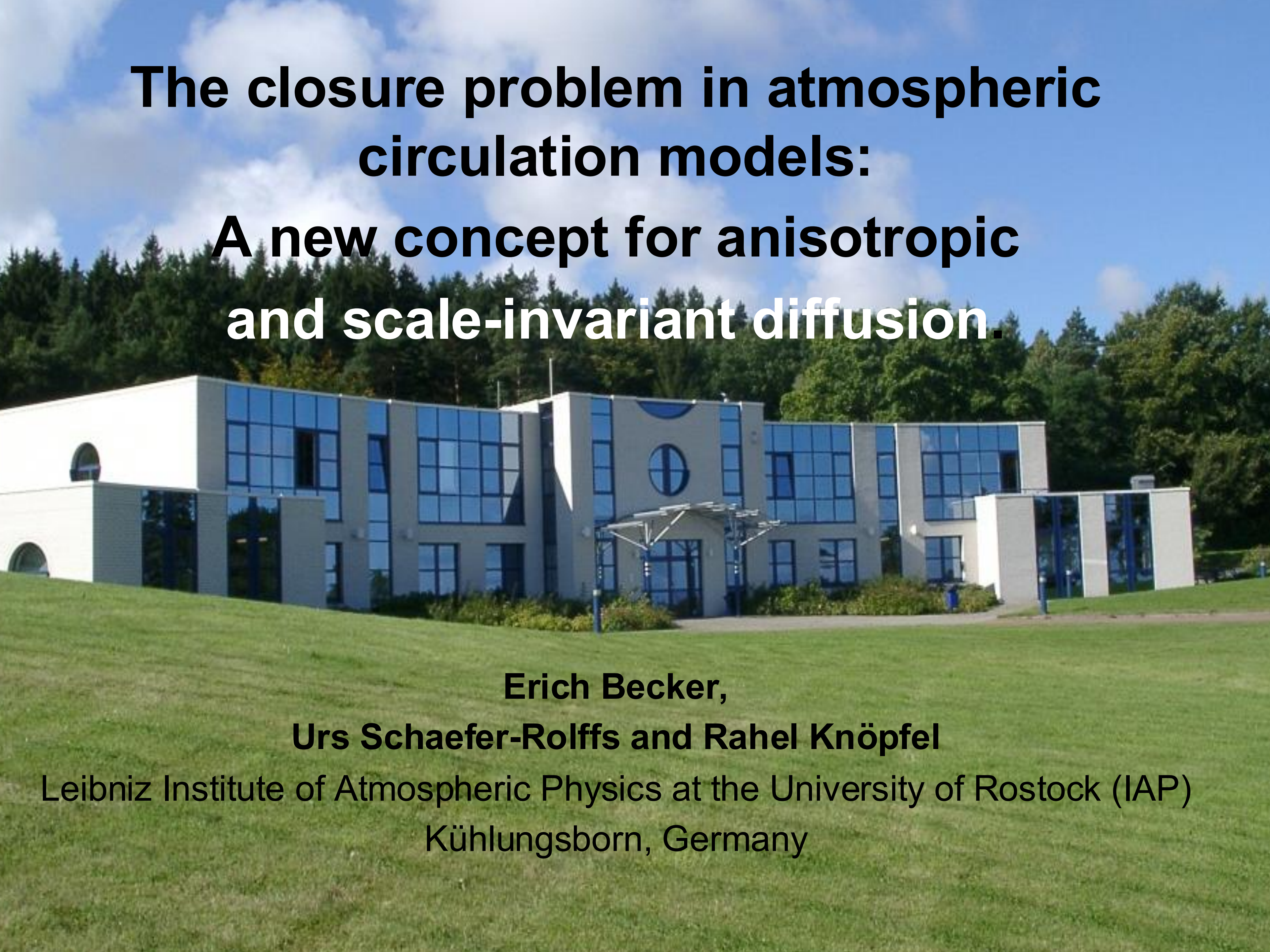

The closure problem in atmospheric circulation models: A new concept for anisotropic and scale-invariant diffusion.

