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Hamiltonian flows in nonlinear optics: physics at ultrashort timescales

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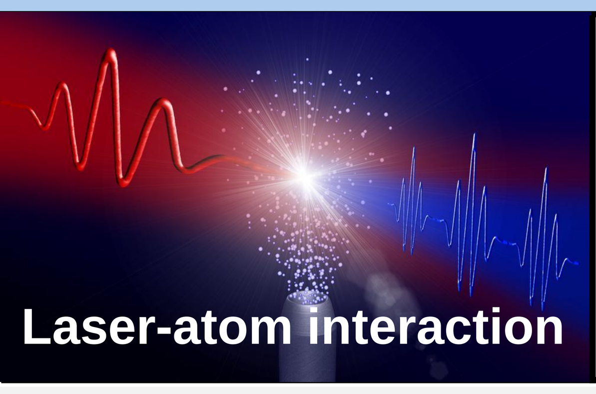
Emmy Noether's groundbreaking works led us to understand "energy" as a dynamical invariant of physical systems with time translational symmetry. Moreover, written in terms of canonical phase space coordinates, it is the generator of time translations, both, in classical physics, and quantum mechanics. As the most prominent example of a Hamiltonian flow, we discuss the Schrödinger equation and its nonlinear generalizations. We emphasize its ubiquitous role in modelling and simulation of laser-matter and nonlinear self-interaction of light, and give a brief account of related research activities at Weierstrass Institute.

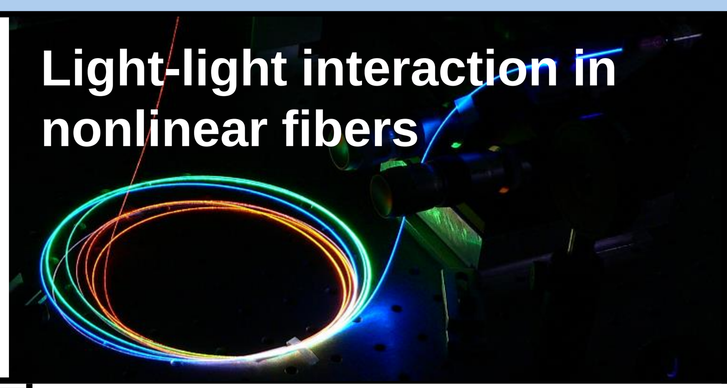


Multiscale methods

Time scales: $t_0 = t,\ t_1 = \varepsilon t$

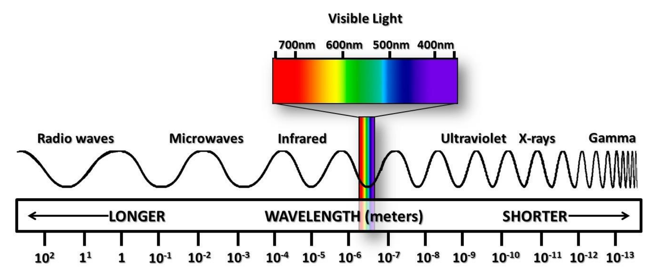
$$\frac{\partial}{\partial t} = \frac{\partial}{\partial t_0} + \varepsilon \frac{\partial}{\partial t_1} + O(\varepsilon^2)$$

$$x(t) = X_0(t_0, t_1) + \varepsilon X_1(t_0, t_1) + O(\varepsilon^2)$$



Soliton perturbation theory

$$E(z,t) = \mathcal{E}\left(t - \frac{z}{v_g}\right) e^{i(\beta(\omega_S)z + \gamma P_0 z - \omega_S t)} + \epsilon(z,t)$$



New frequencies

Hamiltonian flow in classical mechanics

Trajectory in 2N-dimensional phase space: $\boldsymbol{x}\colon \mathbb{R} \to \mathbb{R}^{2N}$

Equation of motion:

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{J}\nabla H(\boldsymbol{x},t), \quad \boldsymbol{J} = \begin{pmatrix} \boldsymbol{0} & \boldsymbol{1}_{N\times N} \\ -\boldsymbol{1}_{N\times N} & \boldsymbol{0} \end{pmatrix}$$

Case $N = 1$: $\quad \boldsymbol{x}(t) = (q(t), p(t))^T,$

$$\dot{q}(t) = \frac{\partial H}{\partial p},\ \dot{p}(t) = -\frac{\partial H}{\partial q}$$

