

Shape optimization of light structures

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We prove rigorous results about the vanishing-mass limit of the classical problem to find a shape with minimal elastic compliance. Contrary to all previous results in the mathematical literature, which utilize a soft mass constraint by introducing a Lagrange multiplier, we here consider the hard mass constraint. Our results are the first to establish the convergence of approximately optimal shapes of (exact) size $\varepsilon \rightarrow 0$ to a limit generalized shape represented by a (possibly diffuse) probability measure. This limit generalized shape is a minimizer of the limit compliance, which involves a new integrand, namely the one conjectured by Bouchitté in 2001 and predicted heuristically before in works of Allaire & Kohn and Kohn & Strang from the 1980s and 1990s. This integrand gives the energy of the limit generalized shape understood as a fine oscillation of (optimal) lower-dimensional structures. Its appearance is surprising since the integrand in the original compliance is just a quadratic form and the non-convexity of the problem is not immediately obvious. In fact, it is the interaction of the mass constraint with the requirement of attaining the loading (in the form of a divergence-constraint) that gives rise to this new integrand. Our proofs rest on compensated compactness arguments applied to an explicit family of div-quasiconvex quadratic forms, computations involving the Hashin–Shtrikman bounds for the Kohn–Strang integrand, and the characterization of limit minimizers due to Bouchitté & Buttazzo.