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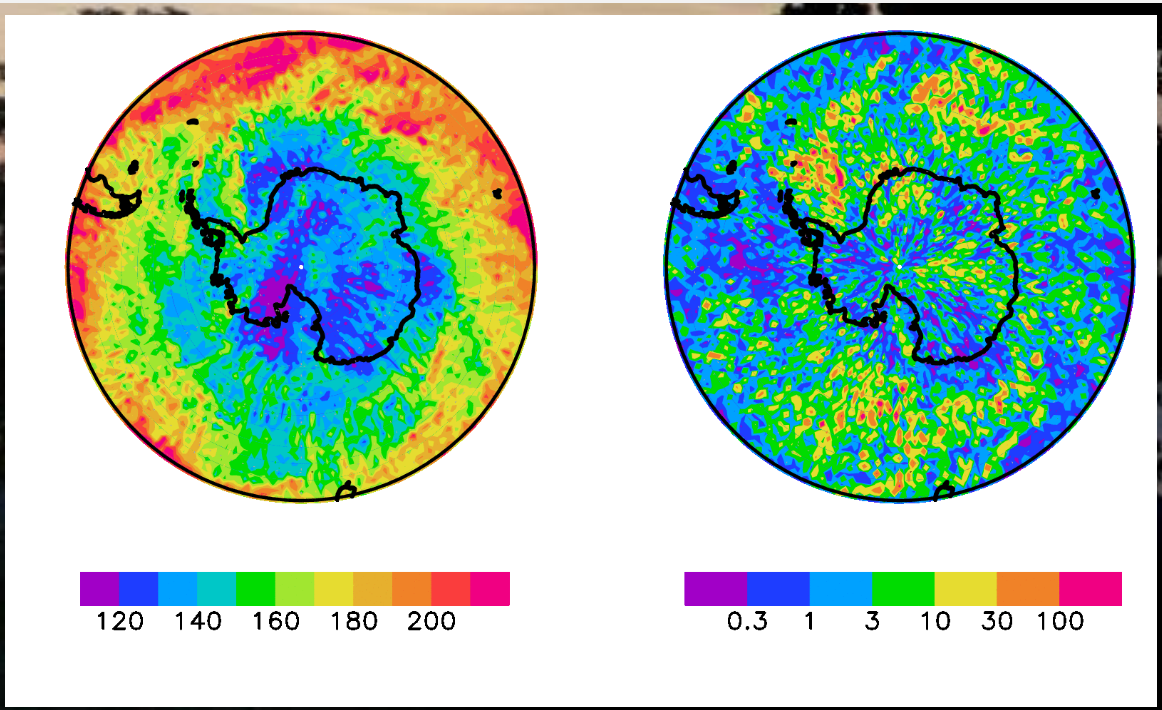
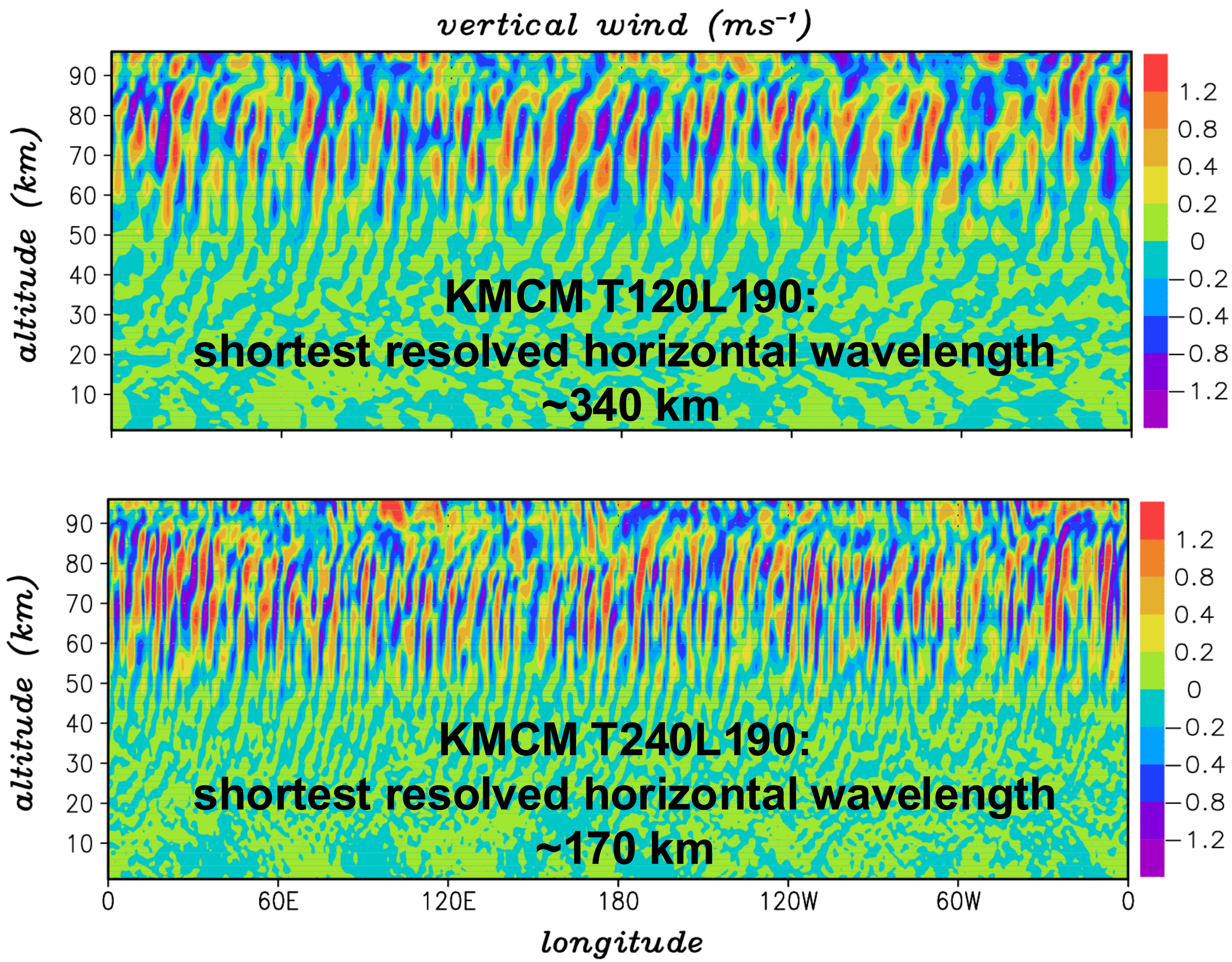
Special focus of model development at IAP:
Representation of gravity waves (GWs) and macro-turbulent diffusion