Laser-atom interaction:

Time-dependent Schrödinger equation for atom in an oscillating electric field:

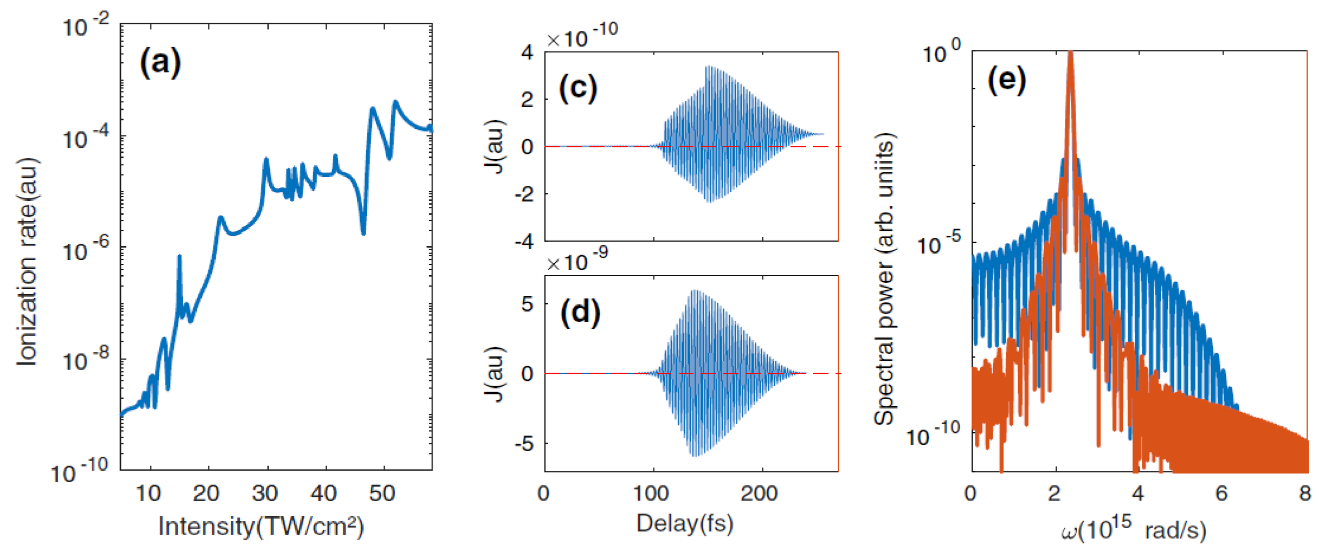
$$i\partial_t \psi(t,\boldsymbol{x}) = H(\boldsymbol{x},t)\psi(t,\boldsymbol{x})$$

Floquet eigenproblem in finite domain + separation of time scales + adiabatic theorem of quantum mechanics (Born&Fock) yields ionizing, resonant states:

$$\Psi_{\mathrm{ad}}(t_0, t_1) = e^{\int_{-\infty}^{t_1} dt' \lambda(t')} \Phi_0(t_0, t_1).$$

$\Phi_0(t_0, t_1)$: Ionizing **Stark resonance** with Lyapunov-stable Floquet multiplier $e^{2\pi\lambda/\omega}$, $\mathrm{Re}(\lambda) < 0$.

Persistent plasma currents and THz frequency combs:



- **Resonant ionization enhancements (Freeman)**
- **Sub-cycle ionization dynamics induces persistent plasma current and THz emssion**

