Mathematical Analysis of a System for Biological Network Formation

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Motivated by recent physics papers describing rules for natural network formation, we study an elliptic-parabolic system of partial differential equations proposed by Hu and Cai. The model describes the pressure field thanks to Darcy's type equation and the dynamics of the conductance network under pressure force effects with a diffusion rate D representing randomness in the material structure. We prove the existence of global weak solutions and of local mild solutions and study their long term behavior. It turns out that, by energy dissipation, steady states play a central role to understand the pattern capacity of the system. We show that for a large diffusion coefficient D, the zero steady state is stable. Patterns occur for small values of D because the zero steady state is Turing unstable in this range; for $D = 0$ we can exhibit a large class of dynamically stable (in the linearized sense) steady states.