Reason:
GWs and turbulence are first-order dynamical processes in the mesosphere/lower thermosphere (MLT, 50-110 km)

Snapshot of temperature (K) and dissipation (K/d) around 85 km in January

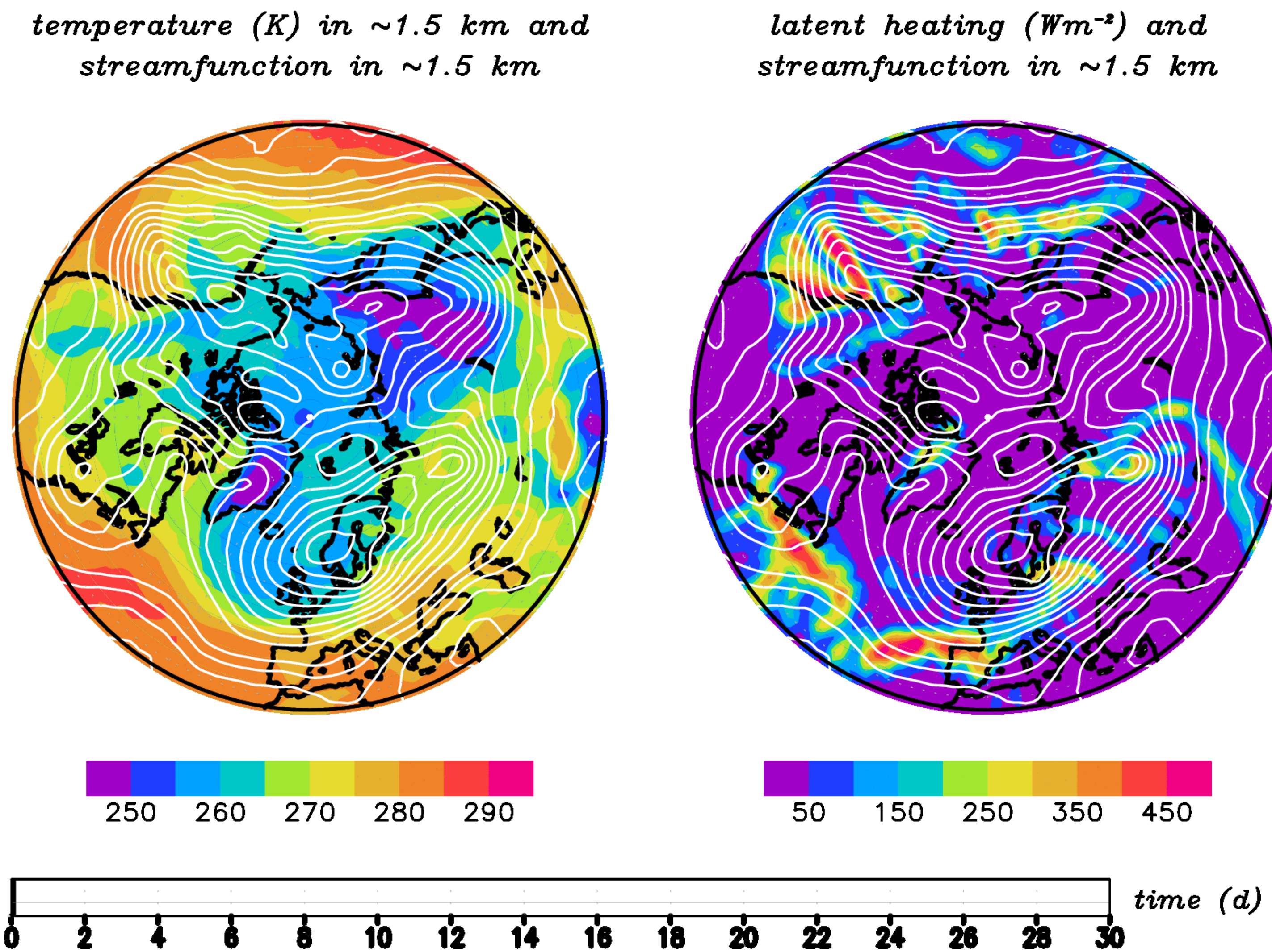


Snapshots of the simulated vertical wind in the summer hemisphere



For higher resolution, the simulated gravity waves have smaller spatial scales (and higher frequencies).

