonetti(1), **Giovanna Bonfanti**(2), **and Riccarda Rossi**(2)

(1) University of Pavia, Italy
(2) University of Brescia, Italy

The mathematical investigation of Contact Mechanics has been extensively developed over the last decades and it includes *adhesive contact* and *delamination* models. Their analysis is relevant for several mechanical and engineering problems, ranging from fractures in brittle materials, to the investigation of earthquakes, to the study of layered composite structures in machine designing and manufacturing. Indeed, the interface regions between the various laminates are fundamental for the strength and stability of the structural elements, and the degradation of the adhesive substance on such regions may lead to material failure. It turns out that the damage theory can be successfully used for describing adhesive contact between solids, in terms of a suitable internal variable accounting for the state of the adhesion. This approach is in fact mainly due to M. Frémond (see [1]). It is closely related to the theory of phase change problems in nonsmooth thermomechanics [6]. These problems have recently attracted remarkable attention, and been widely investigated, both in the case of *rate-independent* evolution for the adhesion parameter and, in the case of *rate-dependent*, or *viscous*, evolution, [2]–[4].

We discuss and present some recent results on a model for adhesive contact with friction between a thermo-viscoelastic body and a rigid support. A PDE system, consisting of the evolution equations for the temperatures in the bulk domain and on the contact surface, of the momentum balance, and of the equation for the internal variable describing the state of the adhesion, is recovered. Unilateral boundary conditions are rendered by a generalization of the Signorini law to the case of adhesive contact. In addition dissipative frictional effects are accounted for due to the presence of non-smooth boundary operators. The existence of global-in-time weak solutions to the associated initial-boundary value problem is proved , mainly by passing to the limit in a carefully tailored time-discretization scheme. Finally, we discuss an asymptotic dimensional reduction analysis for a damage model written in thin domains which formally justify the limit equations we have used for surface damage in contact problems [5].

References

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A topological obstruction related to nematic shells: Morse's index formula for VMO vector fields

Giacomo Canevari(1), **Antonio Segatti**(2), **and Marco Veneroni**(2)

(1) University of Oxford, United Kingdom
(2) University of Pavia, Italy

In this talk, we consider the following problem. Given a compact manifold $N \subseteq \mathbb{R}^d$ with boundary, which are the maps $\mathbf{g}\colon \partial N \to \mathbb{S}^{d-1}$ that can be extended to a unit vector field $\mathbf{u}$, defined on N, belonging to — say — the Sobolev space $W^{1,p}$? This question is motivated by the analysis of variational models for a thin film of nematic liquid crystals spread on a surface. In the continuous setting, the answer depends on a topological obstruction, namely, Morse's index formula. Inspired by Brezis and Nirenberg's work on the topological degree [1, 2], we extend Morse's formula to the class of VMO (Vanishing Mean Oscillation) functions. This yields a characterisation of the boundary data $\mathbf{g}$ that admit an extension with the required properties.

References

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$\mathcal{A}$-free Rigidity and Applications to the Compressible Euler System

Elisabetta Chiodaroli

École polytechnique fédérale de Lausanne

In [4], Székelyhidi and Wiedemann showed that any measure-valued solution to the in- compressible Euler equations in several space dimensions can be generated by a sequence of exact solutions. This means that measure-valued solutions and weak solutions are substantially the same for incompressible Euler, thus leading to a very large set of weak solutions.

In this talk we address the corresponding problem for the compressible Euler system: can every measure-valued solution to the compressible Euler equations be approximated by a sequence of weak solutions? We show that the answer is negative: generalizing a well-known rigidity result of Ball and James [1], we give an explicit example of a measure- valued solution for the compressible Euler equations which can not be generated by a sequence of distributional solutions. We also give an abstract necessary condition for measure-valued solutions to be generated by weak solutions, relying on work of Fonseca and Müller [3]. The dichotomy between weak and measure-valued solutions in the com- pressible case is in contrast with the incompressible situation. The results presented are joint work with E. Feireisl, O. Kreml and E. Wiedemann [2].

References

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Analyses and control for a class of Cahn–Hilliard type phase field systems modelling tumor growth

Pierluigi Colli
University of Pavia, Italy

A diffuse interface model of tumor growth proposed in [6] has been recently investigated and discussed in [5] and [1] from the viewpoint of existence of solutions, uniqueness and global attractor.

