

A variational approach to data-driven problems in fluid mechanics

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In this talk, we discuss a data-driven approach to viscous fluid mechanics. Typically, in order to describe the behaviour of fluids, two different kinds of modelling assumptions are used. On the one hand, there are first principles like the balance of forces or the incompressibility condition. On the other hand there are material specific constitutive laws that describe the relation between the strain and the viscous stress of the fluid. Combining both, one obtains the partial differential equations of fluid mechanics like the Stokes or Navier-Stokes equations. The constitutive laws are obtained by fitting a law from a certain class (for example linear, power law, etc.) to experimental data. This leads to modelling errors.

Instead of using a constitutive relation, we introduce a data-driven formulation that has previously been examined in the context of solid mechanics and directly draws on material data. This leads to a variational solution concept, that incorporates differential constraints coming from first principles and produces fields that are optimal in terms of closeness to the data. In order to derive this formulation we recast the differential constraints of fluid mechanics in the language of constant-rank differential operators. We will see that the data-driven solutions are consistent with PDE solutions if the data are given by a constitutive law. Furthermore we show a Γ -convergence result for the functionals arising in the data-driven fluid mechanical problem which implies that the method is well-adapted to the convergence of experimental data through increasing experimental accuracy.

This is based on joint work with Christina Lienstromberg (Stuttgart) and Stefan Schiffer (Leipzig).