Sequence from a an idealized simulation of the general circulation: Meridional heat exchange due to weather vortices



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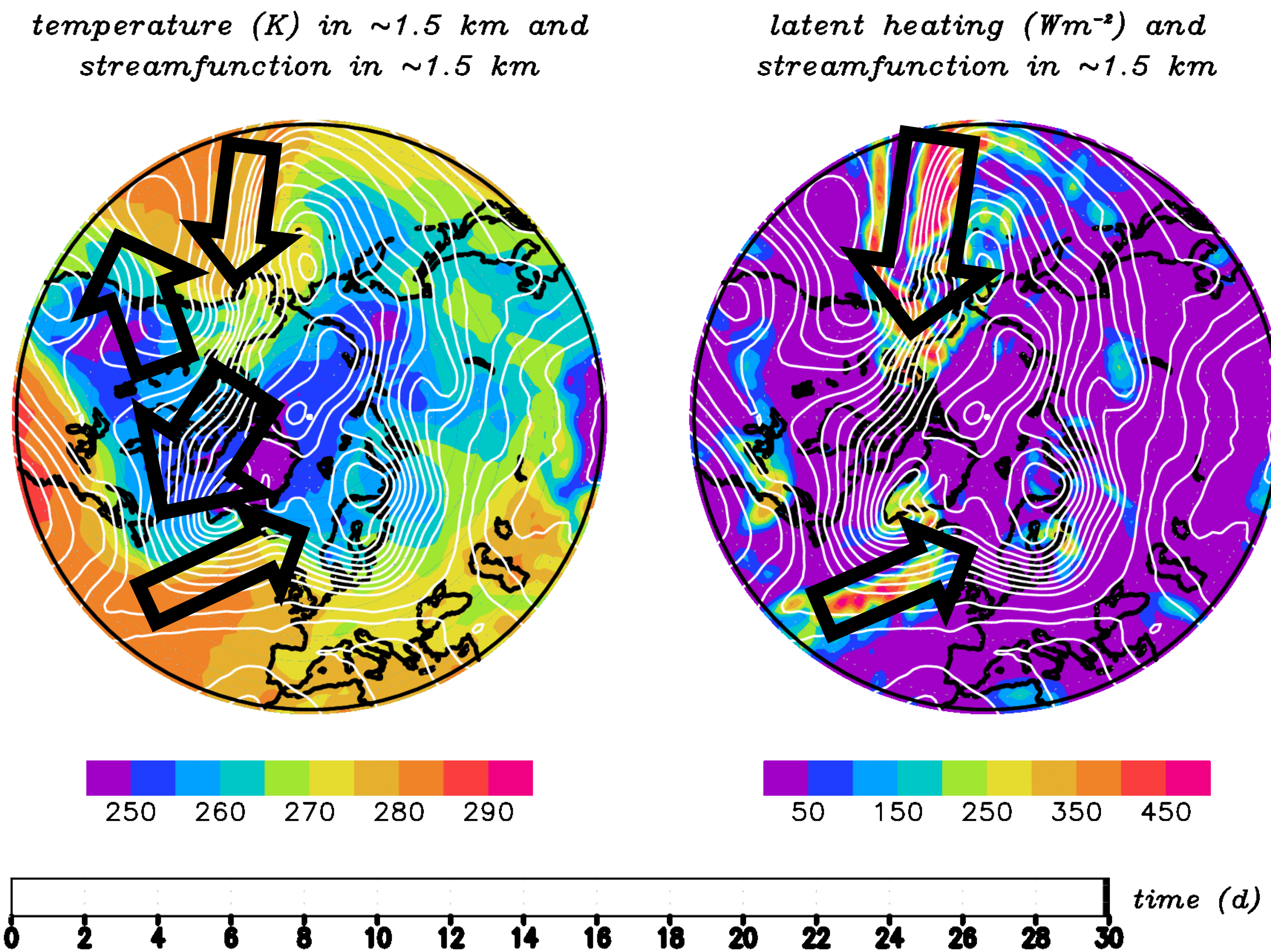
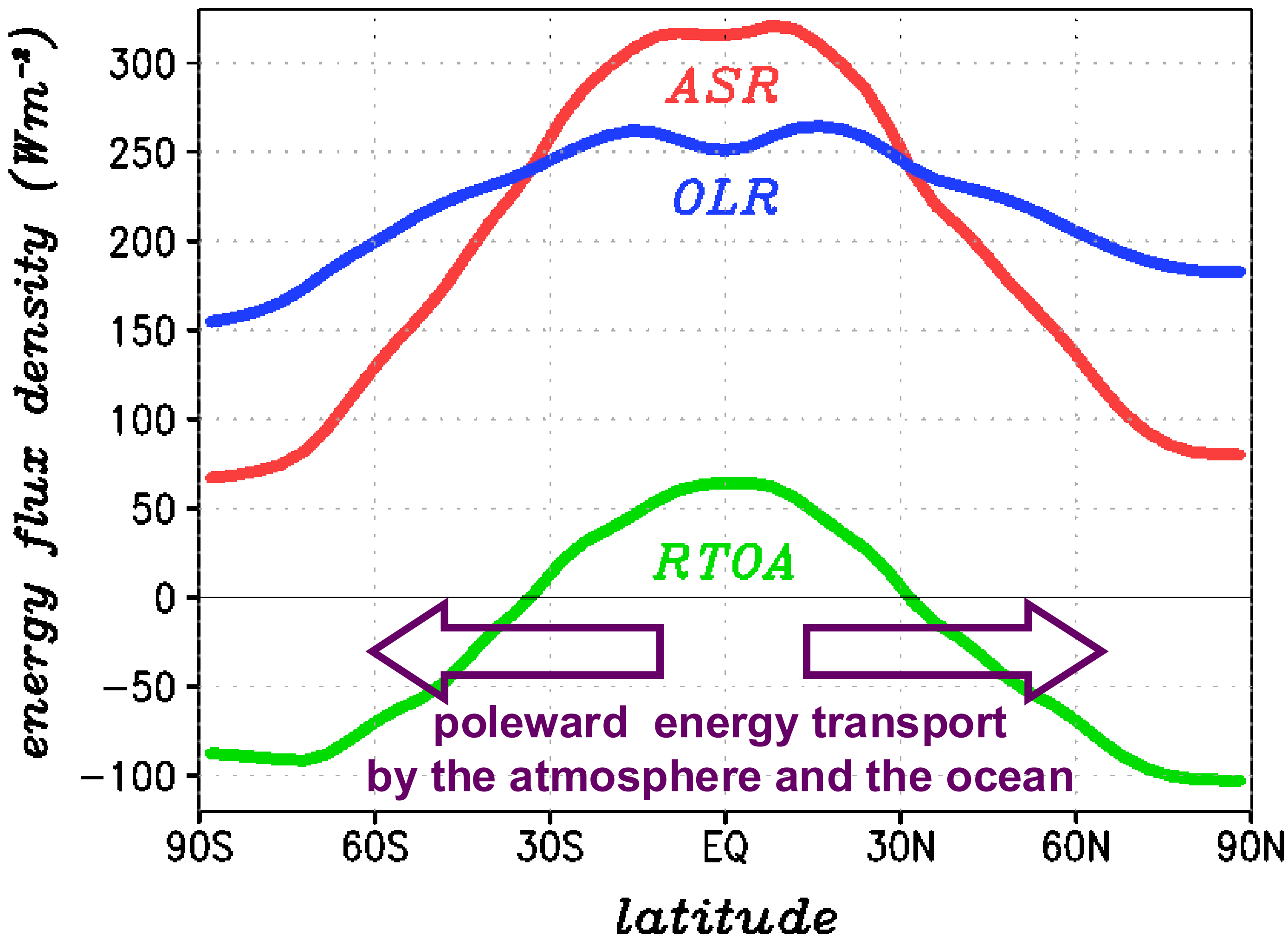
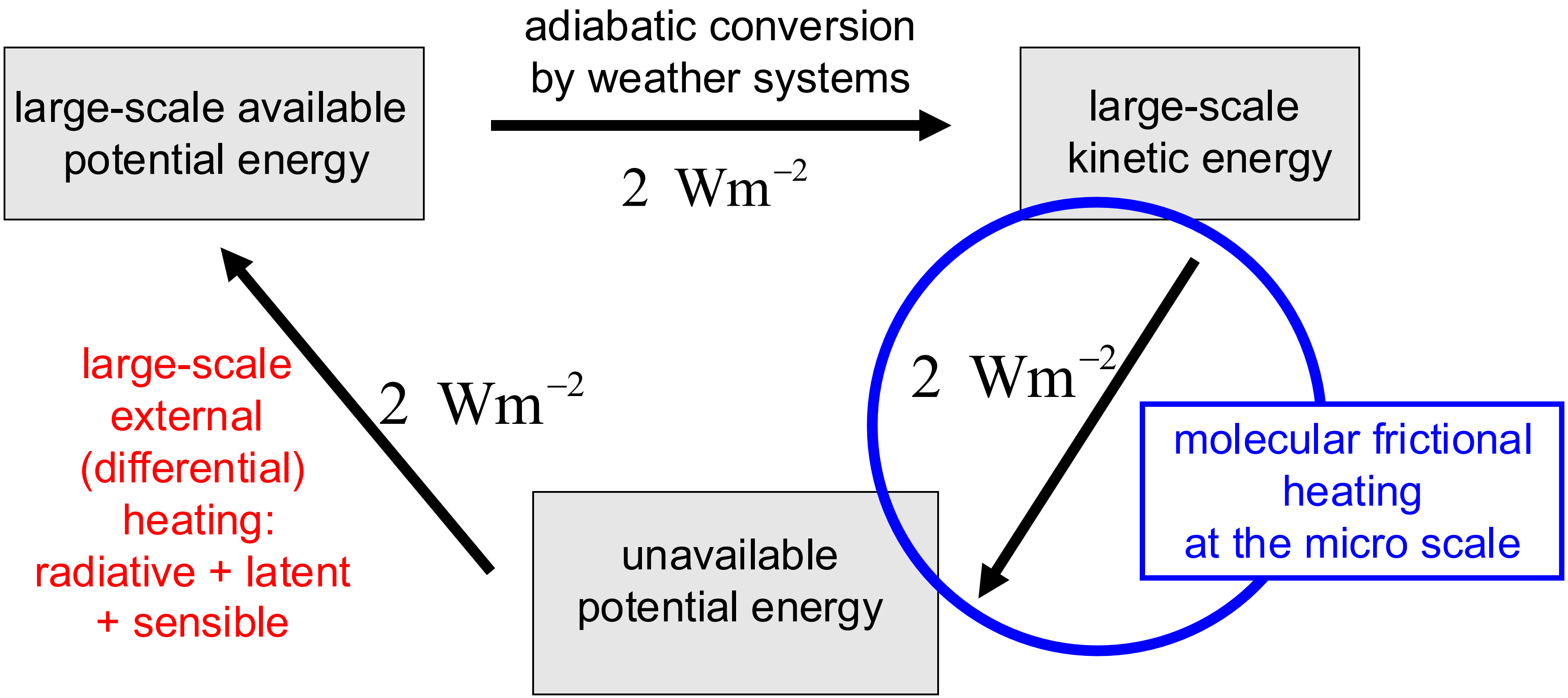