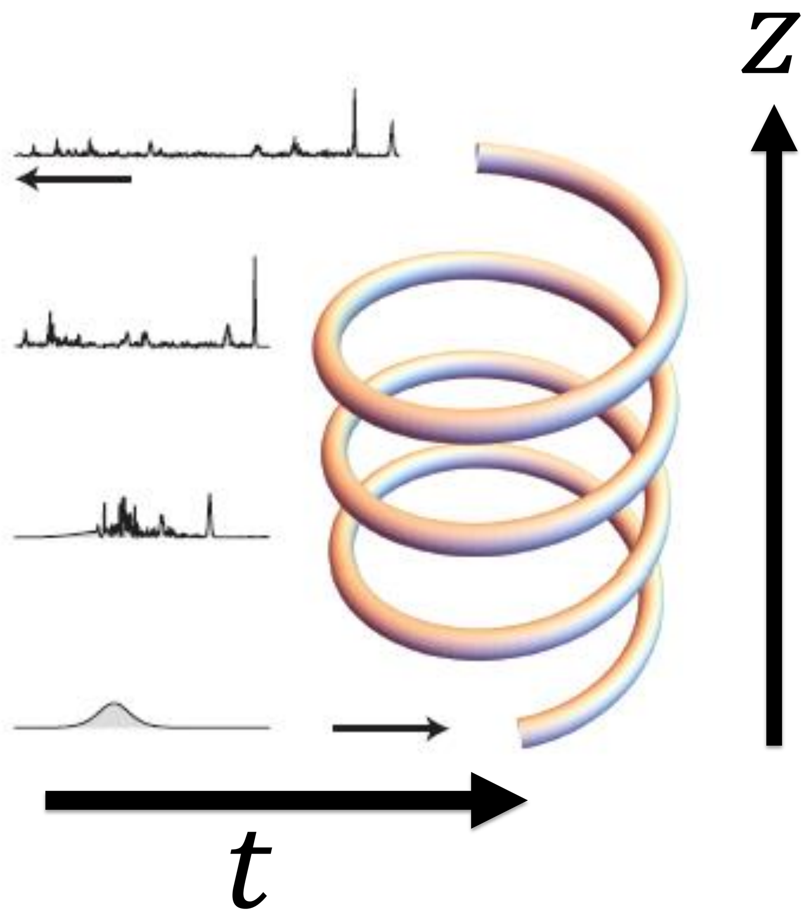
Pulse propagation in optical fibers

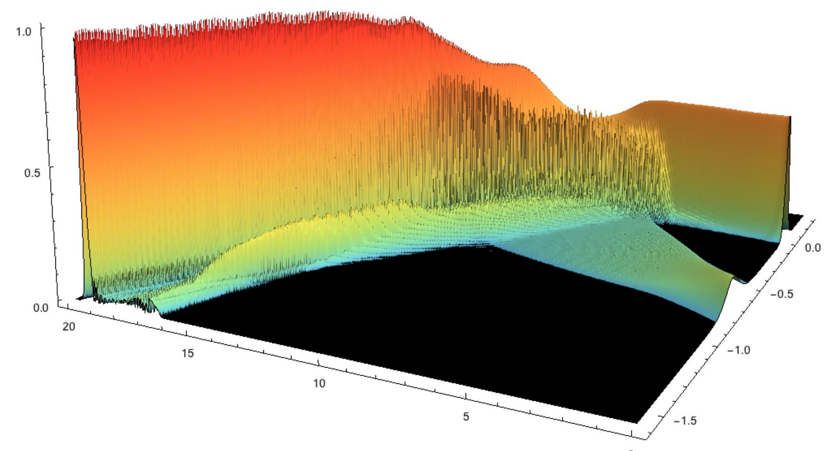
For periodic boundary condtions in t, dynamical equation for Fourier modes $\mathcal{E}_\omega$ of the complex electric field $\mathcal{E}(z,t) = \sum_\omega \mathcal{E}_\omega(z) e^{-i\omega t}$:

$$i\partial_z \mathcal{E}_\omega + |\beta|\mathcal{E}_\omega + \frac{\omega^2}{c^2|\beta|} \sum_{123|\omega} \chi^{(3)}_{123\omega} \check{\mathcal{E}}_{\omega_1} \check{\mathcal{E}}_{\omega_2} \check{\mathcal{E}}_{\omega_3} = 0.$$

