Math for the Digital Factory

Combinatorial Optimization Aspects of Robot Tour Planning

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DFG Research Center MATHEON
Mathematics for key technologies

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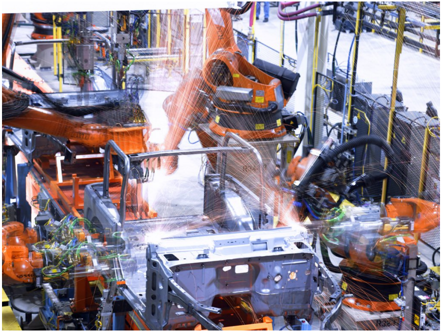
DFG Research Center MATHEON Project:
Automatic reconfiguration of robotic welding cells



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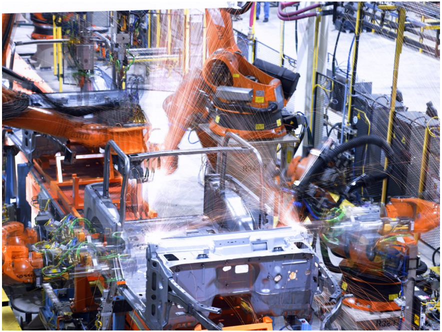

Problem:

- Robots perform spot welding tasks on single component
- Some points can only be processed by specific robots
- Robots must not collide
- Given cycle time

Discrete Part

- task assignment
- sequencing of weld points

Requires

- distances between weld points
- collision information

Continuous Part

- path planning
- collision detection and avoidance

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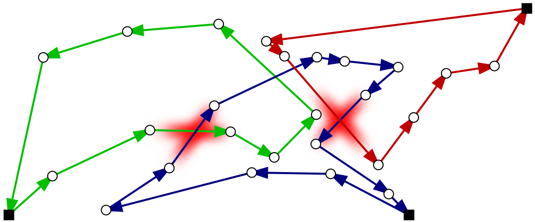
Vehicle Routing Problem with collision constraints

- Representation as a graph for each robot:
  - nodes ⇔ weld points that can be visited
  - arcs ⇔ paths between two weld points
- Each arc has a travel time



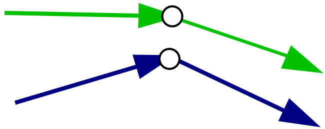
Collisions: Certain moves of two robots must not be made at the same time

Scheduled Tours for the robots:

- A tour with integer start and end times for each arc:
  - $end_a - start_a =$ traversal time of a
  - If $end_a < start_b$ we wait in node v

Collisions between robots at the same time:

- Both robots waiting: node-node collision $\Downarrow$
- One robot moving and one waiting: node-arc collision $\Downarrow$
- Both robots moving: arc-arc collision



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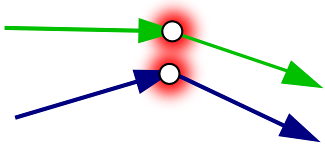

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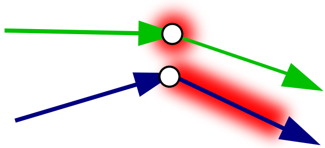

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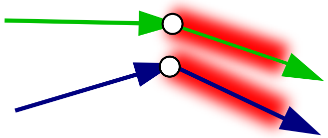

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- collisions are too restrictive
- slight changes of trajectories to avoid collisions

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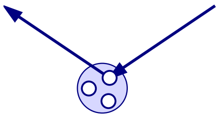

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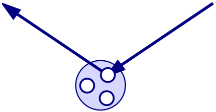

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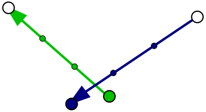

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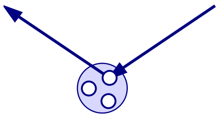

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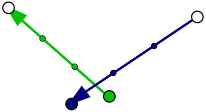

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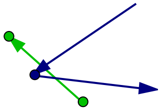

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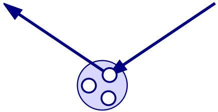

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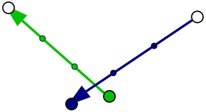

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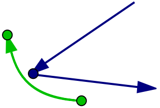

For each feasible scheduled tour $t \in \mathcal{T}$ there is a 0/1–variable x_t

$$\min \sum_{t \in \mathcal{T}} c_t x_t \tag{WCP}$$

$$\text{s.t.} \sum_{t \in \mathcal{T}} \delta_{vt} x_t = 1 \qquad \forall v \in V$$

$$\mathbf{x} \ \textbf{ is collision free} \tag{1}$$

$$x_t \in \{0, 1\} \qquad \forall t \in \mathcal{T} \tag{2}$$

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\end{aligned}
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Using branch-and-price approach:

- Constraints (1) and (2) are enforced by branching in a branch and bound framework.
- For (1), conflicting arcs are forced/forbidden in certain time windows
- Pricing: Elementary shortest path with negative costs and time windows

Resource Constrained Shortest Path

- Sub-problem for many VRP branch-and-price approaches
- State of the art: Bidirectional labeling algorithms

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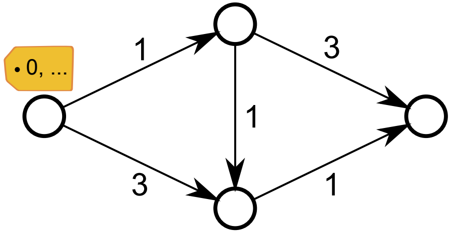