The model consists of a Cahn–Hilliard equation for the tumor cell fraction coupled to a reaction-diffusion equation for a function representing the nutrient rich extracellular water volume fraction.

In particular, if we consider an admissible variant of the system containing two further viscosity terms with small coefficients, it is interesting to see what happens as such coefficents tend to zero: rigorous asymptotic analyses are performed in [2, 3].

On the other hand, a distributed optimal control problem is studied in [4]: the distributed control u plays in the right-hand side of the reaction-diffusion equation and it can be interpreted as a nutrient supply or a medication, while the cost functional aims to keep the tumor cell fraction under control during the evolution.

The talk will try to make an overview of the related results.

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[2] P. Colli, G. Gilardi, E. Rocca, J. Sprekels, *Vanishing viscosities and error estimate for a Cahn–Hilliard type phase field system related to tumor growth*, Nonlinear Anal. Real World Appl. **26** (2015), 93–108.

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E-convergence to the quasi-steady-state approximation in systems of chemical reactions

Karoline Disser[(1)], **Matthias Liero**[(1)], **and Jonathan Zinsl**[(2)]

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(2) Technische Universität München, Germany

We give a simple proof of effective limit equations for systems of ODEs modeling chemical reactions with mass-action kinetics on different time scales. The limit dynamics of some reactions taking place at an infinite rate, known as the quasi-steady-state approximation, can be considered either as a lower-dimensional system of ODEs or as a full-dimensional system including an algebraic constraint.

We show that the entropic gradient structure of the system carries over to the limit, in the sense that the constraint is enforced by a pseudometric on the full space.

References

[1] D. Bothe, Instantaneous limits of reversible chemical reactions in presence of macroscopic convection, *J. Differential Equations* **193** (2003), 27–48.

[2] K. Disser, M. Liero, and J. Zinsl, On the evolutionary Γ-convergence of gradient systems modeling slow and fast chemical reactions, *WIAS Preprint* (2016).

The peridynamic model in nonlocal elastodynamics

Etienne Emmrich and Dimitri Puhst

Technische Universität Berlin, Germany

Peridynamics is a nonlocal continuum theory which avoids spatial derivatives. It is believed to be suited for the description of fracture and other material failure, and to model multiscale problems. In this talk, we introduce the peridynamic model and discuss several aspects of its mathematical analysis. We review recent results on the existence of solutions to the peridynamic equation of motion for a large class of nonlinear pairwise force functions modeling isotropic microelastic material. We also discuss how these results are related to the general theory of nonlinear evolution equations of second order in time. Finally, we study the limit of vanishing nonlocality.

Relative energies and problems of stability in fluid dynamics

Eduard Feireisl

Czech Academy of Sciences, Prague, Czech Republic

We develop the concept of Dafermos' relative entropy/energy in the context of fluid dynamics, in particular, for compressible viscous fluids. We discuss possible applications of the method to various problems: Flows in thin channels, weak-strong uniqueness, singular limits, stochastic perturbations and/or convergence of numerical schemes.

Modeling error estimates in fluid mechanics: An a posteriori approach

Julian Fischer

Max Planck Institute for Mathematics in the Sciences, Leipzig

We present some recent developments in modeling error estimation in fluid mechanics, i.e. estimation of errors caused by the choice of the mathematical model for the fluid in consideration. Among other topics, in our interest is the use of ideally incompressible fluid models to describe slightly compressible fluids: For example, we present an explicit error estimate for the error caused by approximating the compressible Navier-Stokes equation by the incompressible Navier-Stokes equation. The estimates are based on relative entropy techniques and an a posteriori error estimation approach, relying on the explicit knowledge of the (exact or numerical) solution to the simplified model.

References

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Recent results on some diffuse-interface models for incompressible binary fluids with nonlocal interaction

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In the talk we shall present the last results on some diffuse-interface models for flow and phase separation of binary fluids which are based on the coupling of the Navier-Stokes equations with the nonlocal Cahn-Hilliard equation. The nonlocal Cahn-Hilliard/Navier-Stokes (CHNS) system has been studied analytically in a series of papers (cf. [4, 7, 8, 9, 10, 6, 11]) We shall first review a recent result on existence of dissipative global weak solutions for a nonlocal CHNS type system which describes the situation where the two fluids have different densities (cf. [5]). This system represents the nonlocal version of a model derived by H. Abels, H. Garcke and G. Grün in [3] and studied analytically by H. Abels, D. Depner and H. Garcke in [1, 2]. We shall mention the main difficulties connected with this model as far as, in particular, regularity and uniqueness are concerned. This will lead us to consider this system with singular double-well potential and degenerate mobility in 2D. For the nonlocal CHNS system with degenerate mobility, double-well singular potential and matched densities a result concerning existence of strong solutions in 2D will then be presented. This will concern, in particular, the physically relevant and mathematically challenging situation where the viscosity depends on the order parameter. Finally, with the regularity result at hand, we shall study an associated optimal distributed control problem and derive first order necessary optimality conditions. These last results are contained in a work in progress with M. Grasselli and J. Sprekels.

