Stability results for sum of sets in $\mathbb{R}^n$

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Given a Borel A in $\mathbb{R}^n$ of positive measure, one can consider its semisum $S = (A + A)/2$. It is clear that S contains A, and it is not difficult to prove that S and A have the same measure if and only if A is equal to his convex hull minus a set of measure zero. We now wonder whether this statement is 'stable': if the measure of S is close to the one of A, is A close to his convex hull? More in general, one may consider the semisum of two different sets A and B, in which case our question corresponds to proving a stability result for the Brunn-Minkowski inequality. When $n = 1$, one can approximate a set with finite unions of intervals to translate the problem onto $\mathbb{Z}$, and in the discrete setting this question becomes a well studied problem in additive combinatorics, usually known as Freiman's Theorem. In this talk I'll review some results in the one-dimensional discrete setting, and discuss their extension to arbitrary dimension.