Prof. Dr. Valerii A. Galkin

Scientific Research Institute of System Development RAS, Moscow, Russia
(Head of Dept. for Numerical Modeling of Compex Systems)
and
Polytechnical Institute, Surgut State University, Surgut, Russia
(Director)

Generalized Solutions of the Smolukhovskii Semilinear System of Equations and Their Approximations

We consider the following infinite-dimensional system of first-order semilinear partial differential equations with respect to unknowns $u(x, t)$ (the Cauchy problem for the spatially inhomogeneous Smolukhovskii equation

$$\frac{\partial u_k(x,t)}{\partial t} + v_k \frac{\partial u_k(x,t)}{\partial x} = S_k(u(x,t)), \qquad (1)$$
$$x \in \mathbb{R}, \quad k \in \mathbb{N}, \quad t > 0,$$

with constant coefficients v_k, where the Smolukhovskii operator is

$$S_k(u(x,t)) = \frac{1}{2}\sum_{j=1}^{k-1} \Phi_{k-j,j} u_{k-j} u_j - u_k \sum_{j=1}^{\infty} \Phi_{k,j} u_j, \qquad (2)$$
$$k \in \mathbb{N}.$$

Equation (1) is supplemented with the initial data

$$u_k(x,0) = u_k^{(0)}(x) \geq 0, \quad k \in \mathbb{N}, \quad x \in \mathbb{R}. \qquad (3)$$

Such problems arise in modeling of coagulation-fragmentation processes.

It should be emphasized that the presence of infinitely many different values in the set of coefficients v_k may cause the emergence of non-differentiable singularities in the solution with respect to the space-time variables for arbitrarily smooth compactly supported initial data (2) (this does not occur in finite dimensional problems).

The proof of the existence of a functional non-negative solution of the Cauchy problem (1)–(3) is based on the weak compactness of the family of solutions of the finite-dimensional semilinear systems. The existence theorem for generalized solutions of the Cauchy problem (1)–(3) is proved on the basis of uniform bounds in $L_1 \cap L_\infty$ for the norms of u_i in combination with Tartar's method of compensated compactness.

We construct an approximate solution of the Cauchy problem (1), (3) by using a simulation method corresponding to the physics of the coagulation process described by the Smolukhovskii equations (1), (2).