References

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Cahn-Hilliard-Navier-Stokes systems with moving contact lines

C.G. Gal(1), **M. Grasselli**(2), **and A. Miranville**(3)

(1) Florida International University, USA
(2) Politecnico di Milano, Italy
(3) Université de Poitiers, France

The motion of an isothermal mixture of two immiscible and incompressible fluids subject to phase separation can be described by the well-known model H (see [4] and references therein). This is a diffuse interface model which consists of the Navier-Stokes equations for the average velocity $\mathbf{u}$ which are subject to a force depending on the difference ϕ of the relative concentrations of the two fluids. The evolution of ϕ is governed by a convective Cahn-Hilliard equation. The system is also known as Cahn-Hilliard-Navier-Stokes (CHNS) system. In a simplified setting (matched densities) the CHNS system reads

(1) $$\partial_t \mathbf{u} + \mathbf{u}\cdot\nabla\mathbf{u} - \mathrm{div}\,(2\nu D(\mathbf{u})) + \nabla p = \epsilon\mu\nabla\phi + \mathbf{h}(t),$$

(2) $$\mathrm{div}\,\mathbf{u} = 0,$$

(3) $$\partial_t \phi + \mathbf{u}\cdot\nabla\phi - \Delta\mu = 0, \quad \mu = -\epsilon\Delta\phi + \epsilon^{-1}F'(\phi),$$

in $\Omega \times (0,+\infty)$. Here Ω is a bounded domain in $\mathbb{R}^{\mathrm{N}}$, $\mathrm{N} = 2, 3$, $\nu > 0$ is the kinematic viscosity of the mixture, $D(\mathbf{u})$ is the linear deformation tensor, $\mathbf{h}$ is a given external body force, $\epsilon > 0$ is related to the thickness of the (diffuse) interface separating the two fluids and F is a suitable double well potential (e.g., $F(s) = \frac{1}{4}(s^2-1)^2$, $s \in \mathbb{R}$).

In the existing literature there are many theoretical results on system (1)-(3) equipped with no-slip (or periodic) boundary conditions for $\mathbf{u}$ and no-flux (or periodic) boundary conditions for ϕ and μ. It was observed (see, e.g., [1]) that the moving contact line, defined as the intersection of the fluid-fluid interface with the solid wall, is incompatible with the no-slip boundary condition (cf. [2] and its references). More precisely, there is a velocity discontinuity at the moving contact line, and the tangential force exerted by the fluids on the solid surface in the vicinity of the contact line becomes infinite. Thus, in immiscible two-phase flows, none of the mentioned boundary conditions can account for the moving contact line slip velocity profiles obtained from simulations. Therefore, new boundary conditions are required to describe the observed phenomena. An example of such conditions are the so-called generalized Navier boundary conditions (GNBC) (see [5] for a variational derivation).

We intend to discuss the existence of a global weak solution to system (1)-(3) endowed with GNBC accounting for some boundary diffusion for ϕ (see [3]). If time permits, we shall also show that any weak energy solution converges to a single equilibrium.

References

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Global-in-time well-posedness for a phase field system describing rate-dependent damage phenomena

M. Hassan Farshbaf-Shaker and Christian Heinemann

Weierstrass Institute, Berlin, Germany

In this talk we are going to investigate global-in-time well-posedness for a pde/inclusion system in 2D which models damage processes in viscoelastic media according to Kelvin-Voigt rheology. The system consists of the following relations

(4a) $$\mathbf{u}_{tt} - \operatorname{div}\big(\mathbb{C}(\chi)\varepsilon(\mathbf{u}) + \mathbb{D}(\chi)\varepsilon(\mathbf{u}_t)\big) = \boldsymbol{\ell} \qquad \text{in } \Omega \times (0,T),$$

