On off-diagonal behavior of the generalized Stokes operator

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Let $L = -\nabla \cdot \mu \nabla$ denote a second-order elliptic operator in divergence form and let $(\mathrm{e}^{-tL})_{t \geq 0}$ denote the corresponding strongly continuous heat semigroup on $\mathrm{L}^2(\mathbb{R}^d)$. If $E, F \subset \mathbb{R}^d$ denote measurable sets with $\operatorname{dist}(E, F) > 0$ and if $f \in \mathrm{L}^2(\mathbb{R}^d)$ is supported in E, then by the strong continuity of the semigroup, one finds that

$$\|\mathrm{e}^{-tL} f\|_{\mathrm{L}^2(F)} \to \|f\|_{\mathrm{L}^2(F)} = 0 \quad \text{as} \quad t \to 0.$$

An estimate that quantifies the convergence rate is often viewed as an off-diagonal estimate and it is well-known, that heat semigroups satisfy the following type of off-diagonal decay

$$\|\mathrm{e}^{-tL} f\|_{\mathrm{L}^2(F)} \lesssim \mathrm{e}^{-\frac{\mathrm{c}\operatorname{dist}(E,F)^2}{t}} \|f\|_{\mathrm{L}^2(E)}.$$

In this talk, we study off-diagonal behavior of the generalized Stokes semigroup $(\mathrm{e}^{-tA})_{t \geq 0}$ that is generated on $\mathrm{L}^2_\sigma(\mathbb{R}^d)$ by the generalized Stokes operator with bounded measurable coefficients μ, formally given by

$$Au := -\operatorname{div}(\mu \nabla u) + \nabla \phi, \quad \operatorname{div}(u) = 0 \quad \text{in } \mathbb{R}^d. \tag{1}$$

In contrast to the elliptic operator L, the operator A exhibits a non-local behavior due to the presence of the pressure function ϕ. This non-locality affects the non-local behavior of the generalized Stokes semigroup e^{-tA} and it is not clear how fast the support of a divergence free vector field f that is supported in a set E is smeared out. In this talk, first results in this direction are presented. We further discuss possible applications and future questions.