Asymptotic behaviour of a Neumann parabolic problem with hysteresis

Abstract

We consider the following evolution problem

$$\frac{\partial}{\partial t}(f(u) + w) - \Delta u \;=\; 0\,, \qquad \text{in } \Omega \times [0, \infty),$$

coupled with the homogeneous Neumann boundary condition

$$\frac{\partial u}{\partial n} \;=\; 0 \quad \text{on } \partial\Omega \times \,]0, \infty[\;,$$

where $\Omega \subset \mathbb{R}^d$, $d = 2$ or 3, with a Lipschitzian boundary, $f : \mathbb{R} \to \mathbb{R}$ is a given function, Δ is the Laplace operator, and $w(x, t)$ is the output of a scalar Preisach operator.

This equation represents a model of flow through a porous medium and it is motivated by problems in soil hydrology.

The main result achieved is the uniform asymptotic convergence of the process to an equilibrium, i.e. if u is the unique solution of the previous model problem, we have that there exists a constant $u_\infty \in \mathbb{R}$ such that

$$\lim_{t\to\infty} \sup_{x\in\Omega} |u(x, t) - u_\infty| \;=\; 0\,.$$

This is a joint work with Prof. Pavel Krejčí.