(4b) $$\chi_t - \Delta\chi_t - \Delta\chi + \partial I_{(-\infty,0]}(\chi_t) + \frac{1}{2}\mathbb{C}'(\chi)\varepsilon(\mathbf{u}) : \varepsilon(\mathbf{u}) + f'(\chi) \ni 0 \qquad \text{in } \Omega \times (0,T)$$

supplemented with initial conditions for $\mathbf{u}$, $\mathbf{u}_t$ and χ, homogeneous Neumann boundary conditions for χ and χ_t and the nonhomogeneous boundary condition

(5) $$\big(\mathbb{C}(\chi)\varepsilon(\mathbf{u}) + \mathbb{D}(\chi)\varepsilon(\mathbf{u}_t)\big) \cdot \nu = \mathbf{b} \quad \text{on } \partial\Omega \times (0,T),$$

where $\boldsymbol{\ell}$ and $\mathbf{b}$ model external volume and boundary forces. Complete degeneration is prevented by assuming that the viscosity tensor $\mathbb{D}(\cdot)$ is uniformly bounded from below by a positive constant.

In order to establish existence and uniqueness of strong solutions we present a special time-discretization scheme and enhanced a priori estimates from [1] designed for boundary conditions of type (5) and for differential inclusions as in (4b) forcing the uni-directionality $\chi_t \leq 0$.

References

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Diffusive interface dynamics in lattices

Michael Helmers[1] **and Michael Herrmann**[2]**,**

(1) Rheinische Friedrich-Wilhelms-Universität Bonn, Germany
(2) Westfälische Wilhelms-Universität Münster, Germany

Microscopic regularizations of ill-posed forward-backward diffusion equations have many applications and provide dynamical models for diffusive interfaces. Typical examples are the Cahn-Hilliard equation, the viscous approximations studied in [1, 4], and spatially discrete gradient systems such as

$$\dot{u}_j = \Delta p_j\,, \qquad p_j = \Phi'(u_j)\,, \qquad j \in \mathbb{Z}\,,$$

where Δ denotes the discrete Laplacian and Φ' is the bistable derivative of a double-well potential. While the Cahn-Hilliard case is well understood, very little is known about the mathematical analysis of viscous or lattice regularizations.

In this talk we choose Φ' as a trilinear function and demonstrate that the dynamics of both moving and standing phase interface is intimately related to a family of entropy inequalities and a hysteretic Stefan condition, see the figure and [2] for a simpler case.

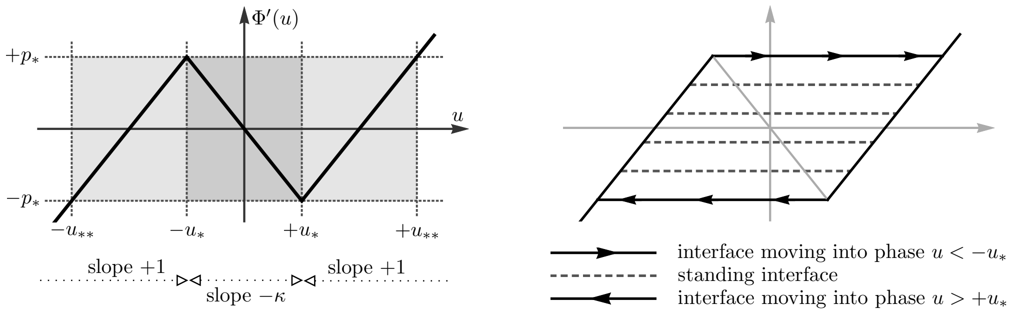

Figure. Cartoon of the nonlinearity Φ' and the macroscopic hysteresis loop.

Theorem 1 (macroscopic evolution in the parabolic scaling limit)**.** *Identifying*

$$u_j(t) = U(\tau,\,\xi)\,, \qquad p_j(t) = P(\tau,\,\xi)\,, \qquad \tau = \varepsilon^2 t\,, \qquad \xi = \varepsilon j\,,$$

the limit $\varepsilon \to 0$ for well-prepared single-interface initial data is governed by

$$\partial_\tau U = \partial_\xi^2 P\,, \qquad U = P + \mathcal{M}[P]\,,$$

where the relay operator $\mathcal{M}$ acts pointwise in ξ on temporal functions. In particular, there exists a unique interface curve $\xi = \xi_*(\tau)$ which separates space-time regions with either $U \leq -u_*$ or $U \geq +u_*$.

