

DYNAMICS OF A 3D RIMMING FLOW

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ABSTRACT

We study the dynamic behaviour of a thin viscous fluid film coating the inner wall of a rotating cylinder – a so-called *rimming flow*. The resulting PDE for the height $h > 0$ of the fluid film is quasilinear, degenerate parabolic and of fourth order.

Various competing effects drive the dynamics of the interface – viscosity, surface tension and gravity. For a positive surface tension parameter $\gamma > 0$ of order one we investigate existence and stability of positive steady states, depending on the influence of gravity ($\delta \geq 0$).

The talk is based on joint works with Juri Jousen (Stuttgart), Janne Laudien (Stuttgart) and Juan Velázquez (Bonn).

DYNAMICS OF A 3D RIMMING-FLOW EQUATION

JURI JOUSSEN FOR SIAM NWCS26

ABSTRACT

A rimming flow is the flow of a fluid partially filling a rotating cylinder. We assume that the fluid forms a capillary-driven thin film coating the inner wall. We formally derive a closed fourth-order degenerate-parabolic evolution equation for the height of the liquid film h :

$$h_t + h_\theta + \gamma \operatorname{div} (h^3 \nabla (\Delta h + h)) = \delta (h^3 \cos(\theta))_\theta$$

with time $t > 0$, cylindrical coordinates $(\theta, z) \in \mathbb{T} \times (0, L)$ and Neumann-type boundary conditions on the cylinder covers. Competing effects relevant to this equation are the cylinder rotation, surface tension $\gamma > 0$, viscosity and gravity $\delta \geq 0$. We investigate the stability of positive steady states. For small gravity $0 < \delta \ll 1$ and short cylinders ($L < \pi$) we show that certain steady states are stable and while they are unstable for long cylinders ($L > \pi$). For cylinders of critical length $L = \pi$ we investigate solutions of the form

$$h(t, \theta, z) = m + a_1 (\delta^2 t) e^{i(\theta-t)} + a_{-1} (\delta^2 t) e^{-i(\theta-t)} + b(\delta^2 t) \cos(z)$$

and formally derive an ODE for the (slow) coefficients $a_1, a_{-1} \in \mathbb{C}, b \in \mathbb{R}$, thus describing the dynamics of such solutions on the slow time scale $\tau = \delta^2 t$.

This talk is based on joint work with Janne Laudien (Stuttgart), Christina Lienstromberg (Stuttgart) and Juan J. L. Velázquez (Bonn).

TWO-TIME SCALE DYNAMICS OF A 2D RIMMING-FLOW EQUATION

JURI JOUSSEN FOR GAMM

ABSTRACT

A rimming flow is the flow of a fluid partially filling a rotating cylinder. We assume that the fluid forms a capillary-driven thin film coating the inner wall. Considering only a cross-section of the cylinder, we formally derive a closed fourth-order degenerate-parabolic evolution equation for the height of the liquid film from the Navier–Stokes equations by means of a lubrication approximation. Competing effects relevant to this equation are the cylinder rotation, surface tension, viscosity and gravity. We investigate the stability of positive steady states. We prove an approximation result showing that the equation exhibits dynamics on two distinct time scales. In the fast time scale, the fluid is transported by the cylinder rotation. For the slow time scale we derive a governing system of ODEs.

This talk is based on joint work with Christina Lienstromberg (Stuttgart) and Juan J. L. Velázquez (Bonn).