On the De Giorgi-Nash-Moser regularity theory for kinetic hypoelliptic operators

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We present new results that extend the De Giorgi-Nash-Moser theory to a class of hypoelliptic equations naturally arising in kinetic theory. These operators are mathematically characterized by two parts: a diffusion part governed by the (fractional) Laplace operator in some set of variables and a first order operator that contains the directions of missing ellipticity. A key ingredient to prove our results is a Poincaré inequality, which we derive from the construction of suitable trajectories. The trajectories we rely on are quite flexible and allow us to consider equations with an arbitrary number of Hörmander commutators and whose diffusive part is either local (second-order) or nonlocal (fractional order). We later combine the Poincaré inequality with a $L^2 - L^\infty$ estimate, a Log-transformation and a classical covering argument (Ink-Spots Theorem) to deduce Harnack inequalities and Hölder regularity along the line of De Giorgi method.

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