The key arguments in the proof are derived by a careful analysis of the lattice ODE and can informally be summarized as follows (see [3] for the details):

(1) Macroscopic interfaces exist according to a *persistence lemma*.
(2) The *waiting lemma* provides an upper bound for the interface speed.
(3) Microscopic phase transitions can be characterized by a *slow-fast splitting*.
(4) Several *fluctuation estimates* control the impact of the mesoscopic oscillations.

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Energy-reaction-diffusion systems

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We derive thermodynamically consistent models of reaction-diffusion equations coupled to a heat equation. While the total energy is conserved, the total entropy serves as a driving functional such that the full coupled system is a gradient flow. The novelty of the approach is the Onsager structure, which is the dual form of a gradient system, and the formulation in terms of the densities and the internal energy. In these variables it is possible to assume that the entropy density is strictly concave such that there is a unique maximizer (thermodynamical equilibrium) given linear constraints on the total energy and suitable density constraints.

We consider two particular systems of this type, namely, a diffusion-reaction bipolar energy transport system, and a drift-diffusion-reaction energy transport system with confining potential. We prove corresponding entropy-entropy production inequalities with explicitely calculable constants and establish the convergence to thermodynamical equilibrium, at first in entropy and further in L^1 using Cziszár-Kullback-Pinsker type inequalities.

Discussion of different time-discretization schemes for rate-independent damage models

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It is well known that rate-independent systems involving nonconvex stored energy functionals in general do not allow for time-continuous solutions even if the given data is smooth in time. Several solution concepts are proposed to deal with these discontinuities, among them the meanwhile classical global energetic approach and the more recent vanishing viscosity approach. Both approaches generate solutions with a well characterized jump behavior. However, the solution concepts are not equivalent. In this context, numerical discretization schemes are needed that efficiently and reliably approximate directly that type of solution that one is interested in. For instance, in the vanishing viscosity context it is reasonable to couple the viscosity parameter with the time-step size. However, the numerical examples from [1] show that even knowing the exact solution it is extremely difficult to choose viscosity and time-discretization parameters in such a way that the correct jump behavior is visible already for rather coarse discretizations. The aim of this lecture is to discuss different time-discretization schemes, to study their convergence and to characterize as detailed as possible the limit curves as the discretization parameters tend to zero. The main focus will lie on alternate minimization schemes that are quite popular in the context of damage models. Switching to a time-reparametrized picture, the behavior at jump points can be made visible and similarities and differences to other approaches will be discussed. The part on alternate minimization schemes is joint work with M. Negri, Pavia, [2].

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Flow in a porous visco-elasto-plastic solid.

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A model for porous media flow with hysteretic pressure-saturation relation involving thermodynamic effects and governed by the system

$$\rho_S u_{tt} = \operatorname{div}\left(B\nabla_s u_t + P[\nabla_s u]\right) + \nabla p - \beta\nabla\theta + g\,, \tag{6}$$

$$G[p]_t = \operatorname{div} u_t + \frac{1}{\rho_L}\operatorname{div}\left(\mu(p)\nabla p\right), \tag{7}$$

$$c_0\theta_t = \operatorname{div}\left(\kappa(\theta)\nabla\theta\right) + \|D_P[\nabla_s u]_t\|_* + |D_G[p]_t| + B\nabla_s u_t : \nabla_s u_t + \frac{1}{\rho_L}\mu(p)|\nabla p|^2 - \beta\theta\operatorname{div} u_t\,, \tag{8}$$

has been derived and existence of global strong solutions in 3D for the isothermal case has been proved in [1]. Existence for the full system under suitable hypotheses is proved in [2]. The unknowns are u (displacement of the solid matrix), p (capillary pressure), and θ (absolute temperature). The system contains four hysteresis operators: The degenerate Preisach operator G describing pressure-saturation hysteresis, P describing elastoplastic hysteresis, and the associated dissipation operators D_P and D_G. The main challenge in the existence proof is related to the degeneracy of G which has been handled by means of a hysteretic version of Moser's iterations.

The permeability μ is assumed to depend only on the pressure. A more realistic case of saturation dependence has been considered [3, 4], but existence results have been obtained only if solid-liquid interaction is neglected and if additional time or space regularizing operators are involved.

