

A mediocre two-component variant of the famous result on equilibration in scalar Fokker–Planck equations

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One of the most striking by-now-classical results on Wasserstein gradient flows is the one about exponential equilibration in nonlinear Fokker–Planck equations: if the nonlinearity satisfies McCann’s condition, and the potential is λ -uniformly convex, then any solution tends to the steady state at exponential rate λ . The proof a la Felix Otto is a remarkable application of uniform displacement convexity. Despite lots of efforts and some progress (particularly by people at WIAS), no result of comparable generality and simplicity has ever been derived for a system of two such Fokker–Planck equations that are coupled by means of cross diffusion.

In this talk, we indicate the main difficulty, which is the total break-down of the displacement convexity as soon as coupling is introduced, no matter how tame. Our main result is that for sufficiently weak coupling, equilibration still happens exponentially fast, with a rate λ , reduced by the coupling strength. The proof is based on the λ -uniform displacement convexity of the decoupled system, and treats the non-convexity as deformation, using non-standard nonlinear functional inequalities. A central challenge is that the steady state is non-explicit and compactly supported.

Some adaptations of the idea are discussed as well: to systems of non-local aggregation equations, to a Keller–Segel system from chemotaxis, and to the second-order destabilized fourth order thin film equation.

This is joint work with Lisa Beck (Augsburg), Christian Parsch (TUM), and Martina Zizza (SISSA).