ASYMPTOTICS FOR SOME VIBRO-IMPACT PROBLEMS WITH A LINEAR DISSIPATION TERM

Abstract. We consider the following measure differential inclusion

$$(S) \qquad \ddot{x}(t) + \gamma\,\dot{x}(t) + \partial\Phi(x(t)) \ni 0, \qquad t \in \mathbb{R}_+$$

where $\gamma \geq 0$, $\Phi : \mathbb{R}^d \to \mathbb{R} \cup \{+\infty\}$ is a lower semicontinuous convex function and $\partial\Phi$ is the subdifferential of Φ. When $\Phi = \psi_K + f$ with $f : \mathbb{R}^d \to \mathbb{R}$ a smooth convex function and ψ_K the indicator function of a closed convex set $K \subset \mathbb{R}^d$, the inclusion (S) describes the motion of a discrete mechanical system with d degrees of freedom, submitted to the frictionless unilateral constraint $x \in K$ and moving under the action of the conservative force $-\nabla f(x)$ and the viscous friction $-\gamma\,\dot{x}$. The mechanical consistency of the model leads to the notion of *dissipative solutions* for which the kinetic energy does not increase when the constraint is active.

If $\mathrm{int}\,(\mathrm{dom}\,\Phi) \neq \emptyset$, existence of such solutions with conservation (resp. loss) of energy at impacts can be established. If moreover $\gamma > 0$ and $\Phi_{|\mathrm{dom}\Phi}$ is locally Lipschitz continuous, any dissipative solution to (S) converges, as $t \to +\infty$, to a minimum point of Φ. When Φ is strongly convex, the speed of convergence is exponential.

Assuming as above that $\Phi = \psi_K + f$, suppose that the boundary of K is smooth enough and that the normal component of the velocity is reversed and multiplied by a restitution coefficient $e \in [0, 1]$ while the tangential component is conserved whenever $x(t) \in \partial K$. We prove that any dissipative solution to (S) satisfying the previous impact law with $e < 1$ is contained in the boundary of K after a finite time. The case $e = 1$ is also addressed and leads to a qualitatively different behaviour.