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A diffuse interface tumour model with chemotaxis and active transport

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We consider a thermodynamically consistent diffuse interface model for tumour growth, which couples a Cahn-Hilliard system and a reaction-diffusion equation. The system of PDEs models the growth of a tumour in the presence of a nutrient and surrounded by host tissue. A new feature of the model is the inclusion of transport mechanisms such as chemotaxis and active transport through specific choices of the fluxes. We will first discuss a simplified model and derive some results regarding the well-posedness of the system. Then, we will discuss the more general model, which is a Cahn-Hilliard-Darcy system coupled to a convection-reaction-diffusion equation for the nutrient. The effects of including the transport mechanisms and fluid flow will be demonstrated with numerical computations, and if time permitting, we will discuss some recent results regarding the existence of weak solutions to the general model.

On Entropy-Transport problems and distances between positive measures

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In this talk, we discuss a general class of variational problems involving entropy-transport minimization with respect to a couple of given finite measures with possibly different total mass. Problems of this kind are a natural generalization of classical optimal transportation problems.

For a certain choices of the entropy/cost functionals they provide a family of distances between measures, that lie between the Hellinger and the Kantorovich-Wasserstein ones and have interesting geometric properties. The connection to the original entropy/transport problem relies on convex duality in a surprising way. A suitable dynamic Benamou-Brenier characterization also shows the link of these distances to dynamic processes of gradient-flow type, which exhibit creation and annihilation of masses, e.g. in tumor growth models.

(Joint work with Alexander Mielke and Giuseppe Savaré.)

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A free boundary problem for the flow of viscous liquid bilayers

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This talk addresses modeling and simulation of thin-film flows for viscous bilayers [1]. A formal gradient structure for the corresponding free boundary problem will be presented and used to develop a numerical algorithm to solve the transient problem of the form

$$\partial_t u - \nabla \cdot (m(u)\nabla \pi) = 0 \qquad \text{in } \omega(t)$$
$$\partial_t \mathbf{u} - \nabla \cdot (M(\mathbf{u})\nabla \boldsymbol{\pi}) = \mathbf{0} \qquad \text{in } \Omega \setminus \omega(t)$$

with $\mathbf{u} = (u, u_+)$ and $\boldsymbol{\pi} = (\pi, \pi_+)$ with $\pi = \delta E/\delta u$, $\pi_+ = \delta E/\delta u_+$ and degenerate mobilities m, M, where $\omega(t)$ is part of the unknowns. The proper treatment of the underlying contact line problem and coupling the PDE on ω with the one on its complement are the main mathematical challenges [2]. Numerical solutions will be used to compare with experiments [3], limitations and perspectives will be discussed.

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Wulff shape emergence and sharp $n^{3/4}$ law for crystals

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In this talk the problem of understanding why particles self-assemble in macroscopic clusters with overall polyhedral shape is investigated. At low temperature ground states for a general finite number n of particles of suitable phenomenological energies possibly accounting for two- and three-body atomic interactions are shown to be connected subsets of regular lattices $\mathcal{L}$, such as the triangular and the hexagonal lattice. By means of a characterization of minimal configurations via a discrete isoperimetric inequality, ground states will be seen to converge to the hexagonal Wulff shape as the number n of particles tends to infinity. Furthermore, ground states are shown to be given by hexagonal configurations with some extra particles at their boundary, and the $n^{3/4}$ scaling law for the deviation of ground states from their corresponding hexagonal configurations is shown to hold. Precisely, the number of extra particles is carefully estimated to be at most $K_{\mathcal{L}}\, n^{3/4} + \mathrm{o}(n^{3/4})$, where both the rate $n^{3/4}$ and the explicitly determined constant $K_{\mathcal{L}}$ are proven to be sharp. The new designed method allows to sharpen previous results [1, 5] for the triangular setting [2] and allows to provide a first analytical evidence of the zigzag-edge selectivity and the emergence of the asymptotic Wulff shape for the hexagonal setting [3] in accordance with what is experimentally observed in the growth of graphene flakes [4]. Results presented are in collaboration with Elisa Davoli and Ulisse Stefanelli (Vienna).

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Rate-dependent elastoplasticity at finite strain: existence and approximation results

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We consider a model for elastoplasticity consisting of the elastic equilibrium equation for the elastic deformation field, coupled with the flow rule for the plastic tensor in accord with the theory of generalized standard solids. The plastic law balances dissipation forces with restoring forces. It has in fact the form of a gradient system, driven by a highly nonconvex energy functional due to the geometric nonlinearities arising from the multiplicative decomposition of the strain.