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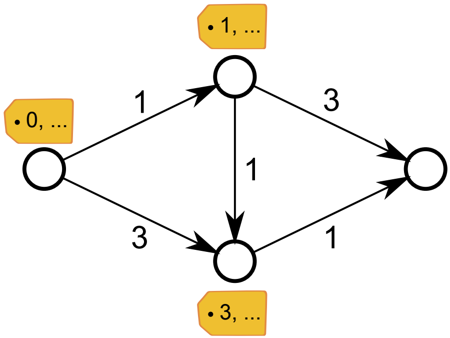

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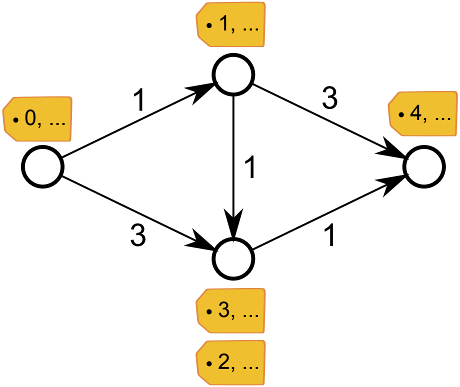

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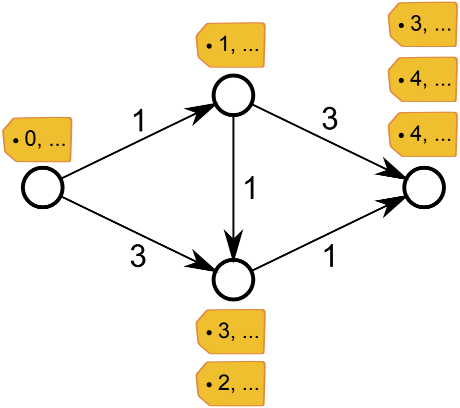

Drawbacks

- Elementary shortest path is expensive
- Algorithms do note scale very well

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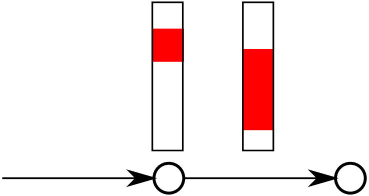

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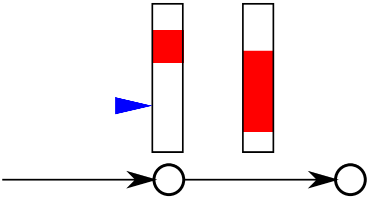

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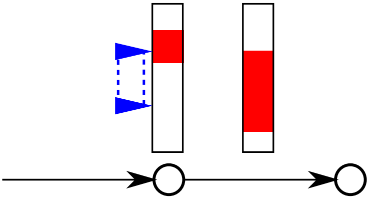

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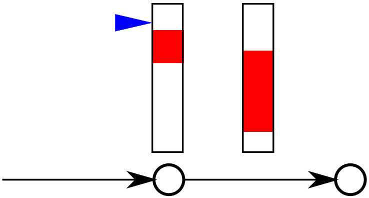
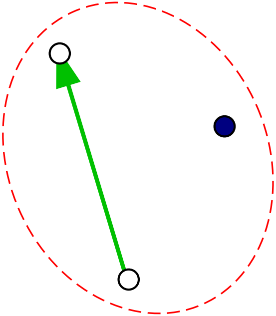


- Take the collision informaiton into account
- Find conflicting edge-node pairs



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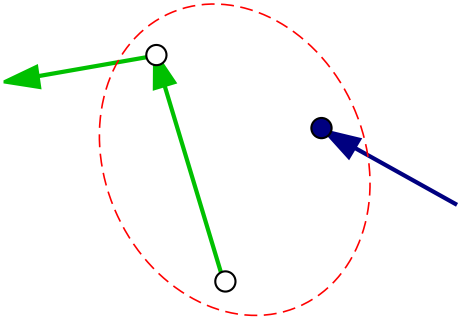

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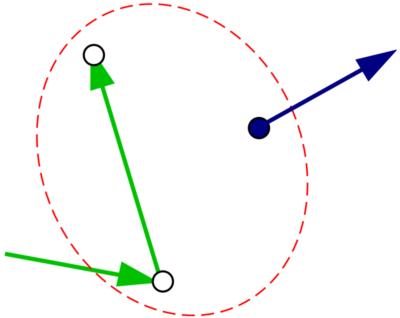

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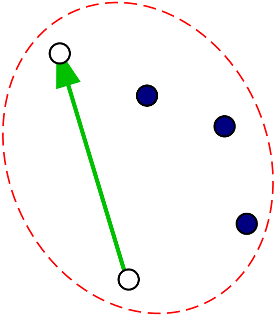

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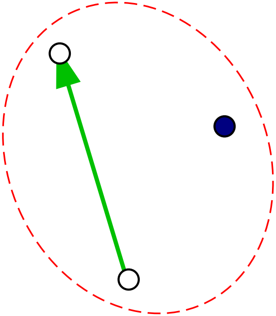

- If this leads to an infeasible node for all robots ⇒ delete the edge
- This is a cut for every infeasible robot

Greedy-partitioning heuristic:

- Assign every node to the closest robot
- Solve the TSP-Problem (approximately) for all robots
- If the cycle time is violated reassign some nodes

LP-rounding heuristic:

- Remove nodes from tours with smaller LP-values
- Concatenate all tours for one robot
- Solve additional sub-MIP to find feasible waiting times:
  - Leads to one binary variable for every conflicting
  - Even for larger instances ($>$ 35 nodes) solving takes less than 1 second

Improving Computational Performance

- Penalizing high collision edges
- Efficient data structures for forbidden periods
- Solve the elementary Shortest Path Problem:
  - Using 2-cycle elimination
  - Using decremental state-space relaxation
- Use Robust Tours instead of Scheduled Tours:
  - Reducing the number of variables
  - Detecting collisions is more complicated
- Solve the pricing problems in parallel on shared memory systems
- Reverse tours in heuristics

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