Simulated mean radiation budget at the top of the atmosphere

ASR = absorbed solar radiation
OLR = outgoing long-wave radiation
RTOA = ASR – OLR = net downward radiation



Lorenz energy cycle



- frictional heating essential for entropy and energy budgets:
  net diabatic heating of the atmosphere = frictional heating (Lorenz, 1967)

- molecular frictional heating occurs at the end of energy cascade through the mesoscales (including GWs) and turbulence

Formulation of a general circulation model with subgrid-scale diffusion consistent with conservation laws

Consistent formulation

$$d_t\mathbf{v} = -f\,\mathbf{e}_z \times \mathbf{v} - \frac{\nabla p}{\rho} + \rho^{-1}\,\partial_z\,(\rho\,K_z\,\partial_z\mathbf{v}) + \rho^{-1}\,\nabla\,(\rho\,K_h\,\mathbf{S}_h)$$

$$d_t h = \frac{d_t p}{\rho} + Q_{rad} + Q_{lat} + \frac{c_p}{\rho}\,\partial_z\,(\rho\,\frac{T}{\Theta}K_z\,\partial_z\Theta\,) + \underbrace{K_z\,(\partial_z\mathbf{v})^2}_{\geq\,0} + K_h\,(\,\mathbf{S}_h\nabla\,)\cdot\mathbf{v}$$

- $\mathbf{S}_h = \mathbf{S}_h^T$, $K_h\,(\,\mathbf{S}_h\nabla\,)\cdot\mathbf{v} \geq 0$, and $\mathbf{S}_h\mathbf{e}_z = 0$ (Becker, 2001, JAS)
- no exchange of mechanical energy between the surface and the the atmosphere $\rightarrow$ energy conserving discretization of vertical momentum diffusion and shear production (finite-difference analogue of the no-slip condition)

  (Burchardt, 2002, OM; Becker, 2003, MWR; Boville & Bretherton, 2003, MWR)

,

Smagorinsky-type horizontal diffusion (Becker & Burkhardt, 2007, MWR) and stress-tensor-based hyperdiffusion (Brune & Becker, 2013, JAS)

$$\rho^{-1}\nabla(\rho K_h \mathbf{S}_h) \longrightarrow \rho^{-1}\nabla(\rho K_h \mathbf{S}_h) + \rho^{-1}\nabla(\rho K_{h0} \mathbf{S}_{hf})$$

$$\mathbf{S}_h = \{(\nabla + \mathbf{e}_z/a)\circ \mathbf{v}\} + \{(\nabla + \mathbf{e}_z/a)\circ \mathbf{v}\}^T - (\nabla\cdot\mathbf{v})\,\mathbf{1}$$

$$K_h = l_h^2\sqrt{|\mathbf{S}_h|^2 + S_0^2} \quad \text{(nonlinear, Smagorinsky-type)}$$

$$\mathbf{v} = -\sum_{n=1}^{N}\frac{a^2}{n(n+1)}\sum_{m=-n}^{+n}\left(\xi_{nm}(\mathbf{e}_z\times\nabla Y_{nm}) + D_{nm}\nabla Y_{nm}\right)$$

$$\mathbf{S}_{hf} = \{(\nabla + \mathbf{e}_z/a)\circ \mathbf{v}_f\} + \{(\nabla + \mathbf{e}_z/a)\circ \mathbf{v}_f\}^T - (\nabla\cdot\mathbf{v}_f)\,\mathbf{1}$$

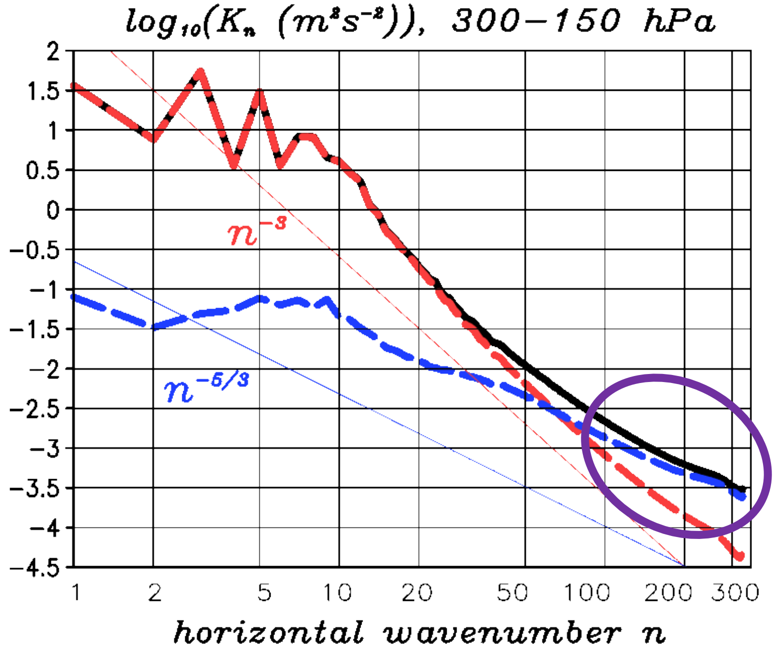
$$K_{h0} = \text{const (vertical coordinate)}$$