In the case of a 1-positively homogeneous dissipation potential, the system is rate-independent. Existence and approximation results have been obtained in [1] in the framework of *energetic solutions*.

In [3] we address the *rate-dependent* case, featuring a dissipation potential with superlinear growth at infinity. Existence results are proved passing to the limit in a time-incremental minimization scheme via variational convergence techniques, developed in [2] for gradient systems driven by nonconvex and nonsmooth energy functionals.

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Variational methods for steady-state Darcy/Fick flow in swelling-exhibitting or poro-elastic solids.

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Some elastic media allow for diffusion of a solvent in the atomic grid or the macromolecular chains causing a swelling (as some metals or polymers) or for a flow of a fluid in pores (as poro-elastic rocks). This rises a nontrivially mechanically-coupled system.

More specifically, confining on the small strains, the momentum equilibrium equation is to be then coupled with the continuity equation for the flow, i.e.:

$$\text{(9a)} \qquad \operatorname{div}\sigma + f = 0 \qquad \text{with} \quad \sigma = \partial_e\varphi(e(u),c),$$

$$\text{(9b)} \qquad \operatorname{div}(\mathbb{M}(c)\nabla\mu) = 0 \qquad \text{with} \quad \mu = \partial_c\varphi(e(u),c),$$

where u is the displacement, $e(u) = \frac{1}{2}(\nabla u)^\top + \frac{1}{2}\nabla u$ the small-strain tensor, c a concentration of a solvent in interatomic grid or the fluid in pores, $\mathbb{M} = \mathbb{M}(c)$ a mobility matrix, σ stress tensor, μ a chemical potential. The free energy $\varphi = \varphi(e,c)$ is to be prescribed at particular cases, and may lead to the Darcy's or the Fick's flows (i.e. the flux proportional to the gradient of the pressure or the concentration, respectively) or their combination. The system (9) is to be accompanied by suitable boundary conditions.

One should distinguish between the general steady-state situations and purely static situation. The former one means that all fields including the specific dissipation rate $\mathbb{M}(c)\nabla\mu\cdot\nabla\mu$ do not depend on time, while the latter means in addition that that the dissipation rate is zero. In the static case, the system enjoys the variational structure: (u,c) is the unique solution of the constrained minimization problem

$$\text{(10)} \qquad \begin{cases} \text{Minimize} & (u,c) \mapsto \displaystyle\int_\Omega \varphi(e(u),c) - f\cdot u \,\mathrm{d}x \\ \text{subject to} & \displaystyle\int_\Omega c \,\mathrm{d}x = C_{\text{total}}, \quad u \in H^1(\Omega;\mathbb{R}^d), \quad c \in L^2(\Omega), \end{cases}$$

where $C_{\text{total}} > 0$ is a constant describing the total solvent content in the body which is now considered isolated. The chemical potential μ is constant (assuming the body Ω is connected) and represents the Lagrange multiplier to the affine constraint in (10).

In the general steady-state case, μ is no constant and the situation is more complicated. The variational structure of a certain subproblem, namely

$$\text{(11)} \qquad \begin{cases} \text{Minimize} & (u,c) \mapsto \displaystyle\int_\Omega \varphi(e(u),c) - f\cdot u - \mu c \,\mathrm{d}x \\ \text{subject to} & u \in H^1(\Omega;\mathbb{R}^d), \quad c \in L^2(\Omega), \end{cases}$$

for μ given can then be combined with the Schauder fixed-point theorem for μ solving a boundary-value problem for (9b) with (u,c) resulting from (11). In fact, a suitable regularization is to be used - the options are to put a gradient on $e(u)$ (i.e. the so-called 2nd-grade nonsimple materials) or on c (i.e. a capillarity) or to consider a gradient phase-field theory.

In the non-static steady-state situation, the flux generates also a heat. If not transferred away fast, it may lead to a substantial variation of temperature. Then the above outlined fixed-point argument is to be augmented by the heat-transfer problem.

Some of these problems bear a generalization for large strains [1]

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A new Visco-Energetic incremental minimization scheme for rate-independent evolution problems

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We study the asymptotic properties of a modified incremental minimization algorithm for rate-independent evolution problems driven by a time-dependent energy. The main novelty of the scheme concerns the dissipation term, typically a distance in the energetic approach, which is perturbed by a suitably rescaled viscous correction.