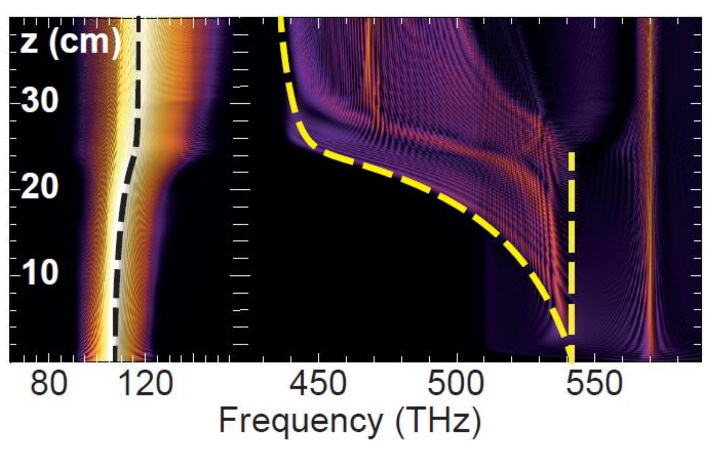
Hamiltonian framework in canonical variables:

$$i\partial_z \mathcal{A}_\omega + \frac{\delta}{\delta \mathcal{A}^*_\omega} \mathrm{H} = 0, \quad \mathcal{E}_\omega = \sqrt{\frac{2\mu_0\omega^2}{|\beta(\omega)|}} \mathcal{A}_\omega$$





Interaction of optical pulses in a nonlinear fiber



Shifting of optical pulses to new frequencies (manipulating light by light)

Employed numerics:

- Operator splitting (Peaceman-Rachford) with fourth-order Numerov approximation of 2nd derivatives
- Fourier split-step
- Pseudospectral Runge-Kutta integrator

Improved Soliton perturbation theory:

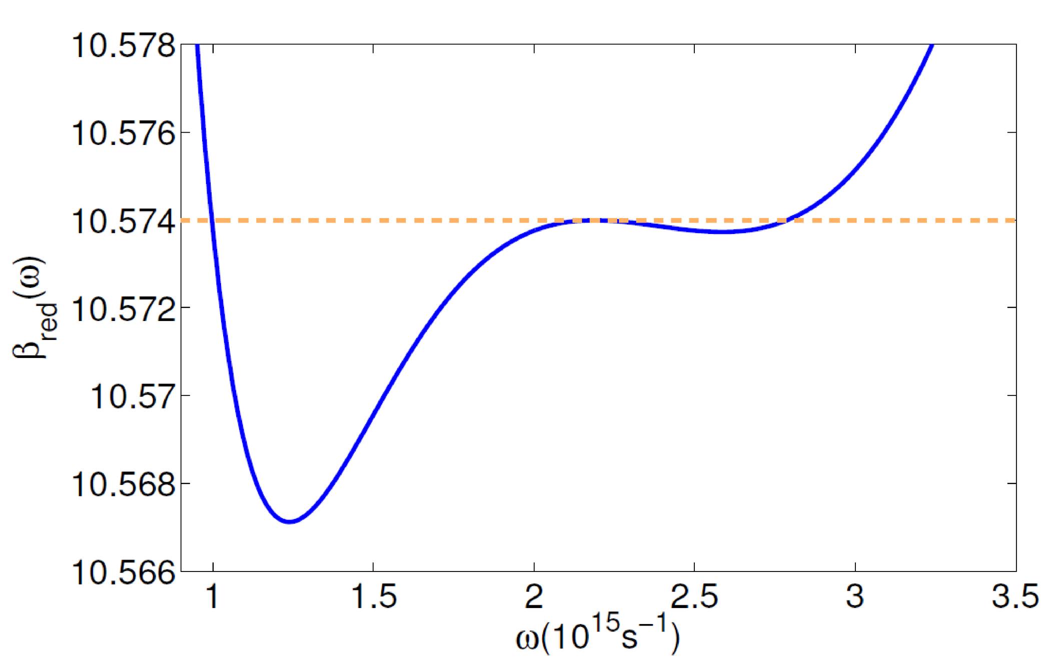
Envelope solition propagating at group velocity v_g + weak radiation background:

$$\mathcal{E}(z,t) = \mathcal{E}_S\left(t - \frac{z}{v_g}\right) e^{i(\beta(\omega_S)z + \gamma P_0 z - \omega_S t)} + \epsilon(z,t)$$

- Emission of phase matched radiation from a moving source (resonant radiation)
- Adiabatic model for evolution of soliton and dispersive pulse parameters
- Analytical model for all-optical switching effects

Resonant radiation in optical fibers and femtosecond filaments: exciting new frequencies

$$\beta_{red}(\omega) = \beta(\omega) - \beta'(\omega_S)(\omega - \omega_S)$$



Reduced propagation constant in nonlinear fiber

Kontakt

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