$$\mathbf{v}_f = -\sum_{n=1}^{N}\text{Filter}(n)\frac{a^2}{n(n+1)}\sum_{m=-n}^{+n}\left(\xi_{nm}(\mathbf{e}_z\times\nabla Y_{nm}) + D_{nm}\nabla Y_{nm}\right)$$

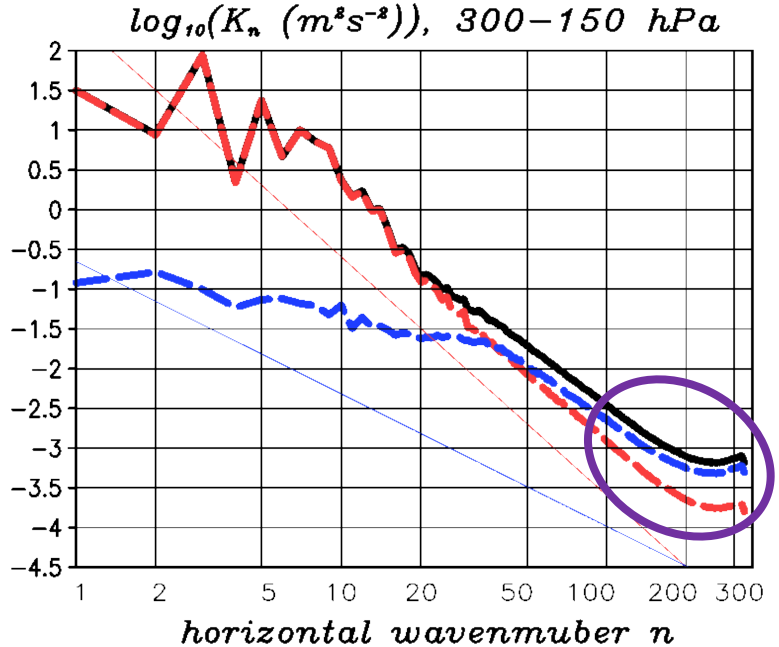
$$K_h(\mathbf{S}_h\nabla)\cdot\mathbf{v} \longrightarrow \underbrace{K_h(\mathbf{S}_h\nabla)\cdot\mathbf{v}}_{\geq 0} + \underbrace{K_{h0}(\mathbf{S}_{hf}\nabla)\cdot\mathbf{v}}_{\text{arbitrary sign}} \geq 0\,?$$

The Classical Smagorinsky Model (CSM) is insufficient to simulate realistic energy spectrum

CSM completed by hyperdiffusion (filter increases from 0 to1 between n=270 and n=300)



CSM with an eight times larger horiz. mixing length



Possible reason for the CSM being an insufficient closure:
The mesoscales in the free atmosphere are subject to a forward energy cascade that is governed by the scaling laws of **stratified turbulence** (e.g., Lindborg, 2006, JFM). Any closure that complies with such a **macro-turbulent inertial range** must be **scale invariant**.

Scale-invariant horizontal and vertical diffusion in the free atmosphere for truncation in the mesoscales

(Schaefer-Rolffs, Knöpfel & Becker, Meteorol. Z, 2015; Schaefer-Rolffs & Becker, MWR, 2012)

diffusion coefficients for momentum

$$K_h = l_h^2 \, |\mathbf{S}_h| \gg K_z = l_z^2 \, |\partial_z \mathbf{v}|$$

$$\mathbf{S}_h = (\nabla \circ \mathbf{v}) + (\nabla \circ \mathbf{v})^T - (\nabla \cdot \mathbf{v}) \, \mathbf{1}$$

$$\tilde{X} = \sum_{n=1}^{N} f(n) \sum_{m=-n}^{+n} X_{nm} Y_{nm}$$ (filtering over the smallest resolved scales)

$$l_h^2 = \left| \widetilde{\mathbf{v} \circ \mathbf{v}} - \tilde{\mathbf{v}} \circ \tilde{\mathbf{v}} - \frac{1}{2} \mathbf{1} \left(\tilde{\mathbf{v^2}} - \tilde{\mathbf{v}}^2 \right) \right| \left| \left(\frac{\tilde{\Delta}}{\Delta} \right)^2 |\tilde{\mathbf{S}}_h| \, \tilde{\mathbf{S}}_h - \widetilde{|\mathbf{S}_h| \, \mathbf{S}_h} \right|^{-1}, \quad l_z \propto l_h^{1/3}$$

diffusion coefficients for any scalar variable c (sensible heat or tracer)