As in the energetic setting, the limit curves (called *Vico-Energetic* solutions) have bounded variation and can be characterized by the combination of energy balance and stability: both take into account the viscous term in the jump characterization and combine the robust stability properties of energetic solutions with a more localized jump behaviour.

The study of the classical example of a linearly perturbed bistable energy clarifies the role of the viscous perturbation and exhibits a one-parameter family of solutions, ranging from the Energetic to the Balanced Viscosity case.

(In collaboration with Luca Minotti)

A Gamma convergence approach to a sharp-interface limit of a phase transition problem, with application to a tumor growth model

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We consider an approximate two-phases model for tumor growth, consisting in a forth order system of PDEs involving a Cahn-Hilliard type equation. We are interested in the sharp interface limit, that is the limit of the solutions as the approximating parameter tends to zero. Benefiting of a gradient flow structure, we employ a technique introduced by Sandier and Serfaty, known as Γ-convergence for gradient flows, allowing us to prove that the solutions tends to a solution of a free-boundary problem. The free boundary evolution can be described and is shown to be very similar to the limit of the Cahn-Hilliard equation solutions.

Non existence and instantaneous extinction for very fast fractional diffusion equations

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In this talk I will discuss about non-existence of solutions of the Cauchy problem in $\mathbb{R}^N$ for the nonlinear parabolic equation involving fractional diffusion $\partial_t u + (-\Delta)^s \phi(u) = 0$, with $0 < s < 1$ and very singular nonlinearities ϕ .

On a nonstandard viscous nonlocal Cahn-Hilliard system

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We consider a nonlocal version of a nonstandard system of Cahn–Hilliard type whose local analogue was introduced by P. Podio-Guidugli as a model for phase separation and diffusion of atomic species on a lattice [see Ric. Mat. 55 (2006), 105-118]. The local variant has been the subject of a series of papers by P. Colli, G. Gilardi, P. Podio-Guidugli, and the author, in the past years. In this lecture, we present an analysis of the more challenging nonlocal variant of the model. Besides existence and uniqueness, we establish strong stability results and, if time permits, an associated optimal control problem.

Finite plasticity in $P^T P$

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The classical approach to finite plastic deformations prescribes the deformation gradient of the medium to be multiplicatively decomposed as FP where F stands for the elastic and P for the plastic strain, respectively. The requirement of frame-indifference imposes that the hyperelastic stored energy density is given in terms of the symmetric Cauchy-Green tensor $F^T F$ only. Moving from this fact, I shall comment on the possibility of formulating the full finite plasticity model in terms of the corresponding plastic metric tensor $P^T P$. This situation is indeed common in applications and bears some relevant advantages with respect to the classical formulation in terms of P [2].

I will comment on the global existence of quasistatic evolutions for this model and on the possibility of rigorously ascertain the small-deformation limit via evolutive Γ-convergence techniques [1].

This work is in collaboration with Diego Grandi (Vienna).

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From adhesive contact to brittle delamination in visco-elastodynamics

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This contribution addresses two models describing the rate-independent fracture of a material compound along a prescribed interface in a visco-elastic material. This unidirectional process is modeled in the framework of Generalized Standard Materials with the aid of an internal delamination parameter. In the context of (fully) rate-independent systems within the energetic formulation it has become a well-established procedure to obtain solutions of the brittle model via an adhesive-contact approximation based on tools of evolutionary Gamma-convergence. This means that the non-smooth, local brittle constraint, confining displacement jumps to the null set of the delamination parameter, is approximated by a smooth, non-local surface energy term. Here, we discuss the extention of this approach for systems that couple the rate-independent evolution of the delamination parameter with a viscous and dynamic evolution of the displacements in the bulk. This is joint work with Riccarda Rossi (Brescia) .

Liquid crystal inertia in the Q-tensor framework

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One of the most intriguing aspects of liquid crystals, from both a physical and mathematical point of view concerns the presence of inertia, manifested as a second-order material derivative.

Usually, based on physical considerations, the inertia is considered to be negligible and dropped from the equations, which conveniently simplifies the equations significantly.

We consider one of the simplest cases in which the inertia is kept, within the Qian-Sheng formalism, and provide a basic well-posedness result. We further consider some simple instances of a mysterious type of solution: the Ericksen twist waves, and analyze some of its aspects.

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