

GLOBAL EXISTENCE AND LARGE TIME BEHAVIOUR OF NONLINEAR
REACTION-DIFFUSION SYSTEMS

Bao Quoc Tang

KARL-FRANZENS-UNIVERSITÄT, GRAZ, AUSTRIA

In this talk, I will discuss global existence and large time behaviour of nonlinear reaction-diffusion systems for the form

$$\begin{cases} \partial_t u_i - \operatorname{div}(D_i(x, t)\nabla u_i) = f_i(u), & x \in \Omega, \\ \nabla u_i \cdot \nu = 0, & x \in \partial\Omega, \\ u_i(x, 0) = u_{i0}(x), & x \in \Omega \end{cases}$$

which dissipate mass and/or entropy, i.e.

$$\sum_{i=1}^m f_i(u) \leq 0 \quad \text{and/or} \quad \sum_{i=1}^m f_i(u) \log u_i \leq 0.$$

Such systems typically arise from chemical reactions or population dynamics, and due to the potentially high order of nonlinearities, the global existence of solutions is highly non-trivial. In fact, it is shown that, even with the dissipation of total mass, any super quadratic growth rate of nonlinearities is able to exhibit blow-up in finite time. In the first part of the talk, we show for the case $D_i(x, t) \equiv d_i$ and the nonlinearities are at most quadratic, the global existence and boundedness of solutions in all dimensions. This is done by exploiting Hölder continuity of some parabolic equations of non-divergence form with rough coefficients. When the diffusion coefficients $D_i(x, t)$ might be discontinuous, an L^p -energy type method is introduced to obtain global bounded solutions, under some intermediate sum conditions. The second part of the talk deals with convergence towards equilibrium for systems arising from (weakly) reversible chemical networks. The main tool is the entropy method, which provides somewhat explicit rates and is sufficiently robust to be applicable to renormalised solutions.

Keywords: Reaction-diffusion systems; Mass/entropy dissipation; Global existence; Convergence to equilibrium; Entropy methods.