Dynamic behavior of growth processes: Phase separation, self-similarity, and oscillations

André Schlichting (WWU Münster)

The talk reviews some growth processes describing the evolution of clusters consisting of atomic parts called monomers. The growth and shrinkage can only occur by adding and removing single monomers, specified through a rate kernel depending solely on the involved clusters' size.

First, I discuss the exchange-driven growth model, obtained as the mean-field limit of stochastic particle systems (zero-range process). Under a detailed balance condition on the kernel, the model's longtime behavior can be described entirely. Here, the total mass density, determined by the initial data, acts as an order parameter in which the system shows a phase separation.

Next, we consider the model for a family of product kernels, which do not satisfy a detailed balance condition. After a suitable rescaling to self-similar variables, the equation becomes a discrete Laplace with a power-law as diffusion coefficient, which in particular degenerates at the origin and grows at infinity. We will see that the solution converges to a stretched exponential self-similar profile.

Lastly, we consider the now-classic Becker-Döring system to which an injection of monomers and a depletion of large clusters is added. These equations have been extensively used to model chemical-physical systems, especially bubbleator dynamics. The model approximates a transport equation with a conservation law entering the boundary condition by formal asymptotics. For the limit model, a Hopf bifurcation is shown, indicating temporal oscillations in the model.

Joint works with Constantin Eichenberg, Barbara Niethammer, Robert Pego, and Juan Velazquez.