

A Brownian motion on the Wasserstein space

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The space of probability measures on a metric space carries a natural distance induced from the problem of optimal transportation, turning the space of measures into a formal infinite dimensional Riemannian manifold. It is tempting to ask whether some version of a Brownian motion on this manifold can be defined. We show that for the case of probability measures on the real line this is indeed possible. Depending on a temperature parameter we obtain a diffusion process on the space of mass distributions which for varying temperatures interpolates between the two extremes of deterministic heat and Brownian motion in single point respectively. Central to the construction is the introduction of a Gibbs measure on the spaces of probability distributions with the Boltzmann entropy as Hamiltonian.