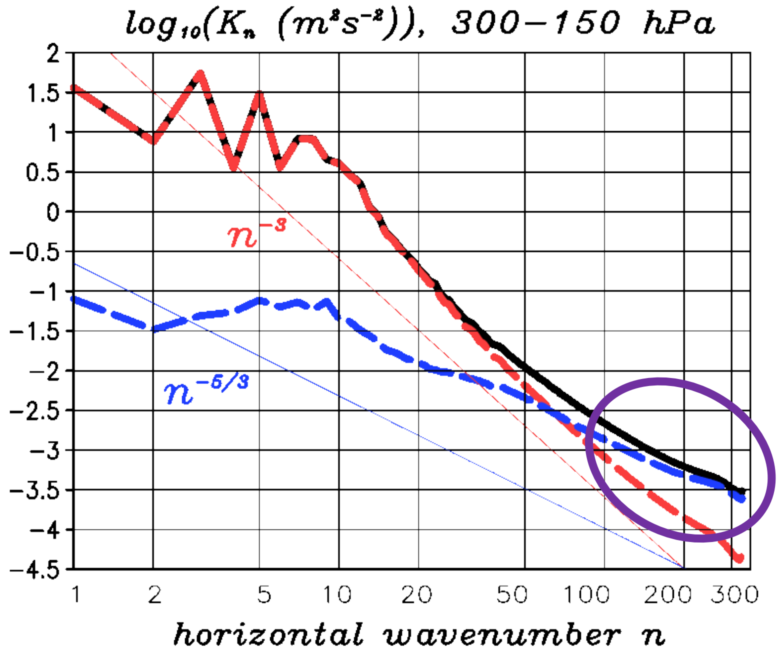
$$K_{c\,h} = l_{c\,h}^2 \, |\mathbf{S}_h| \gg K_{c\,z} = l_{c\,z}^2 \, |\partial_z \mathbf{v}|$$

$$l_{c\,h}^2 = \left| \widetilde{c\mathbf{v}} - \tilde{c}\tilde{\mathbf{v}} \right| \left| \left(\frac{\tilde{\Delta}}{\Delta} \right)^2 |\tilde{\mathbf{S}}_h| \, \widetilde{\nabla c} - \widetilde{|\mathbf{S}_h| \, \nabla c} \right|^{-1}, \quad l_{c\,z} \propto l_{c\,h}^{1/3}$$

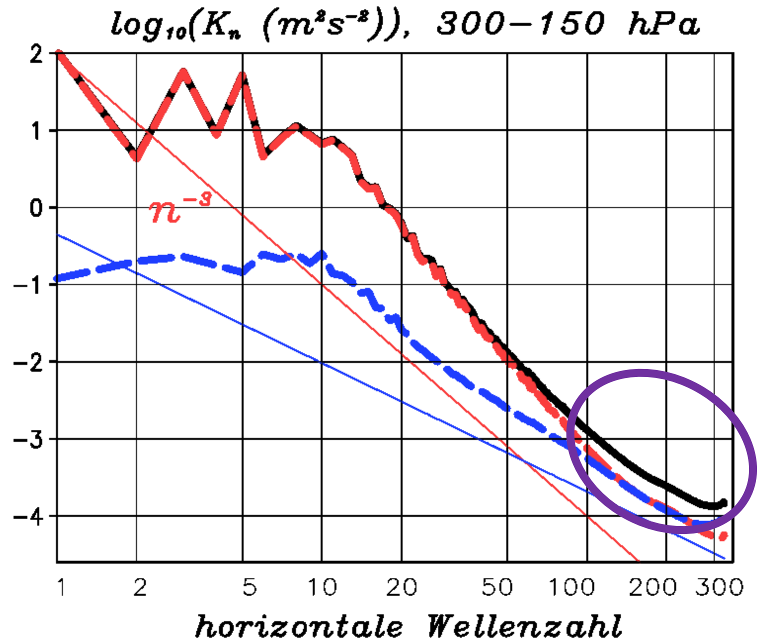
The new closure derives the horizontal and vertical mixing lengths for momentum and scalars from the constraint of **scale invariance** and the aspect ratio of **stratified turbulence.**
We solve the Germano identity (Germano et al., 1991, Phys. Fluids) in terms of the tensor norms such that the scheme fulfills the Second Law.

Application of the new Dynamic Smagorinsky Model (DSM)

CSM completed by hyperdiffusion (filter increases from 0 to1 between n=270 and n=330)



DSM (testfilter from n=270 to n=330)



Summary

The forced-dissipative nature of the general circulation of the atmosphere can be characterized by the Lorenz energy cycle. The dissipation of kinetic energy takes place in the boundary layer, as well as in the free atmosphere.

In the free atmosphere, the energy cycle includes a horizontal cascade of kinetic energy energy through the mesoscales down to the Ozmidov scale that is (likely) governed by the aspect ratio of stratified turbulence, $l_z \sim l_h^{1/3}$ (implying an equivalent horizontal cascade of available potential energy).

Any diffusion scheme in a high-resolution atmospheric circulation model should 1) fulfill the conservation laws, 2) be scale-invariant for momentum and scalars, and 3) invoke a physically based aspect ratio (e.g., stratified turbulence).

Such a closure can be formulated based on the Dynamical Smagorinsky Model and first test simulations are successful.