

The Dirichlet-to-Neumann operator on $C^{1+\kappa}$ -domains

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Consider a second-order elliptic operator $\mathcal{A}: H_{\text{loc}}^1(\Omega) \rightarrow \mathcal{D}'(\Omega)$ in divergence form on an open set Ω in $\mathbf{R}^d$ with bounded real measurable principal coefficients and where all the lower-order coefficients are bounded measurable and complex valued. We merely assume that 0 is not in the spectrum of the corresponding operator A_D with Dirichlet boundary conditions. We show that for every $\varphi \in C(\partial\Omega)$ there are $\Phi \in C(\overline{\Omega})$ and a unique $\mathcal{A}$ -harmonic function u on Ω such that $\Phi|_{\partial\Omega} = \varphi$ and $\Phi - u \in H_0^1(\Omega)$. So u is a generalised solution of the Dirichlet problem and the definition is rather simple. We show that u is bounded on Ω and satisfies $\|u\|_{\infty} \leq c \|\varphi\|_{\infty}$ for a constant $c > 0$ independent of φ . Remarkable, one can choose $c = 1$ if the semigroup $(e^{-tA_D})_{t>0}$ is contractive on $L_{\infty}(\Omega)$. We also show that the map $\varphi \mapsto u$ has the same regular boundary points as the classical Perron solution with respect to the Laplacian.

This is joint work with Wolfgang Arendt and Manfred Sauter.